

# Passive Imaging and monitoring: disorder and correlation.

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Whisper

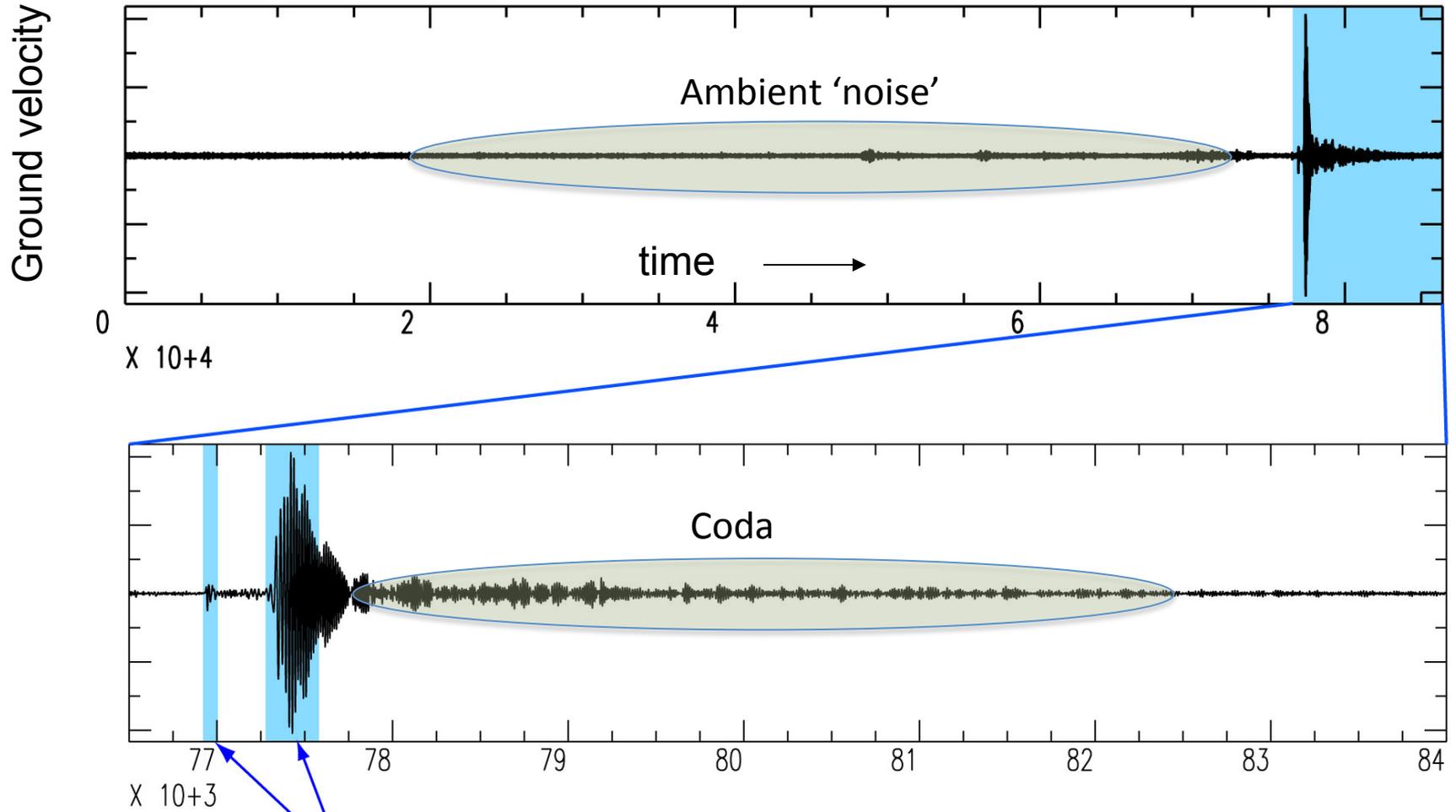
*Towards continuous monitoring of the  
continuously changing Earth*



Institut des Sciences de la Terre

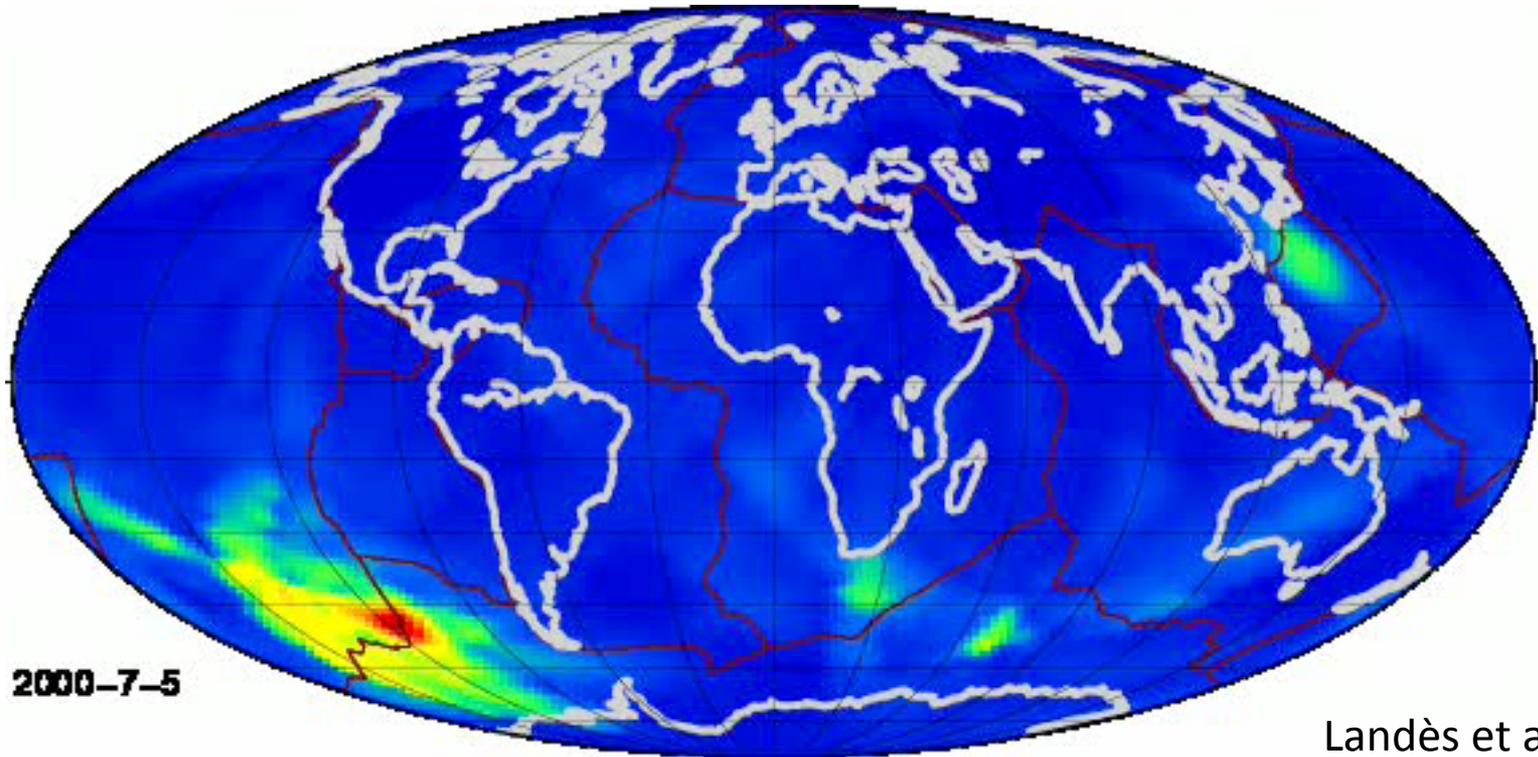


# one day of seismic record



ballistic waves used in traditional tomography

The origin of the noise in the period band 5-10s as seen by seismic arrays



VARIABLE SOURCE LOCATIONS

At higher frequencies: human activity, wind,....

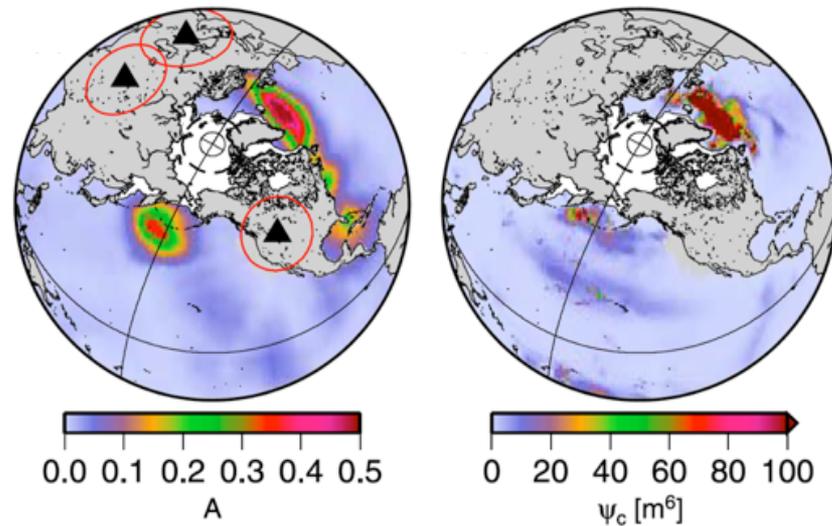
# Global 'noise' sources in the microseism band (extended $\approx 2-50s$ )

Strong contribution from oceanic waves

Example of a global comparison  
(secondary microseism-  
Miche/Longuet-Higgins mechanism)

seismological observations

oceanographic modeling

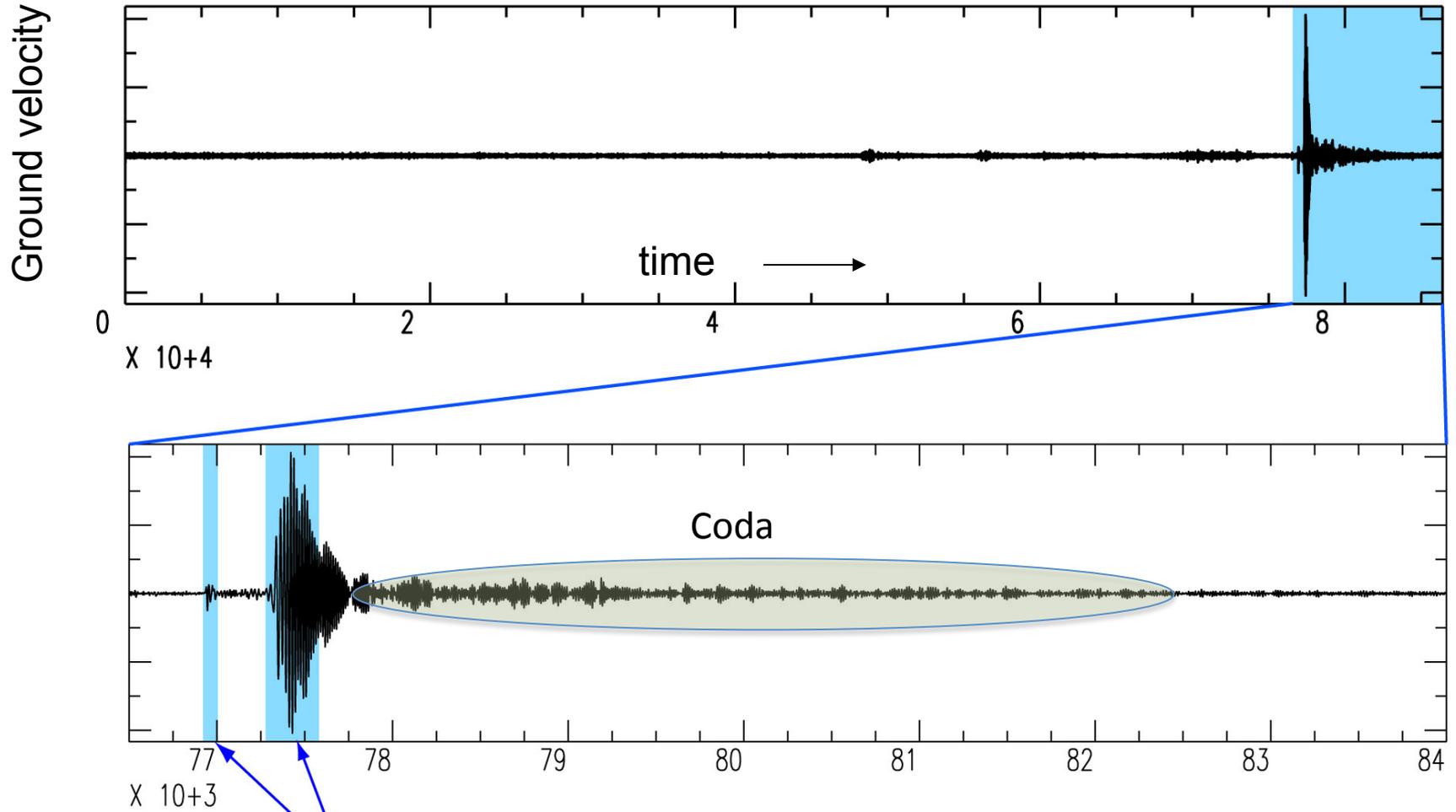


Hillers et al., 2012

Longer periods: infragravity waves, e.g Fukao et al. 2010

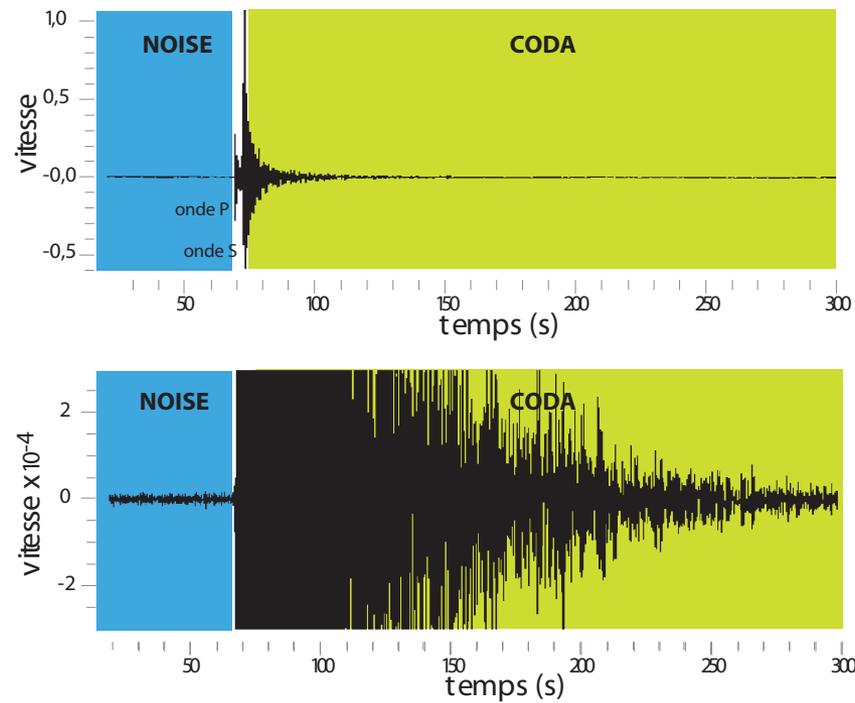
+EARTHQUAKES

# one day of seismic record



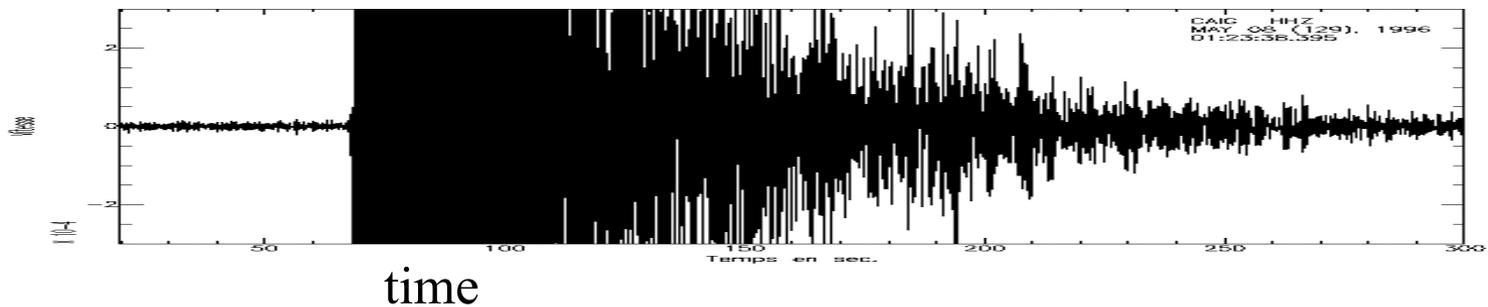
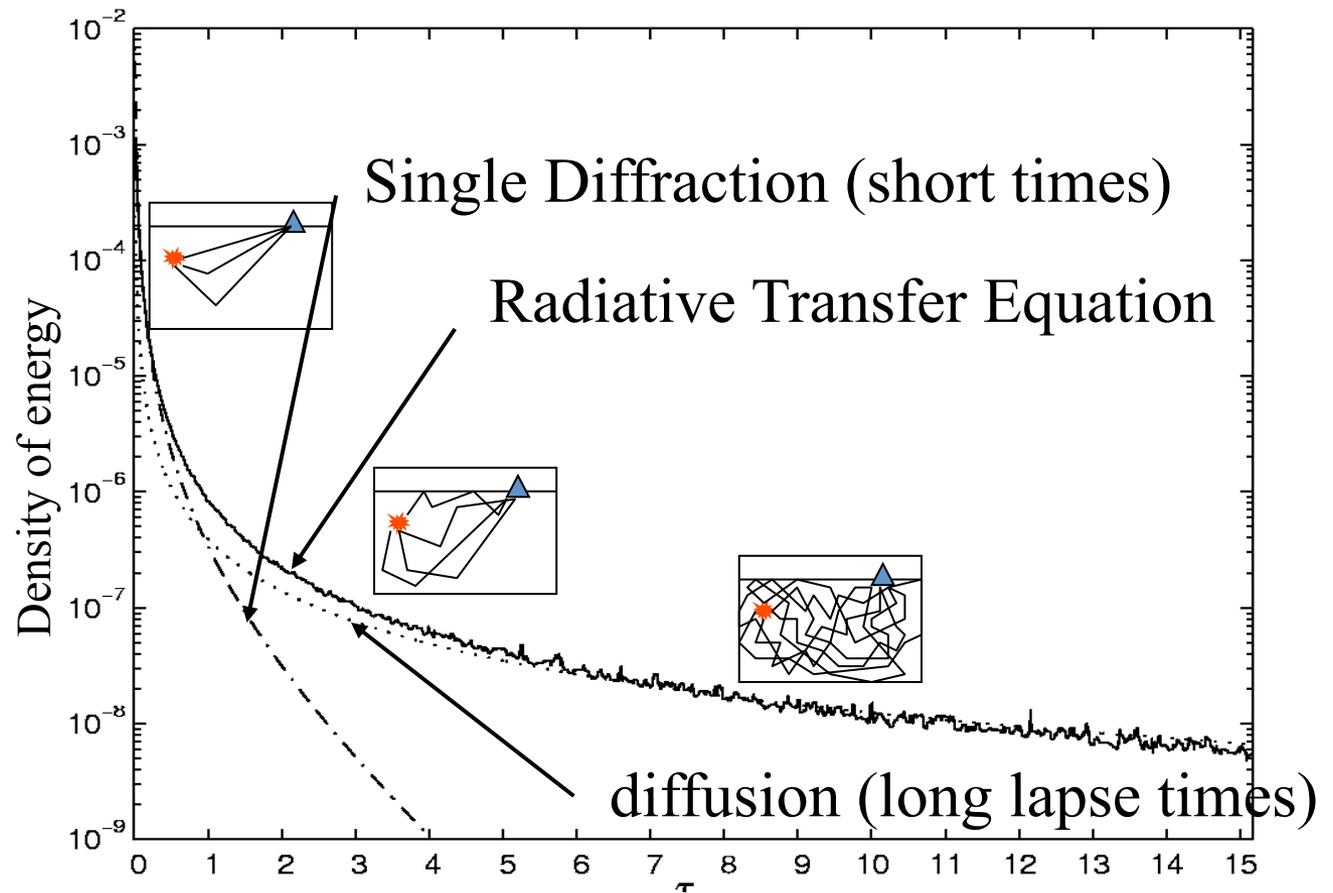
ballistic waves used in traditional tomography

A typical record of a local earthquake (0.1-10Hz)



Coda waves = multiply scattered waves (deterministic speckle)

# Propagation regimes and description of **energy**



Long continuous records, with components belonging to various propagation regimes (ballistic to diffuse)

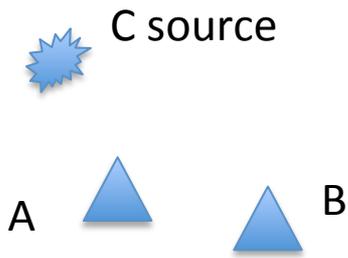
Measurements of the wavefield in space and time

→ Long range field correlations:

**Two point time correlation:**

$$Corr_{SR}(\tau) = S(t) \otimes R(t) = \int_{-\infty}^{+\infty} S(t)R(t - \tau)dt$$

*In the Fourier domain :  $C_{SR}(\omega) = S(\omega).R^*(\omega)$*

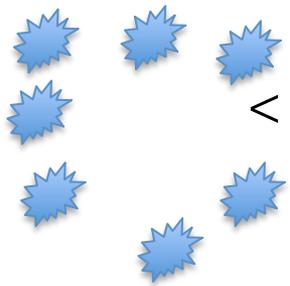


Focusing/virtual source in A

- C source (uncertainties about physics, location..)
- A et B receivers
- Correlation :

$$Corr_{AB}(\tau) = S_{AC}(t) \otimes S_{BC}(t)$$

$$\langle S_{AC}(t) \otimes S_{BC}(t) \rangle_{SOURCES}$$



## Time reversal, correlation and convolution

### Convolution

$$\text{Conv}_{SR}(t) = S(t) * R(t) = \int_{-\infty}^{+\infty} S(\tau)R(t - \tau)d\tau$$

*In the Fourier domain :  $\text{Conv}_{SR}(\omega) = S(\omega).R(\omega)$*

### Time reversal (symmetry of the wave equation)

$$\widehat{R}(t) = R(-t)$$

$$\widehat{R}(\omega) = FT[R(-t)] = R^*(\omega)$$

### Correlation and convolution

$$\text{Corr}_{SR}(\omega) = S(\omega).R^*(\omega) = S(\omega).\widehat{R}(\omega) = \text{Conv}_{S\widehat{R}}(\omega)$$

The correlation between two signals is **equal** to the convolution of one signal with the time reversed of the other.

# Correlation and Time reversal Focusing/virtual source in A

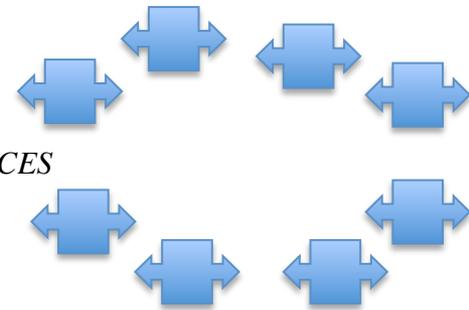
C TR device



- A source
- C receiver ( $S_{AC} = S_{CA}$ )
- C emits the time reversed signal
- B receiver
- Convolution :

$$S_{CA}(t) * S_{BC}(-t)$$

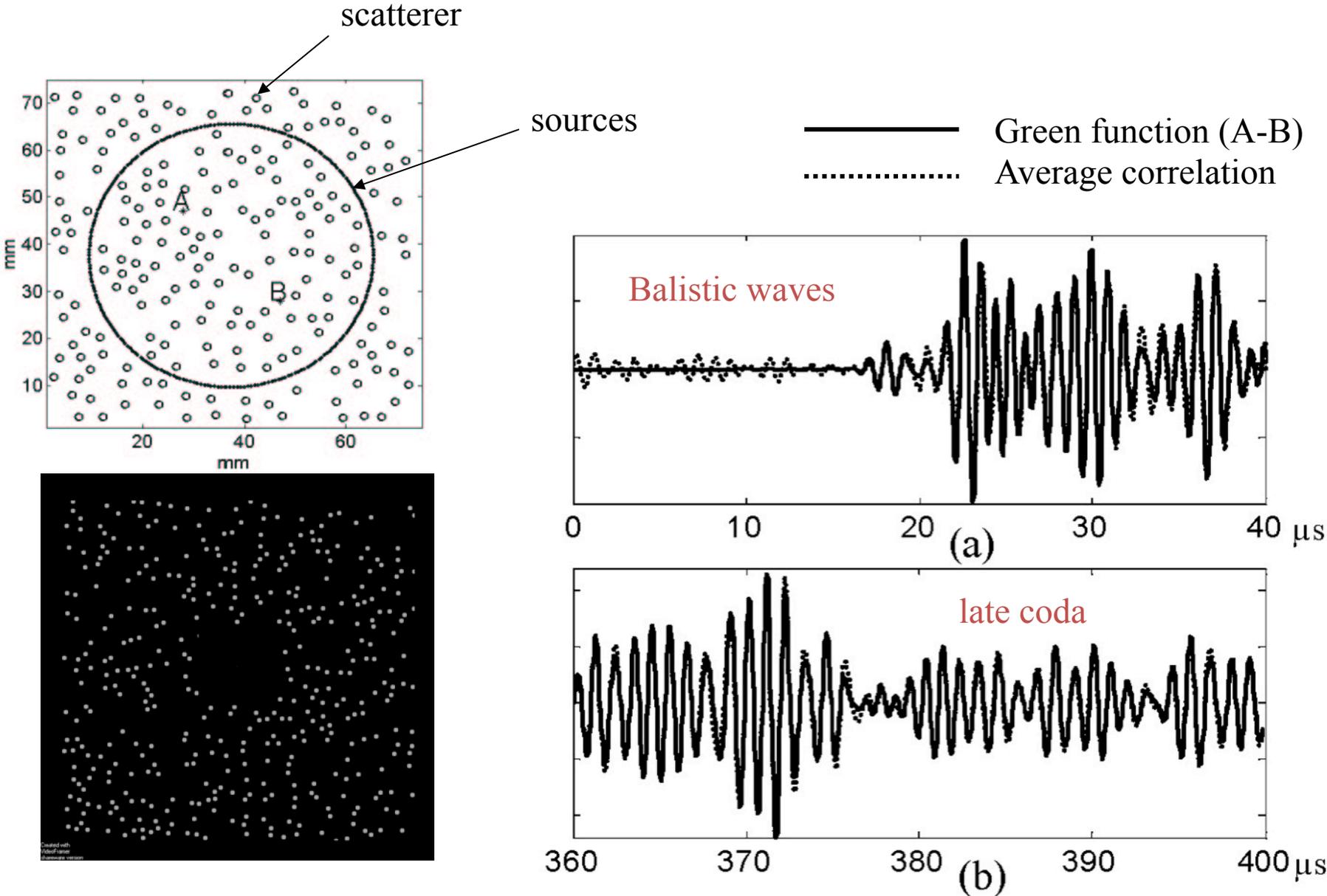
$$\langle S_{CA}(t) * S_{BC}(-t) \rangle_{\text{TIME REVERSAL DEVICES}}$$



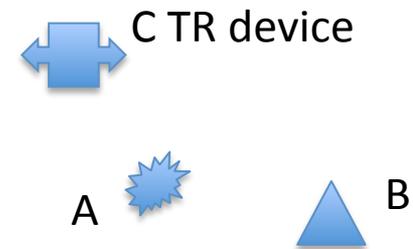
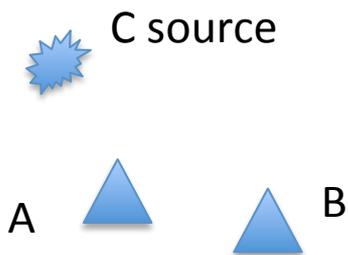
# La piscine à Retournement Temporel

From Nicolas Perrez, Petrobras, USPI.

# A numerical experiment with an open medium (absorbing boundaries):



# Correlation and Time reversal Focusing/virtual source in A

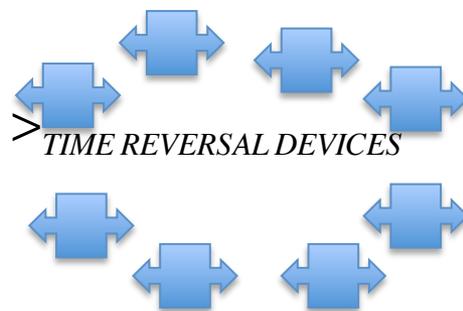
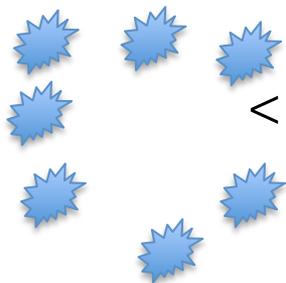


- C source
- A et B receivers
- Correlation

- A source
- C receiver ( $S_{AC} = S_{CA}$ )
- C emits the time reversed signal
- B receiver
- Convolution :

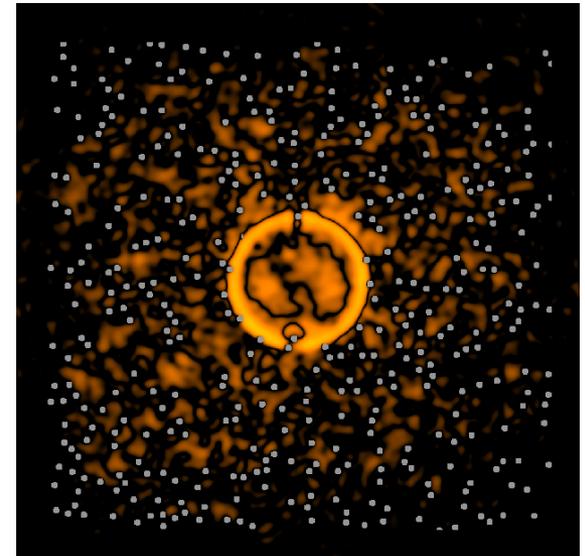
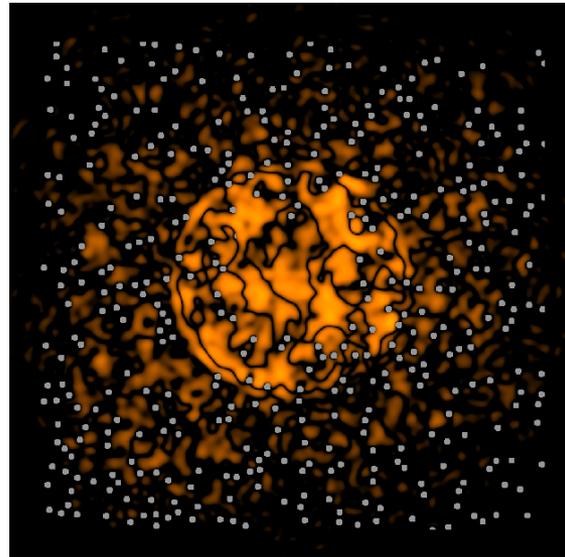
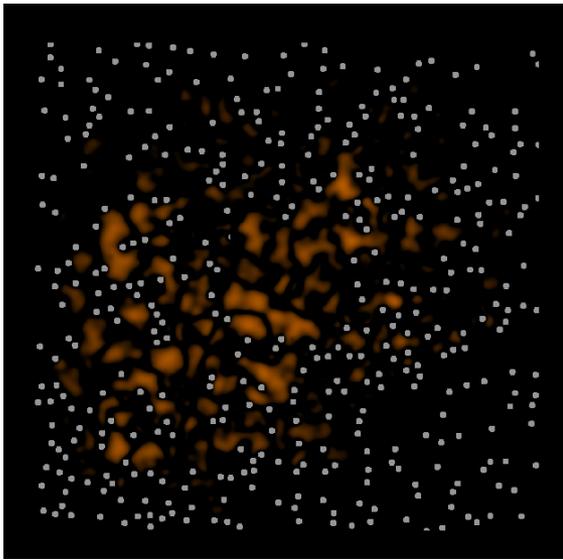
$$Corr_{AB}(\tau) = S_{AC}(t) \otimes S_{BC}(t) = S_{CA}(t) * S_{BC}(-t)$$

$$\langle S_{AC}(t) \otimes S_{BC}(t) \rangle_{SOURCES} = \langle S_{CA}(t) * S_{BC}(-t) \rangle_{TIME REVERSAL DEVICES}$$



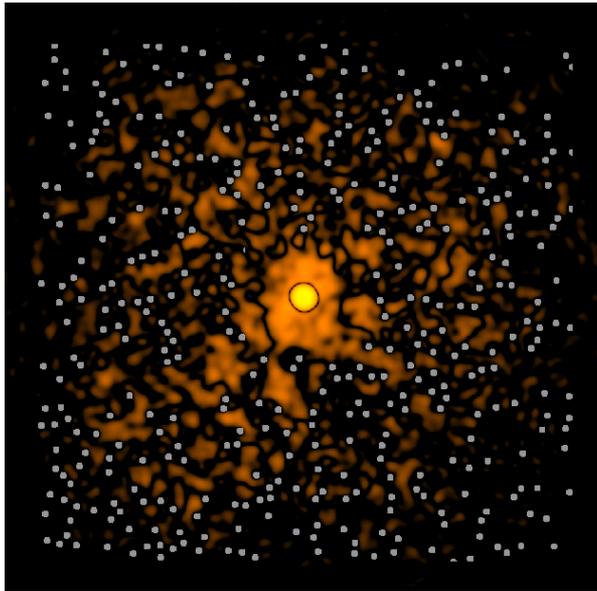
Ambient noise based method is equivalent to using a (broken) time reversal mirror defined by the sources of ambient noise and the scatterers.

Re-emission from the points 'C'  
of the time-reversed signals  
(map of cross-correlations)



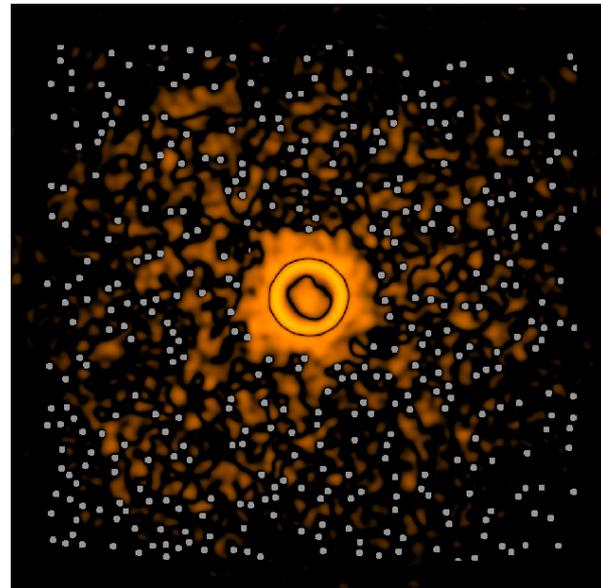
Constructive  
interferences of time-  
reversed field

Converging field  
:  $G(-t)$

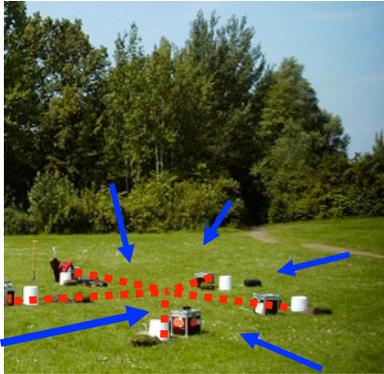


Nearly perfect refocalisation

Re-emission from A :  
 $G(t)$



# Perspective with a previous approach. Spatial autocorrelation coefficient (SPAC): Average spatial coefficient (2D circular array)

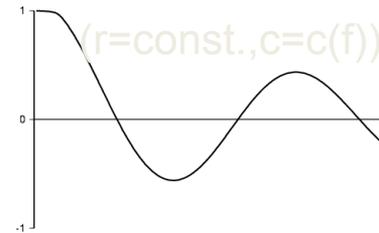


Aki (1957): azimuthal averaged spatial autocorrelation coefficients.  
Single mode, plane waves

$$\bar{\rho}(\mathbf{r}, \omega_0) = \frac{1}{\pi} \int_0^{\pi} \rho(\mathbf{r}, \varphi, \omega_0) d(\theta - \varphi)$$

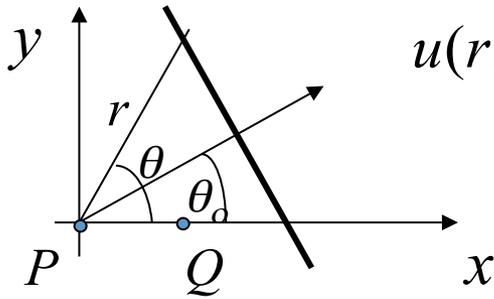
$$\bar{\rho}(r, \omega_0) = J_0\left(\frac{\omega_0 r}{c(\omega_0)}\right)$$

Azimuthal average could be considered with positions of sensors (the original technique) or under the hypothesis that waves are incident from all directions.



A plane wave in 2D in spectral domain:

$$u(r, \theta, \omega) = F(\omega) \exp(-ikr \cos(\theta - \theta_0))$$



Two point correlation:

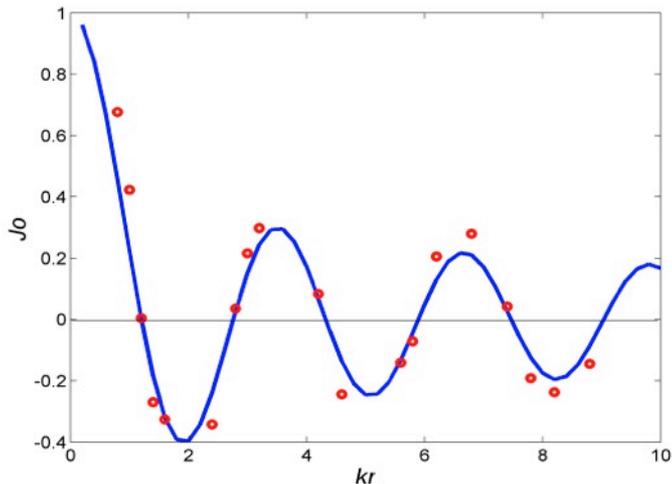
$$\frac{u^P u^{Q*}}{|u^P| |u^Q|} = e^{+ikr \cos \theta_0}$$

$$\langle \rho(r, \omega) \rangle = \left\langle \frac{u^P u^{Q*}}{|u^P| |u^Q|} \right\rangle = \langle e^{ikr \cos \theta_0} \rangle = \frac{1}{2\pi} \int_0^{2\pi} e^{ikr \cos \theta_0} d\theta_0 = J_0(kr)$$

azimuthal average of the cross-correlation

To be compared with the form of the 2D scalar Green function:

$$G = \frac{1}{4i\mu} H_0^{(2)}\left(\frac{\omega r}{\beta}\right) = \frac{1}{4\mu} \left\{ -Y_0\left(\frac{\omega r}{\beta}\right) - iJ_0\left(\frac{\omega r}{\beta}\right) \right\}$$

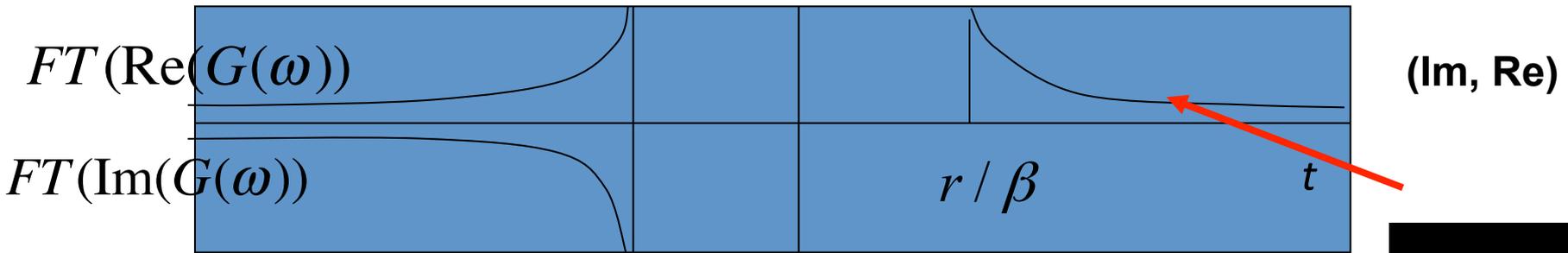
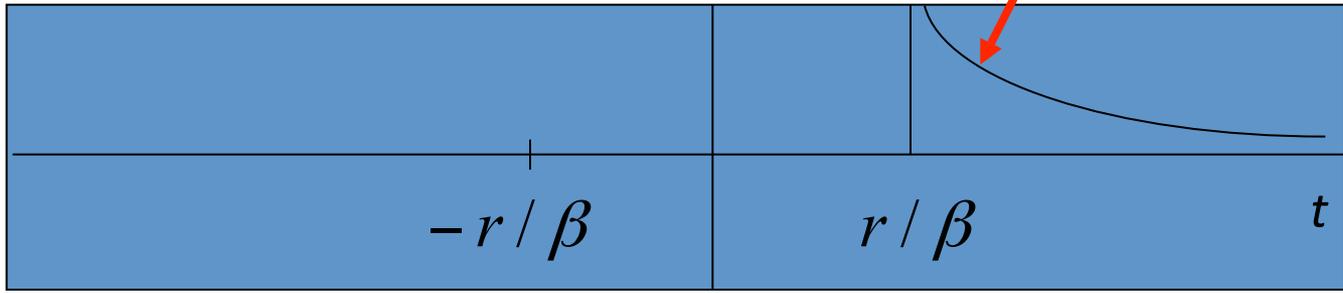


$$\text{Im}(G_{PQ}) = \frac{-1}{4\mu} \left\langle \frac{u_2(P) u_2^*(Q)}{|u_2(P)| |u_2(Q)|} \right\rangle$$

# Causality

$$G = \frac{1}{4i\mu} H_0^{(2)}\left(\frac{\omega r}{\beta}\right)$$

$$G = \frac{1}{2\pi\mu} \frac{H\left(t - \frac{r}{\beta}\right)}{\sqrt{t^2 - \frac{r^2}{\beta^2}}}$$

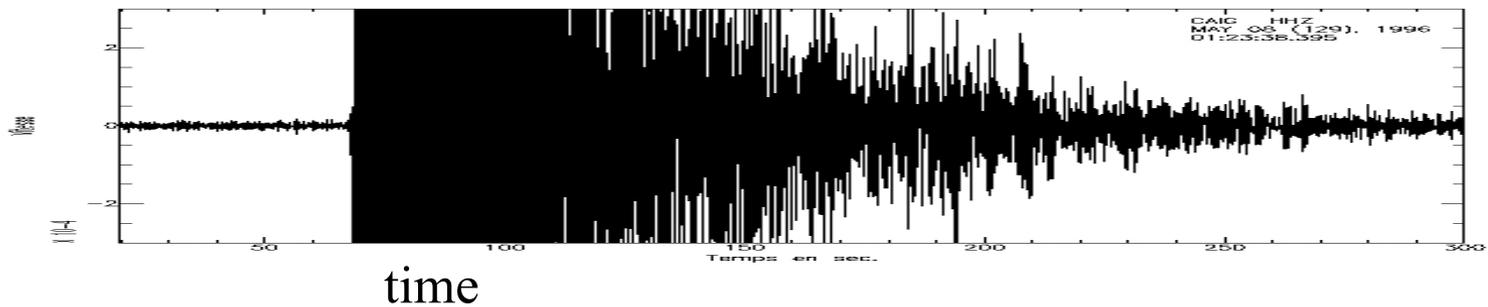
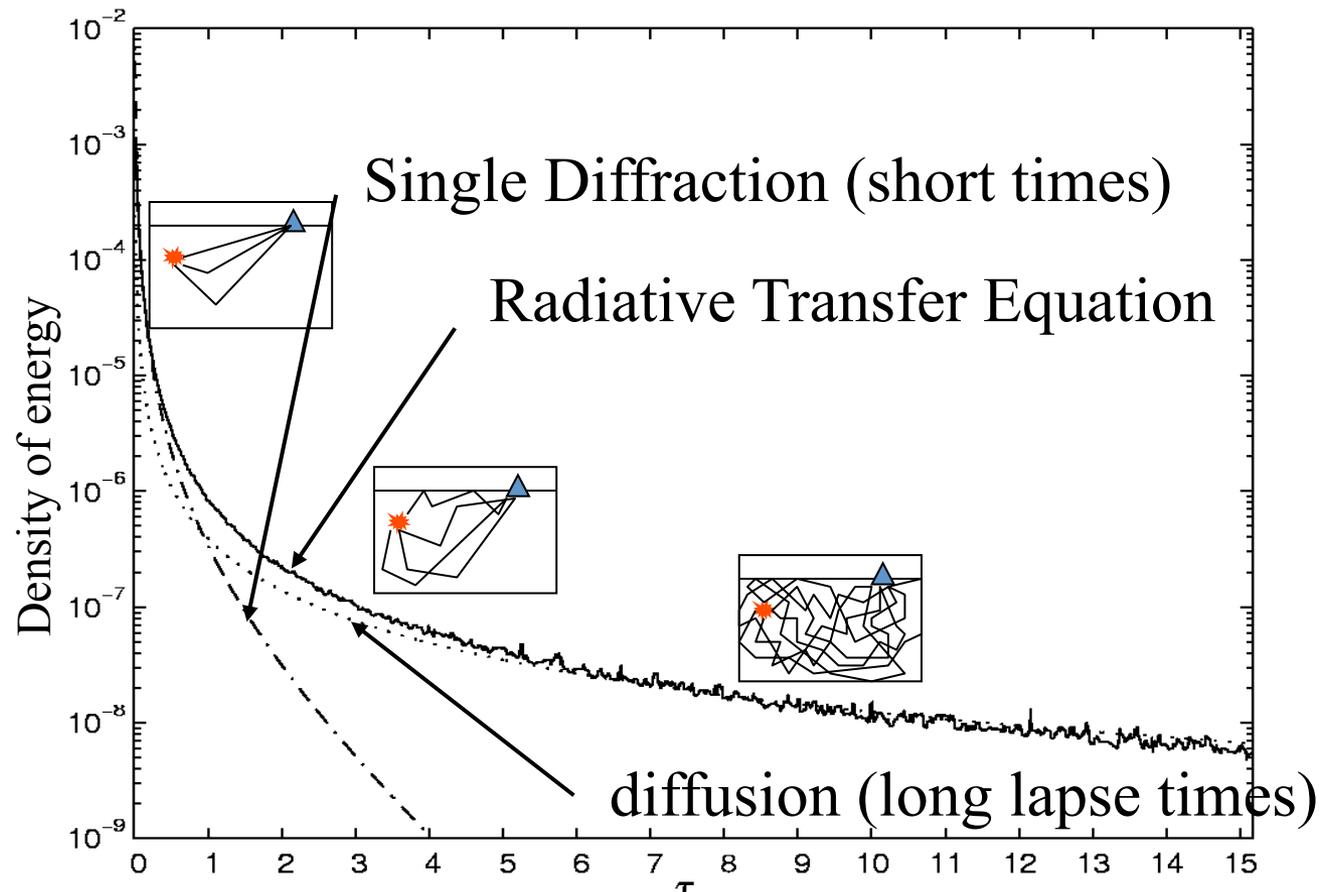


**$G/2$**

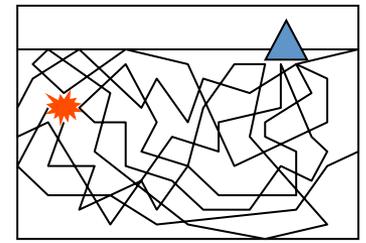
2D single mode (SH,..):

Correlation with isotropic illumination = Green function (+t/-t)

# Propagation regimes and description of **energy**

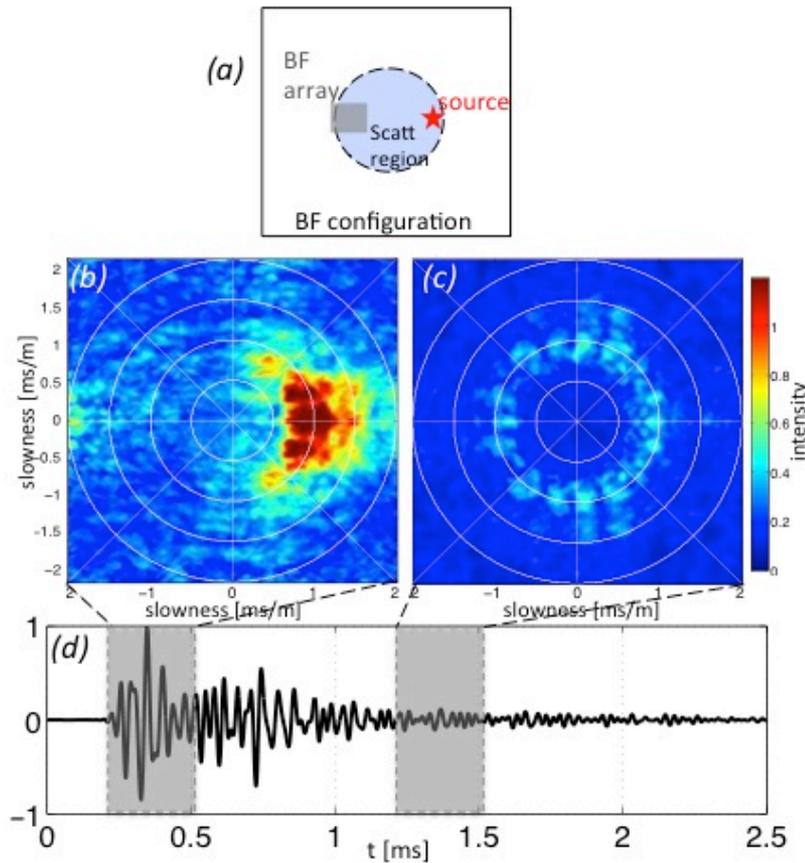


For asymptotically long lapse time (diffusion), the disorder produces a completely randomized wave-field. such that , all the modes of propagation are excited in average to equal energy (the equipartition principle).



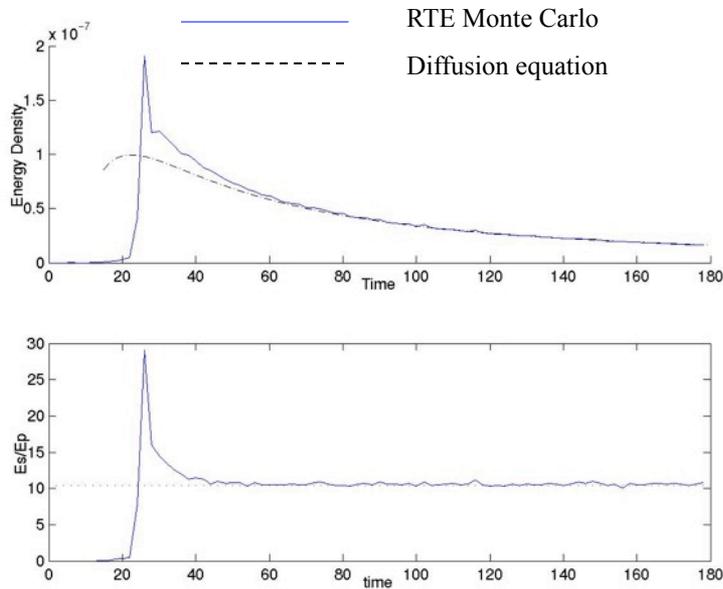
Numerical example: 2D scalar waves

→ intensity isotropy

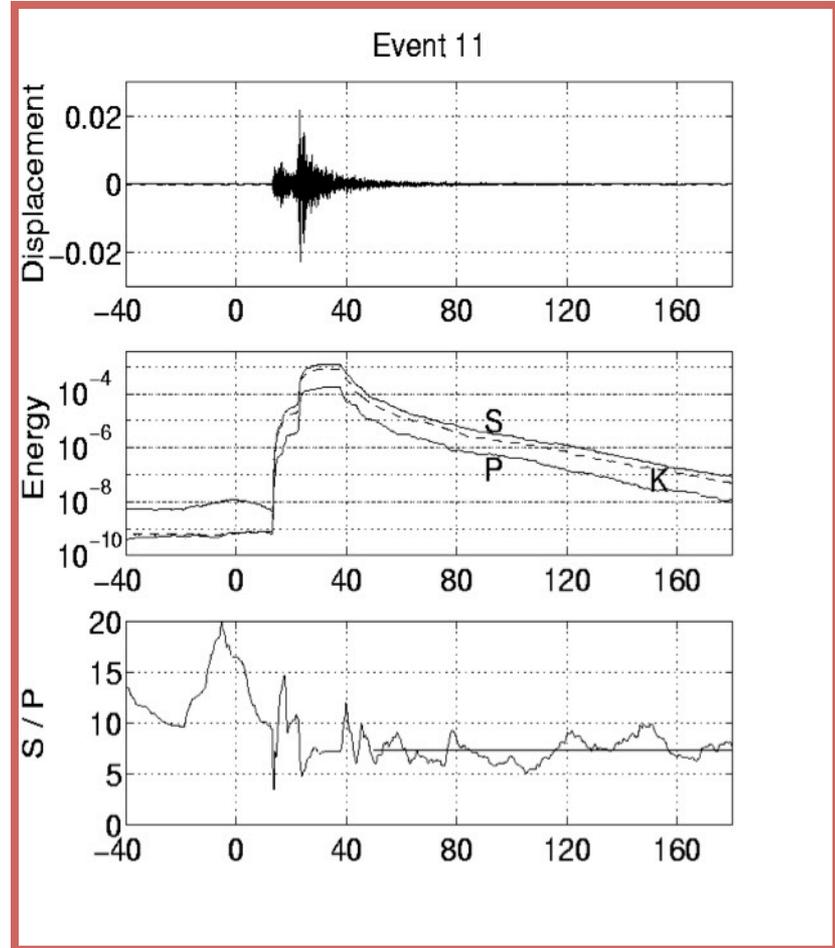


Implication for elastic waves (Weaver, 1982, Ryzhik et al., 1996): P to S energy ratio stabilizes at a value independent of the details of source and scattering!

### Numerical simulation



### Observations



Infinite space  
Energy ratio

$$2D: E_S / E_P = \left( \frac{\alpha}{\beta} \right)^2$$

$$3D: E_S / E_P = 2 \cdot \left( \frac{\alpha}{\beta} \right)^3$$

## P-SV case

## Green function in 2D

$$G_{ij} = \frac{i}{4\rho\omega^2} \left\{ -\delta_{ij} k^2 H_0^{(2)}(kr) + \frac{\partial^2}{\partial x_i \partial x_l} \left[ H_0^{(2)}(qr) - H_0^{(2)}(kr) \right] \delta_{lj} \right\}$$

$$G_{ij}(P, Q) = \frac{-i}{8\rho} \left\{ A \delta_{ij} - B (2\gamma_i \gamma_j - \delta_{ij}) \right\} \quad \gamma_j = \frac{x_j - \xi_j}{r}$$

$$A = \frac{H_0^{(2)}(qr)}{\alpha^2} + \frac{H_0^{(2)}(kr)}{\beta^2} \quad B = \frac{H_2^{(2)}(qr)}{\alpha^2} - \frac{H_2^{(2)}(kr)}{\beta^2}$$

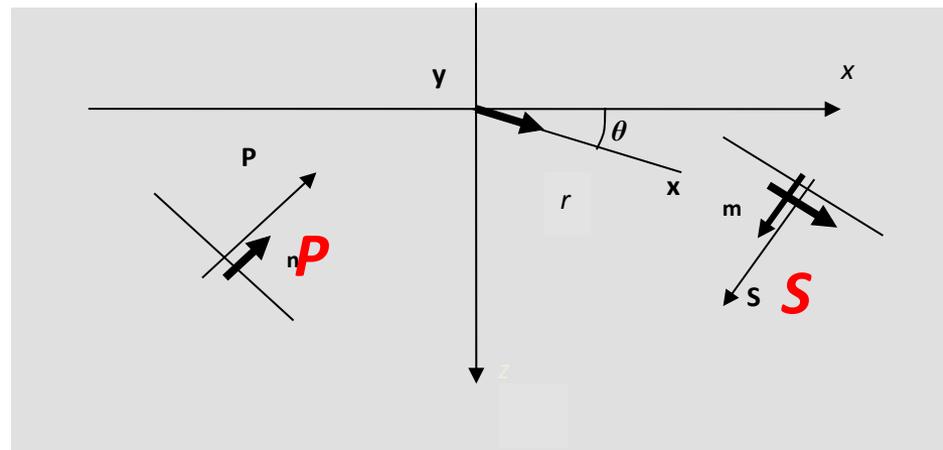
$$\alpha = \sqrt{\frac{\lambda + 2\mu}{\rho}} \quad \beta = \sqrt{\frac{\mu}{\rho}} \quad r = |P, Q|$$

**THE 2D P-SVCASE** 
$$\beta^2 \frac{\partial^2 u_i}{\partial x_j \partial x_j} + (\alpha^2 - \beta^2) \frac{\partial^2 u_j}{\partial x_i \partial x_j} = \frac{\partial^2 u_i}{\partial t^2}$$

**Summation of P and S plane waves:**

$$u_l(\mathbf{x}, \omega, t) = P(\omega, \phi) n_l \exp(-i \frac{\omega}{\alpha} x_j n_j) + S(\omega, \psi) m_l \exp(-i \frac{\omega}{\beta} x_j m_j)$$

**Correlation:**



$$u_l(\mathbf{y}) u_s^*(\mathbf{x}) = (P^2 n_l n_s + SP^* m_l n_s) \exp(\mathbf{i}kr \cos[\phi - \theta]) + (S^2 m_l m_s + PS^* n_l m_s) \exp(\mathbf{i}kr \cos[\psi - \theta])$$

**Azimuthal average assuming:**  $P^2 \alpha^2 = \varepsilon S^2 \beta^2$

$$\langle \bullet \rangle = \frac{1}{4\pi^2} \int_0^{2\pi} d\phi \int_0^{2\pi} \bullet d\psi$$

$$\langle u_i(\mathbf{y}) u_j^*(\mathbf{x}) \rangle = \frac{S^2 \beta^2}{2} \left\{ A \delta_{ij} - B(2\gamma_i \gamma_j - \delta_{ij}) \right\}$$

$$A = \varepsilon \frac{J_0(qr)}{\alpha^2} + \frac{J_0(kr)}{\beta^2} \quad \text{and} \quad B = \varepsilon \frac{J_2(qr)}{\alpha^2} - \frac{J_2(kr)}{\beta^2}$$

**And finally if  $\varepsilon=1$ , i.e. 2D equipartition ratio:**

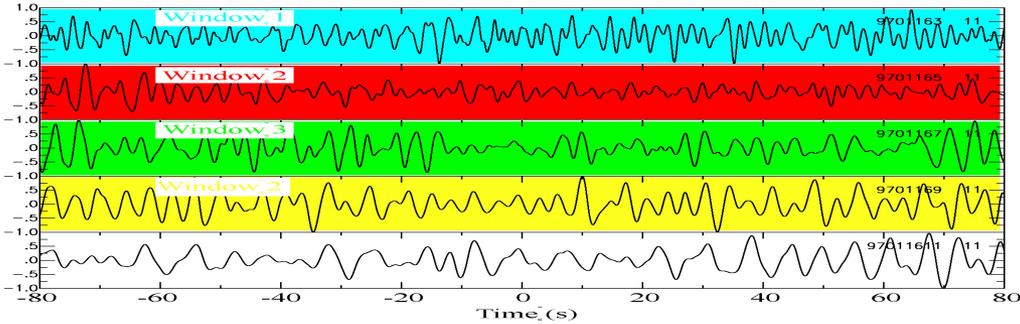
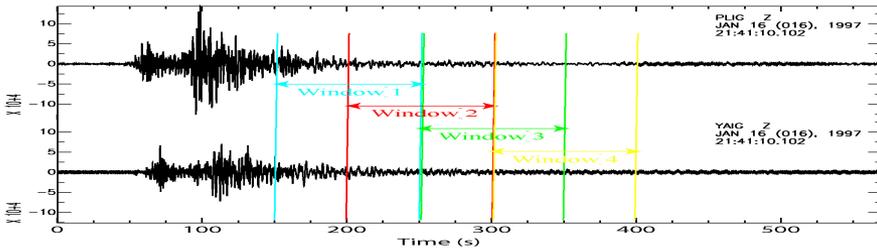
$$2D: \quad E_S / E_P = \left( \frac{\alpha}{\beta} \right)^2$$

$$\langle u_i(\mathbf{y}, \omega) u_j^*(\mathbf{x}, \omega) \rangle \equiv -8E_S k^{-2} \text{Im} \left[ G_{ij}(\mathbf{x}, \mathbf{y}, \omega) \right]$$

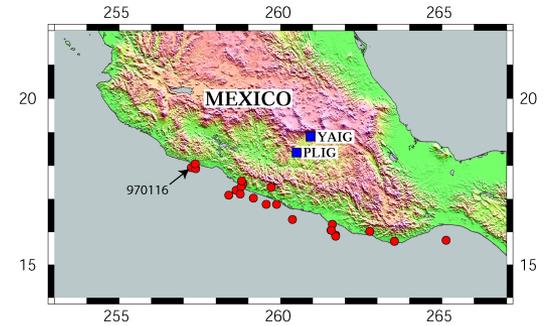
**Formally, same result in 3D (Sánchez-Sesma and Campillo, BSSA 2006)**

# Seismological application: coda waves

Individual cross-correlations: fluctuations dominate.

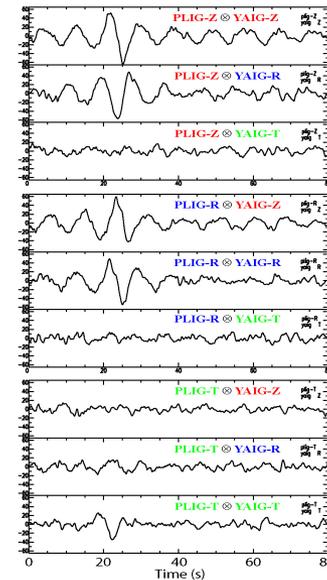


After averaging over 100 EQs →

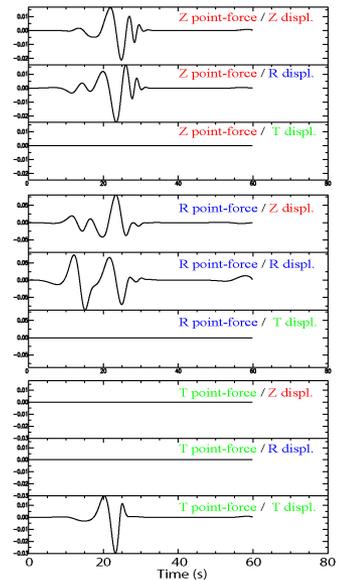


## Emergence of the Green function

Stacks of 196 cross-correlations



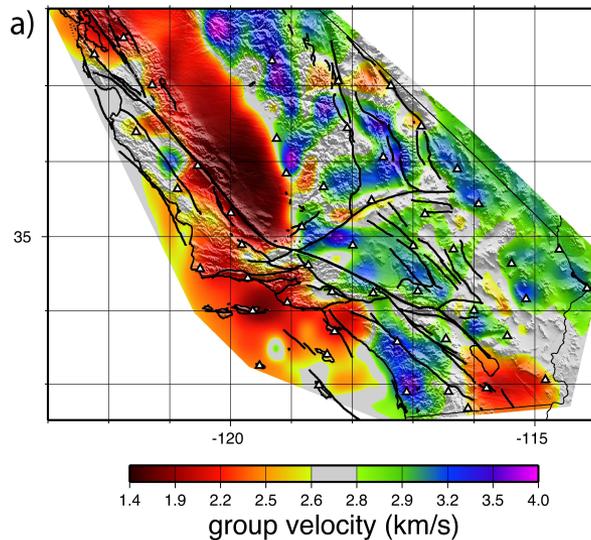
Theoretical Green tensor at 69 km distance



# Cross-correlations of coda and noise records ≈ Green functions = virtual seismograms

-demonstrated for the retrieval of surface waves (e.g. Paul and Campillo, 2001; Campillo and Paul, 2003; Shapiro and Campillo, 2004....) or body waves (e.g. Zhan et al., 2010 ; Poli et al., 2012).

High resolution velocity map of California obtained from ambient noise (Rayleigh) (Shapiro, Campillo, Stehly and Ritzwoller, Science 2005)



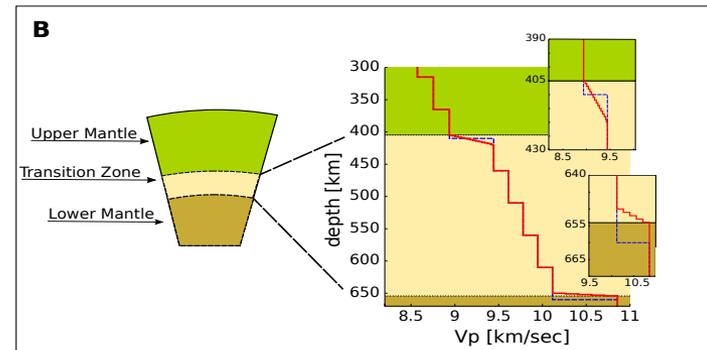
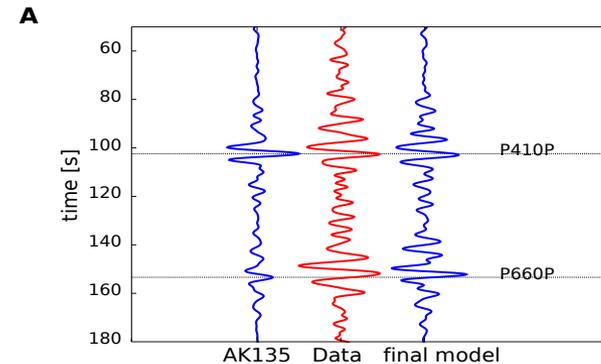
Large N sensor array ⇒  $N^2/2$  correlations

## Earth's mantle discontinuities from ambient noise

( phase transition → (P,T))

Body waves (Poli et al., 2012)

Poli, Campillo, Pedersen. Science 2012



# Arbitrary medium: an integral representation written in the frequency domain

(see e.g. Weaver et al. 2004, or Snieder, 2007)

$$G_{12} - G_{12}^* = \frac{4i\omega\kappa}{c} \int_{\mathcal{V}} G_{1x} G_{2x}^* dV + \oint_S \left[ G_{1x} \vec{\nabla} (G_{2x}^*) - \vec{\nabla} (G_{1x}) G_{2x}^* \right] \vec{dS}$$

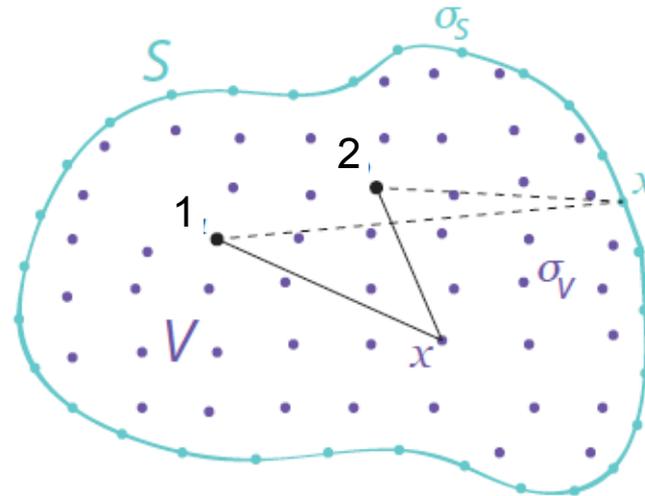
Volume term

Surface term

FT of  $G(-t)$

Absorption coefficient

FT of  $G(t)$



Helmholtz equation  $G_{1x} = G(\vec{r}_1, \vec{x}; \omega)$

$$\Delta G_{1x} + V(\vec{x})G_{1x} + (k + i\kappa)^2 G_{1x} = \delta(\vec{x} - \vec{r}_1)$$

where the potential  $V(\vec{x})$  describes the scattering contribution  
does not extend to infinity.

As for the classical representation theorem, we consider a combination of the fields from source at 1 and 2 and compute the flux:

$$I = \oint_S \left[ G_{1x} \vec{\nabla} (G_{2x}^*) - \vec{\nabla} (G_{2x}) G_{1x}^* \right] \overrightarrow{dS}$$

With the divergence theorem:

$$I = \int_{\mathcal{V}} \vec{\nabla} \left[ G_{1x} \vec{\nabla} (G_{2x}^*) - \vec{\nabla} (G_{1x}) G_{2x}^* \right] dV$$

$$I = \int_{\mathcal{V}} \vec{\nabla} \left[ G_{1x} \vec{\nabla} (G_{2x}^*) - \vec{\nabla} (G_{1x}) G_{2x}^* \right] dV \quad \text{reduces to}$$

$$I = \int_{\mathcal{V}} \left( G_{1x} \Delta G_{2x}^* - \Delta G_{1x} G_{2x}^* \right) dV$$

Using the definition of the GF:

$$\Delta G_{1x} = \delta(\vec{x} - \vec{r}_1) - V(\vec{x}) G_{1x} - (k + i\kappa)^2 G_{1x}$$

we obtain:

$$I = G_{12} - G_{21}^* - \frac{4i\omega\kappa}{c} \int_{\mathcal{V}} G_{1x} G_{2x}^* dV$$

and finally:

$$G_{12} - G_{12}^* = \frac{4i\omega\kappa}{c} \int_{\mathcal{V}} G_{1x} G_{2x}^* dV + \oint_S \left[ G_{1x} \vec{\nabla} (G_{2x}^*) - \vec{\nabla} (G_{1x}) G_{2x}^* \right] \vec{dS}$$

# Representation theorem for correlation: passive imaging

Arbitrary medium: an integral representation written in the frequency domain

$$G_{12} - G_{12}^* = \frac{4i\omega\kappa}{c} \int_V G_{1x} G_{2x}^* dV + \oint_S \left[ G_{1x} \vec{\nabla} (G_{2x}^*) - \vec{\nabla} (G_{1x}) G_{2x}^* \right] \vec{dS}$$

FT of  $G(t)$

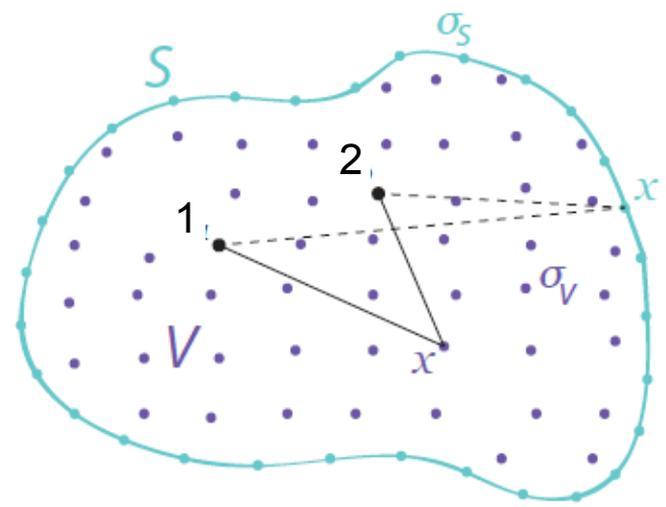
FT of  $G(-t)$

Absorption coefficient

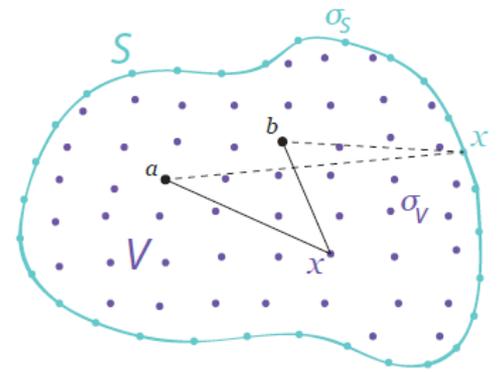
Volume term

Surface term

Source average over « correlation terms »



e.g. Weaver et al., 2004, Snieder 2007,....



Volume term: 
$$G_{12} - G_{12}^* = \frac{4i\omega\kappa}{c} \int_V G_{1x} G_{2x}^* dV$$

$\kappa$  is finite (attenuation)

$S$  is assumed to be sufficiently far away, for its contribution to be neglected (spreading and attenuation)

Surface term: 
$$G_{12} - G_{12}^* = \oint_S \left[ G_{1x} \vec{\nabla} (G_{2x}^*) - \vec{\nabla} (G_{1x}) G_{2x}^* \right] \vec{dS}$$

$\kappa = 0$  (no attenuation)

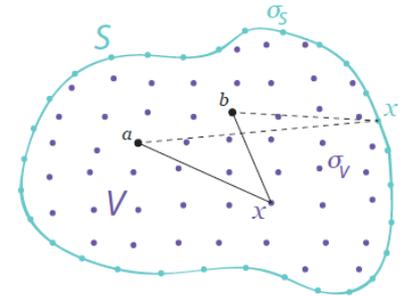
No source in the bulk

Surface term: 
$$\oint_S \left[ G_{1x} \vec{\nabla} (G_{2x}^*) - \vec{\nabla} (G_{1x}) G_{2x}^* \right] d\vec{S}$$

If the surface is taken in the far field of the medium heterogeneities:

$$G_{1x} \sim \frac{1}{4\pi |\vec{x} - \vec{r}_1|} \exp(-ik |\vec{x} - \vec{r}_1|) \quad \text{and} \quad \vec{\nabla} (G_{1x}) \sim i\vec{k} G_{1x}$$

and we obtain a widely used integral relation:



$$\oint_S \left[ G_{1x} \vec{\nabla} (G_{2x}^*) - \vec{\nabla} (G_{1x}) G_{2x}^* \right] d\vec{S} \approx -2i \frac{\omega}{c} \oint_S G_{1x} G_{2x}^* dS$$

Source average over  
« correlation terms »

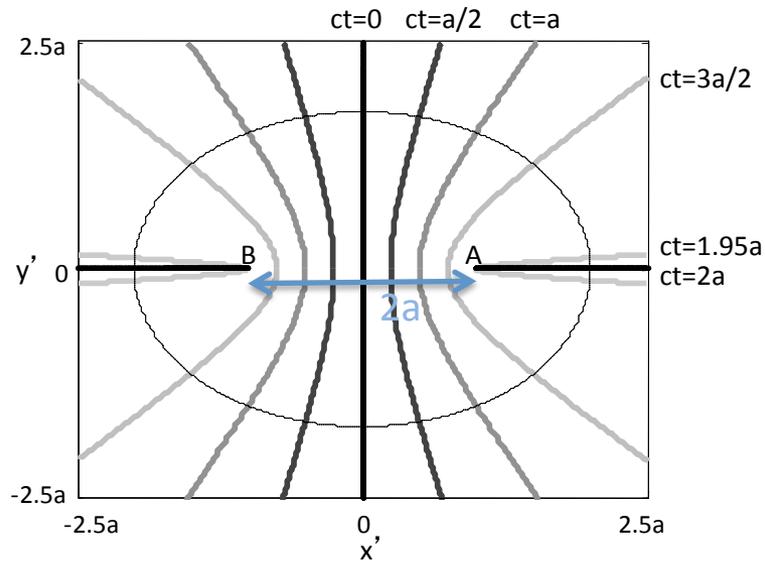
→ Derode et al., 2003: Analogy with Time reversal mirrors

→ Wapenaar 2004

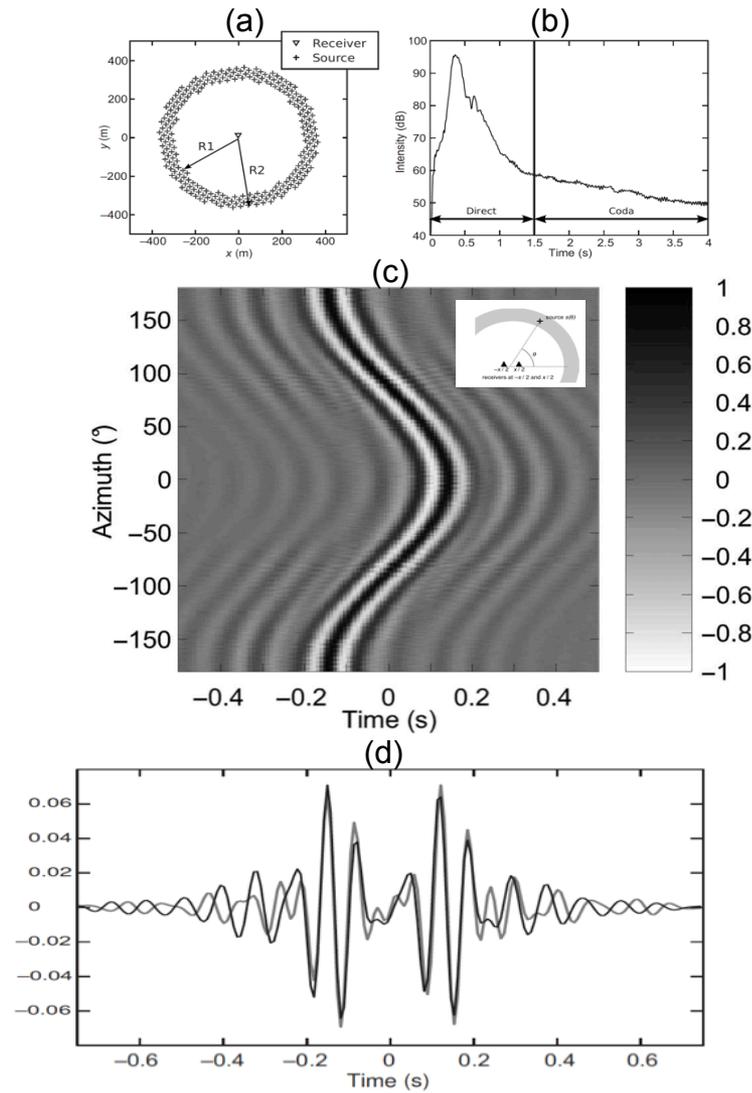
For surface waves: distant sources of noise at the surface of the sphere ( $\approx 2D$  problem)

Location of the sources that contribute to the correlation: the end fire lobes

Difference of travel time between A and B  
wrt the position of the source



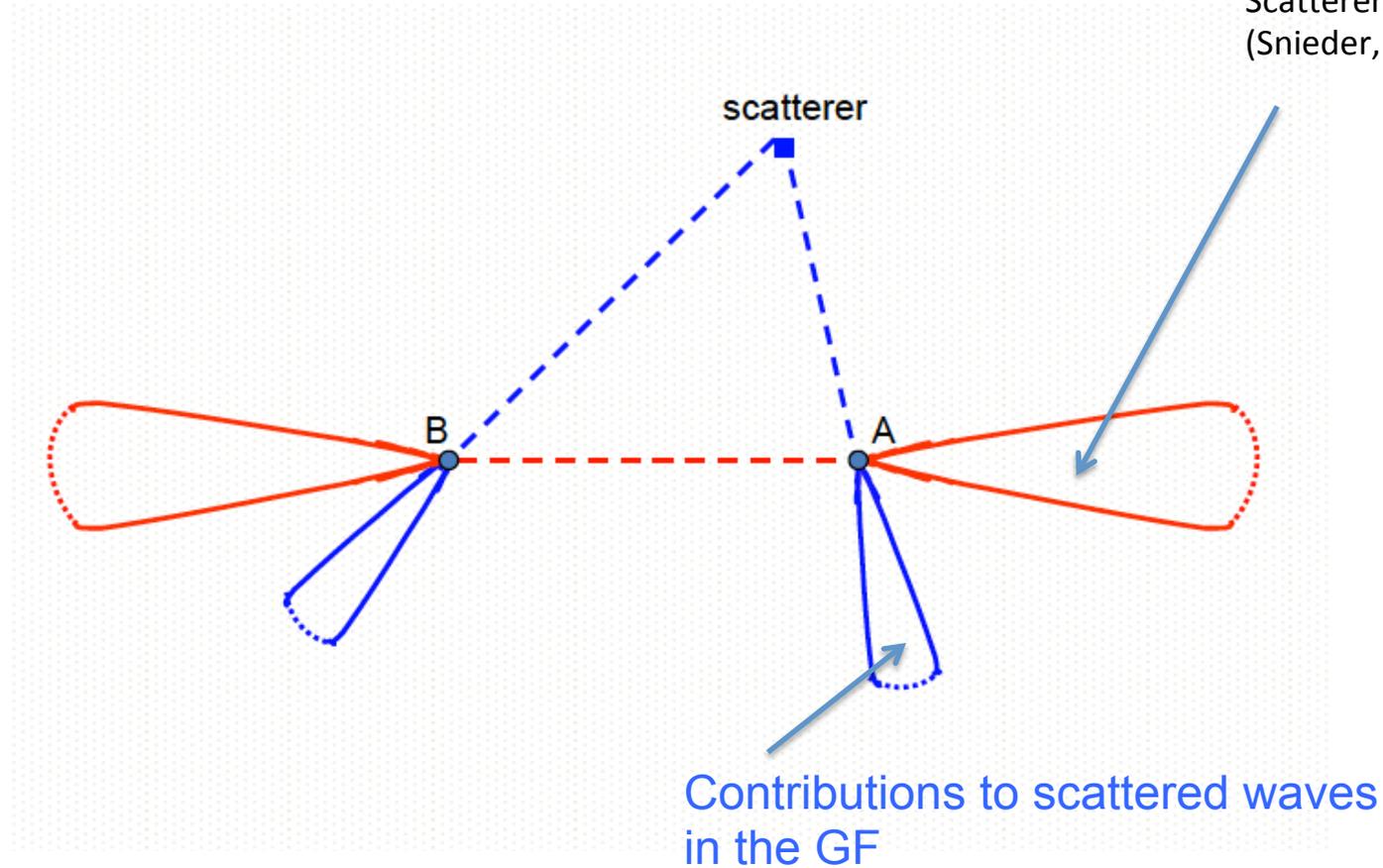
# Stationary phase and end fire lobes



# End fire lobes

Contributions to direct waves  
in the GF

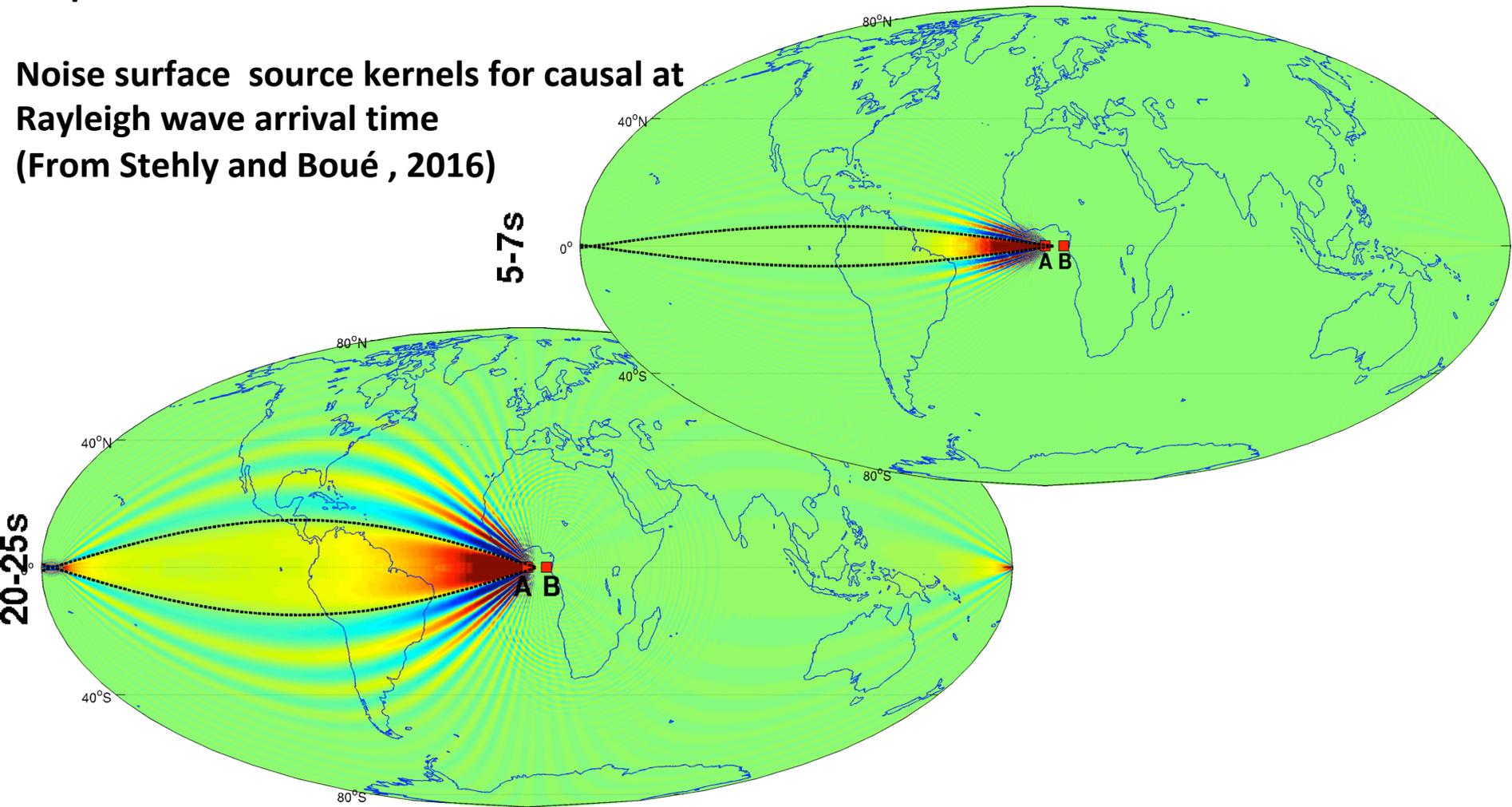
Scatterers or sources  
(Snieder, 2004; Roux et al., 2005 )



Extension to scattered waves

# Amplitude

Noise surface source kernels for causal at  
Rayleigh wave arrival time  
(From Stehly and Boué , 2016)



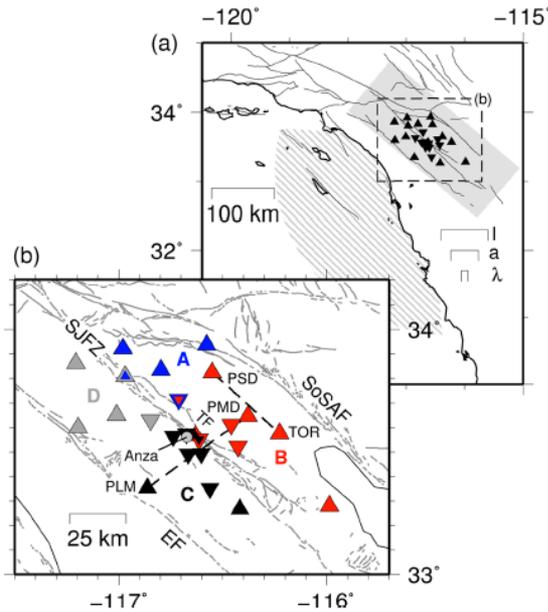
The kernels depends on frequency and interstation distance:

→ Difficulty for attenuation measurements

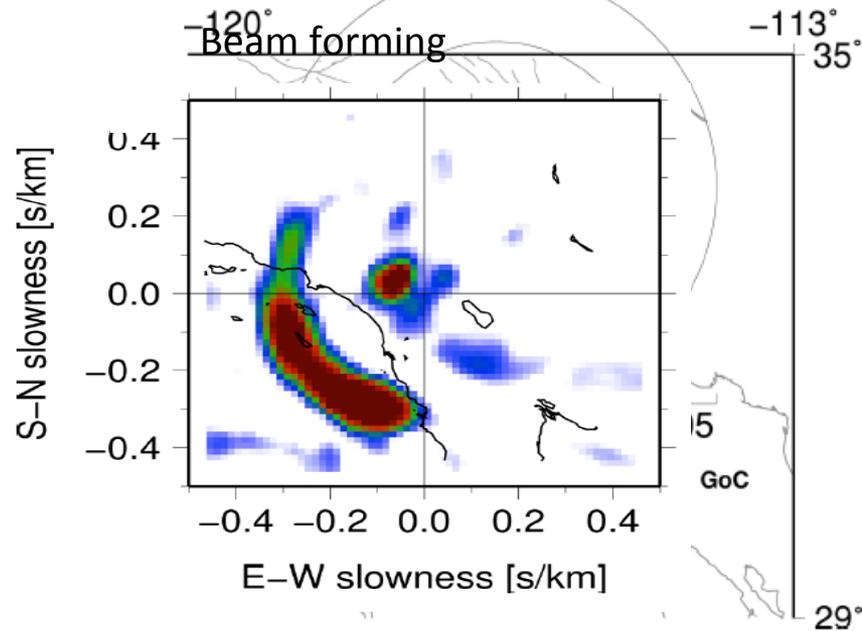
# Problems and accuracy

## Anisotropic intensity of the noise: the example of the San Jacinto fault

*From Hillers et al., 2013 G3*

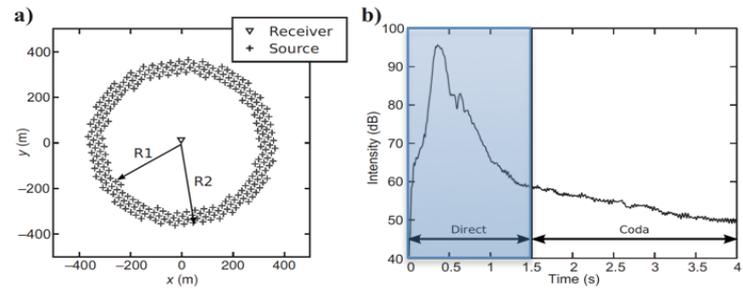


Anisotropic intensity of the noise  
(measured for winter and summer and from  
different components)

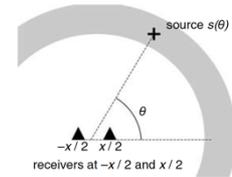


# Correlation of direct waves:

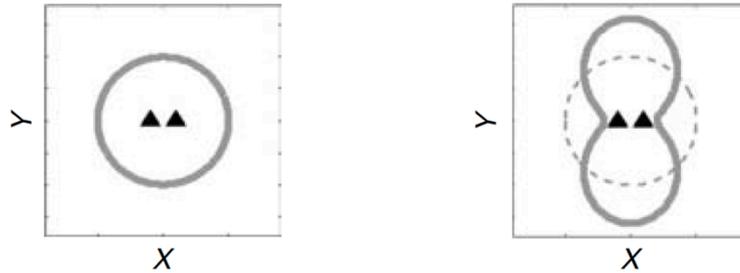
## Bias in the travel time



Increasing anisotropy of the source intensity  $B$

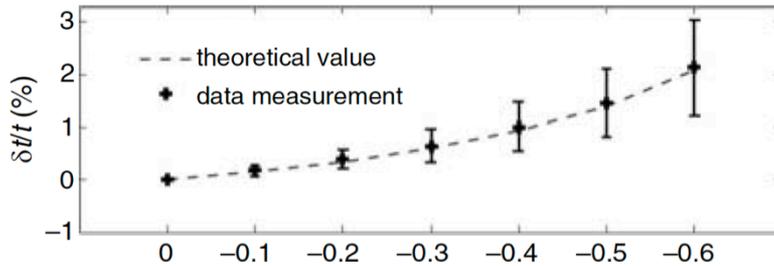


Azimuthal distribution of source intensity



$$B(\theta) = 1 + B_2 \cos(2\theta)$$

Travel time error wrt the observed Green function



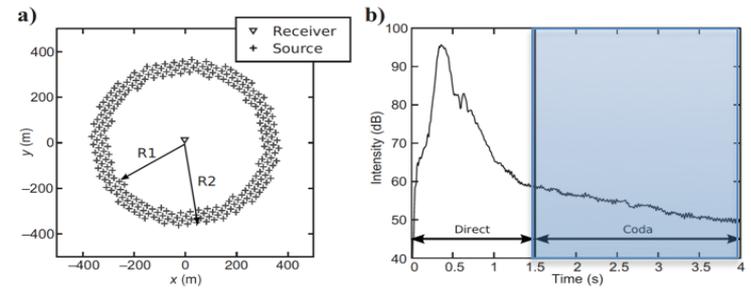
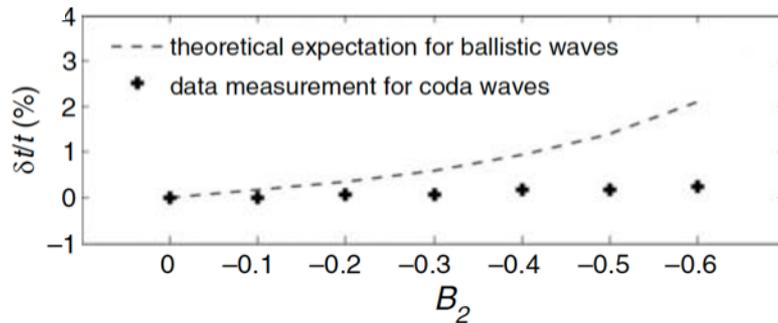
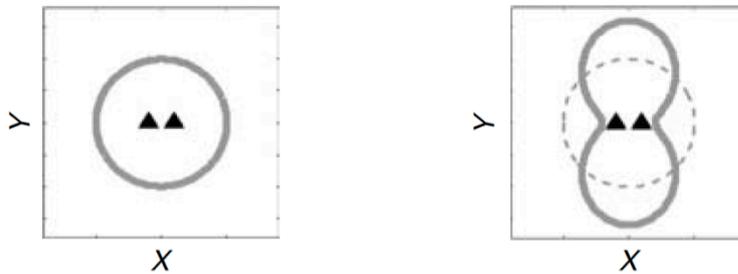
$$\delta t = \frac{1}{2t\omega_0^2 B(0)} \left. \frac{d^2 B(\theta)}{d\theta^2} \right|_{\theta=0}$$

valid with  $t$  (travel time)  $>$   $T$  (period)

In presence of scattering:  
Correlation of coda waves

-isotropy improved by multiple scattering

Increasing anisotropy of the source intensity  $B$

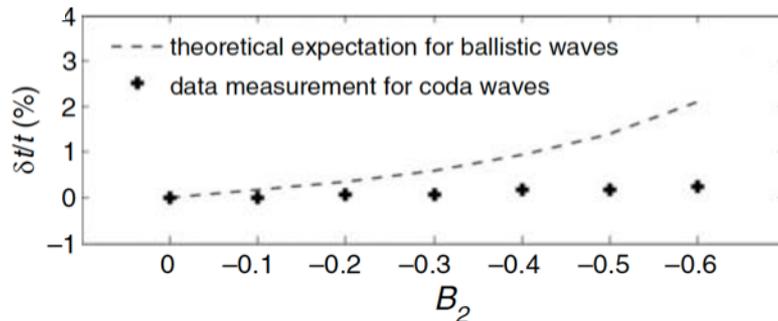
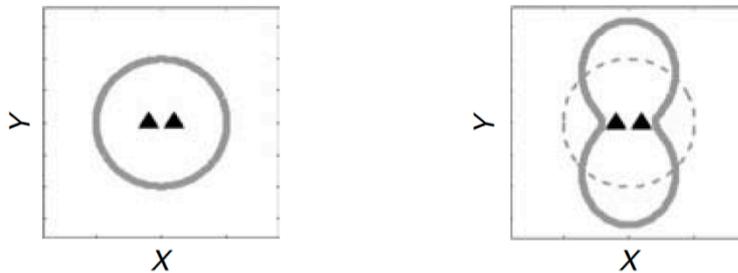


$$B(\theta) = 1 + B_2 \cos(2\theta)$$

No bias in the correlation of coda waves!

In presence of scattering:  
 Correlation of coda waves  
 -isotropy provided by multiple scattering

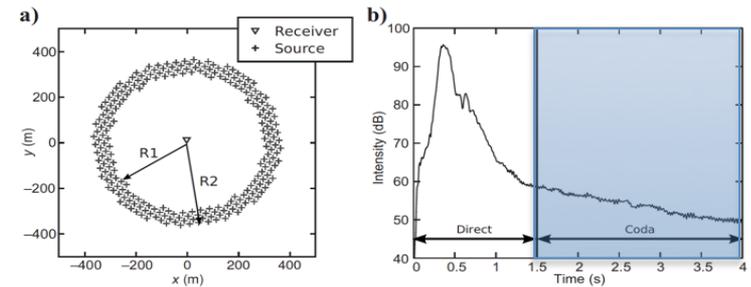
Increasing anisotropy of the source intensity  $B$



Noise records contain direct **and** scattered waves:

→ the biases of direct wave travel times are generally small enough for imaging purpose

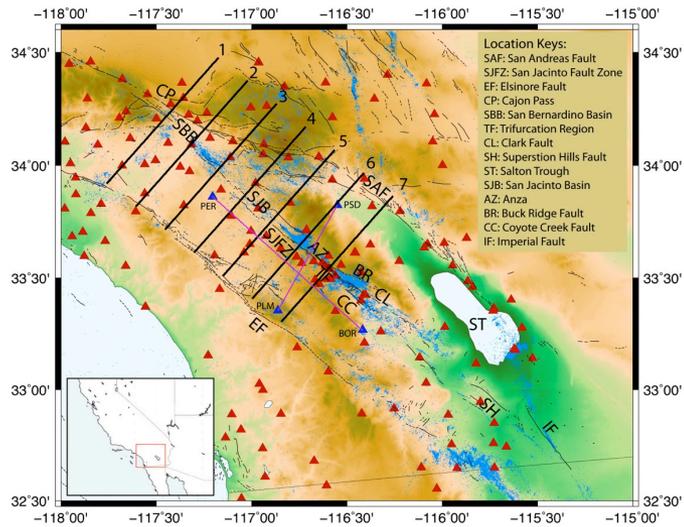
→ Importance of processing strategies



$$B(\theta) = 1 + B_2 \cos(2\theta)$$

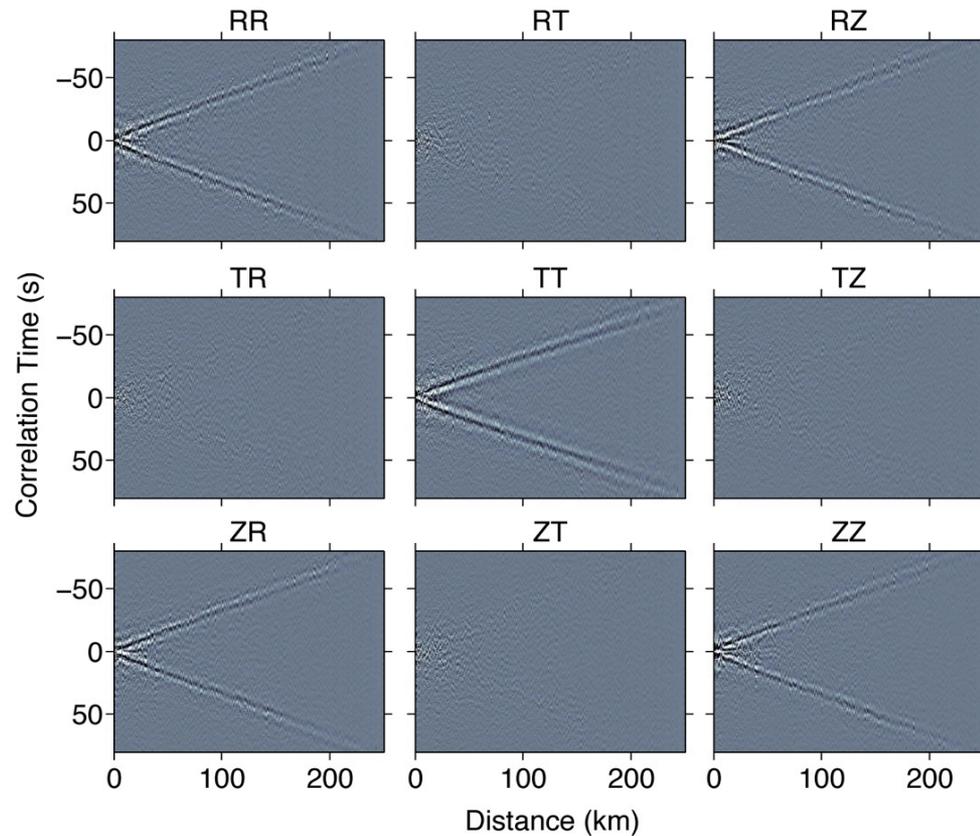
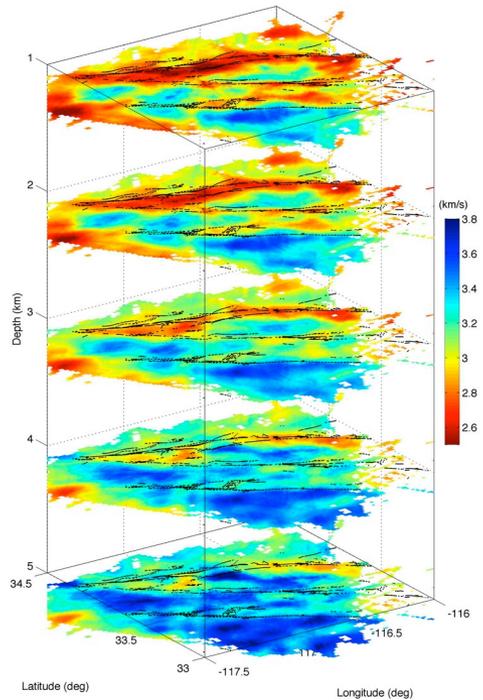
Scattering provides the  
 diversity of incidence  
 directions → isotropization of  
 intensity

No bias in the correlation of  
 coda waves!



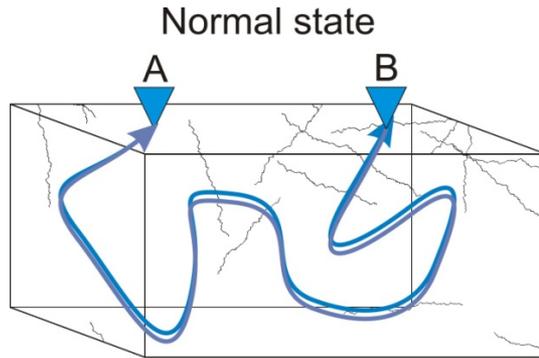
## Shear wave tomography

## 9-component correlations



# Noise based seismic velocity temporal changes

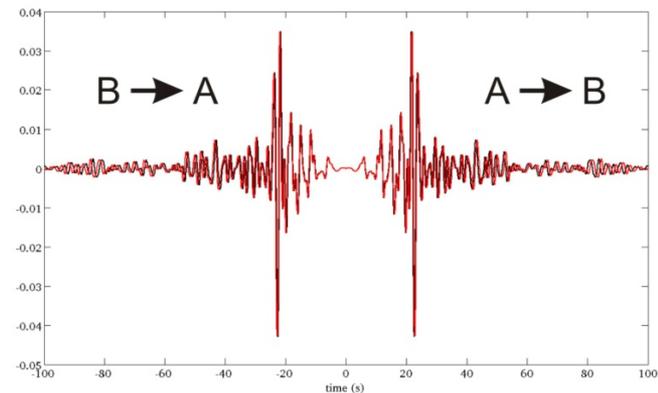
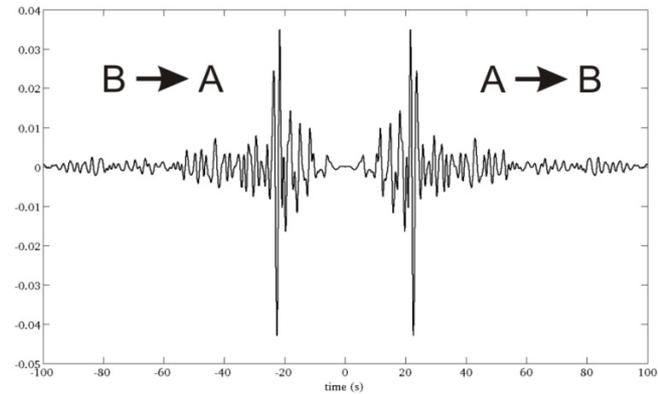
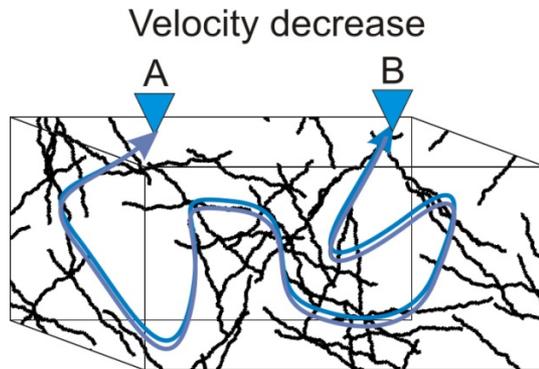
Because seismic noise is continuous in time, it is possible to reconstruct **repeating virtual seismic sources** and perform **continuous monitoring of seismic velocities**.



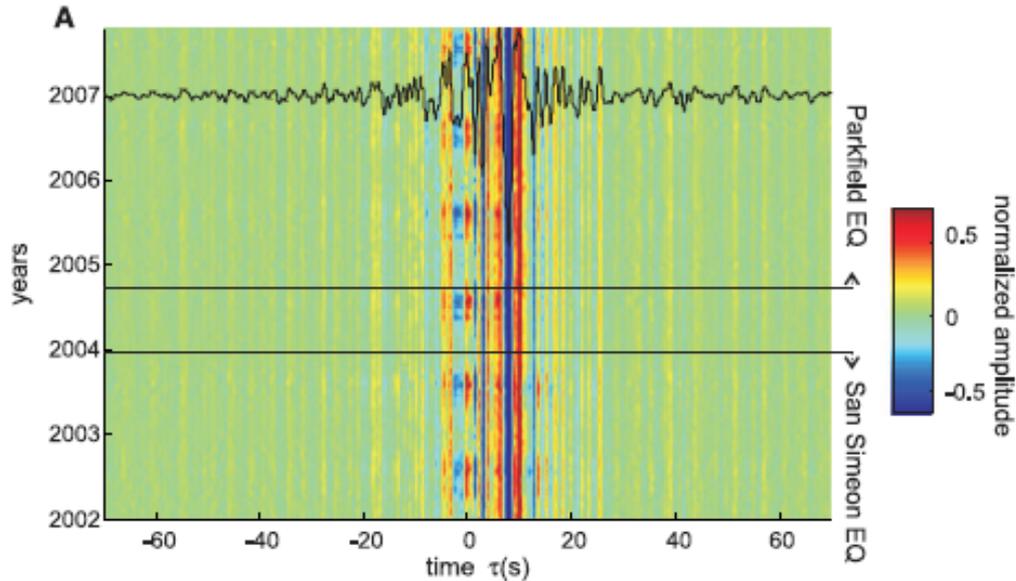
$\Delta V$



A large downward-pointing arrow with the symbol  $\Delta V$  next to it, indicating a change in velocity.



## Correlation functions as approximate Green functions

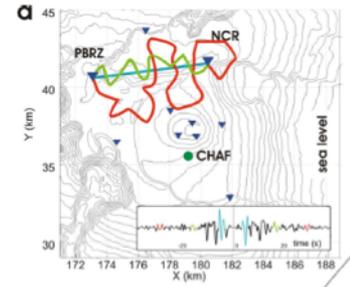
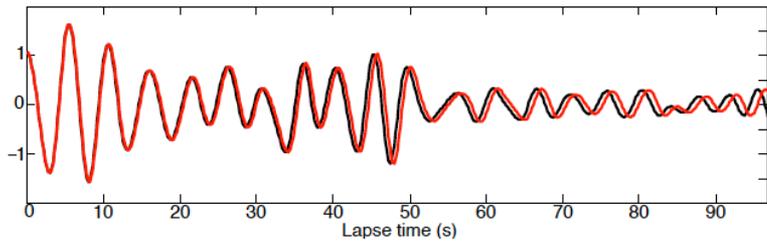


Direct waves are sensitive to noise source distribution (errors small enough for tomography ( $\leq 1\%$ ) but too large for monitoring (goal  $\approx 10^{-4}$ )

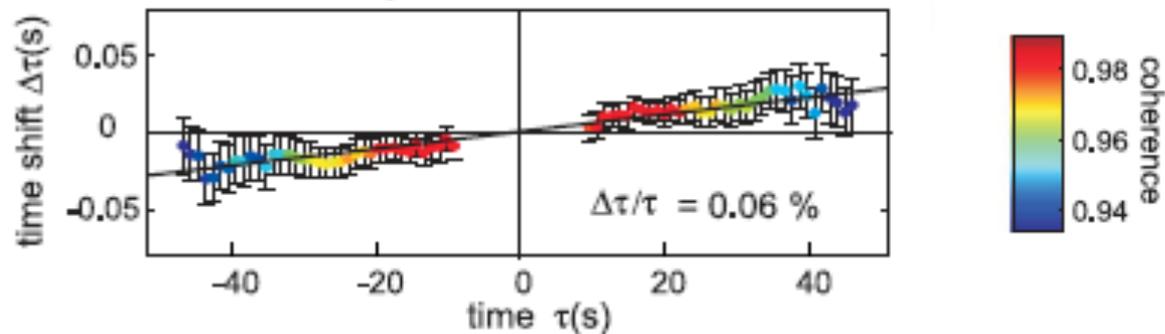
Stability of the ‘coda’ of the noise correlations

# Detecting a small change of seismic speed: coda waves

Comparing a trace with a reference under the assumption of an homogeneous change



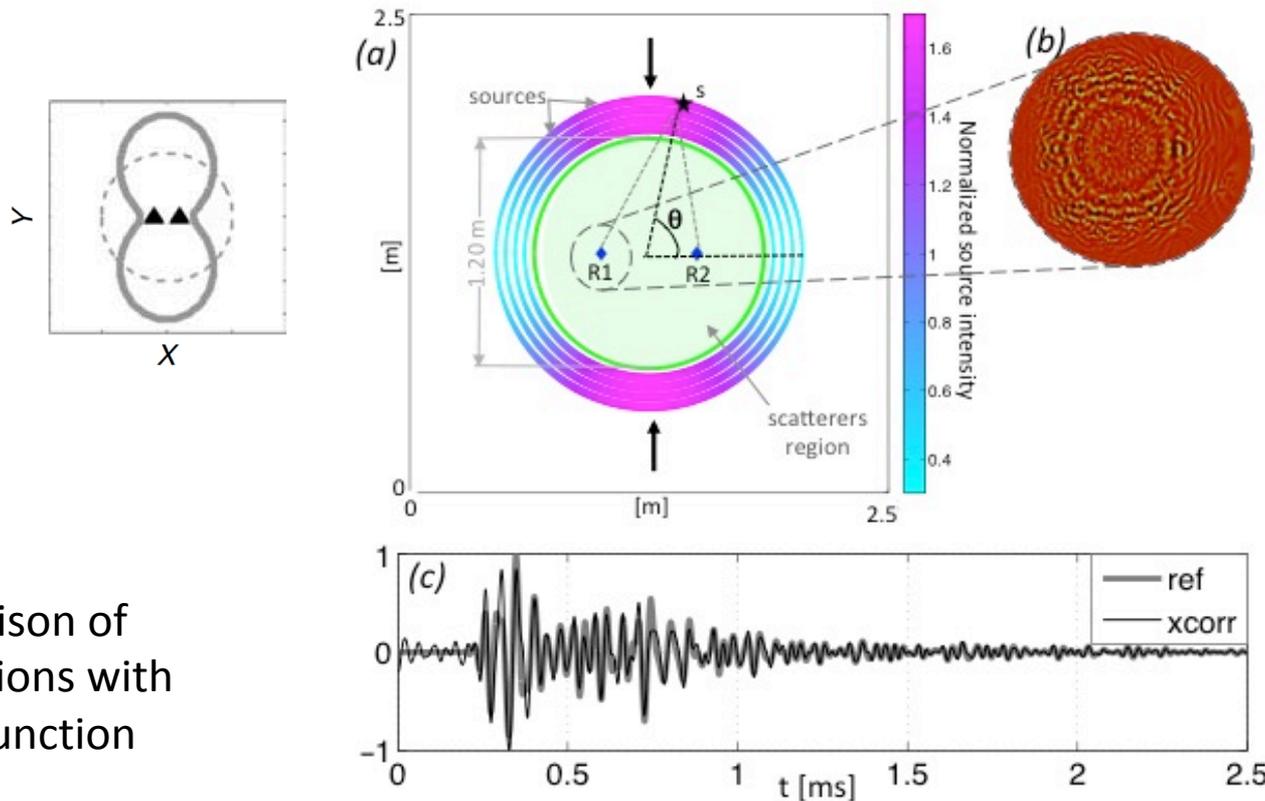
The ‘doublet’ method: moving window cross spectral analysis (phase measurements)



Alternative technique: stretching

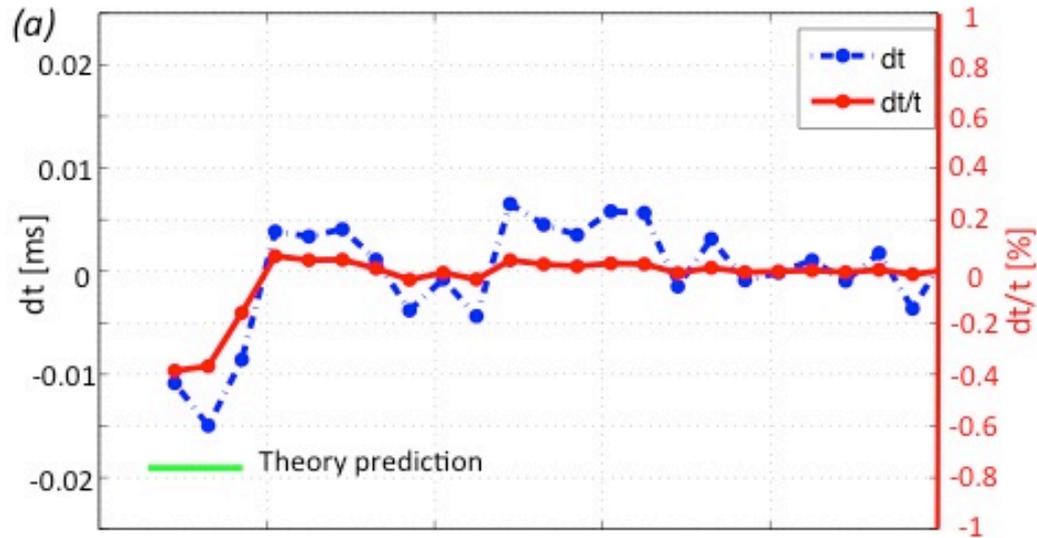
# Measuring slight changes of seismic velocity using coda waves (long travel time) Numerical simulations in a scattering medium

## 2D spectral elements, anisotropic intensity of sources



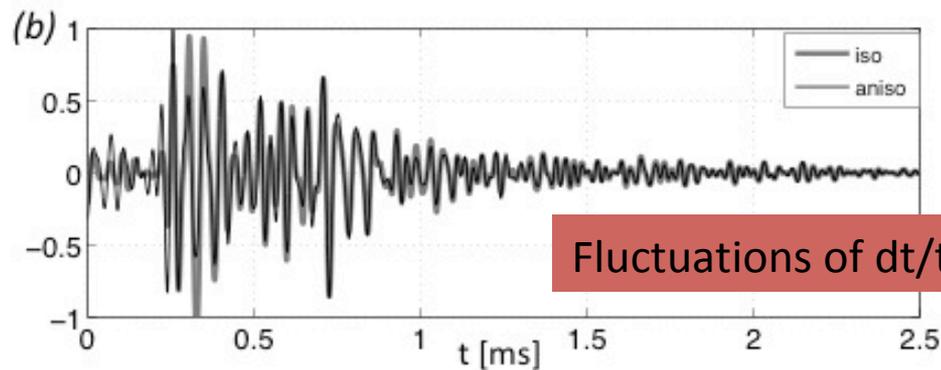
Comparison of correlations with Green function

Measure of the bias induced by a strong anisotropy  
of the wave field  
(delay with respect to the Green function)



Blue: delay

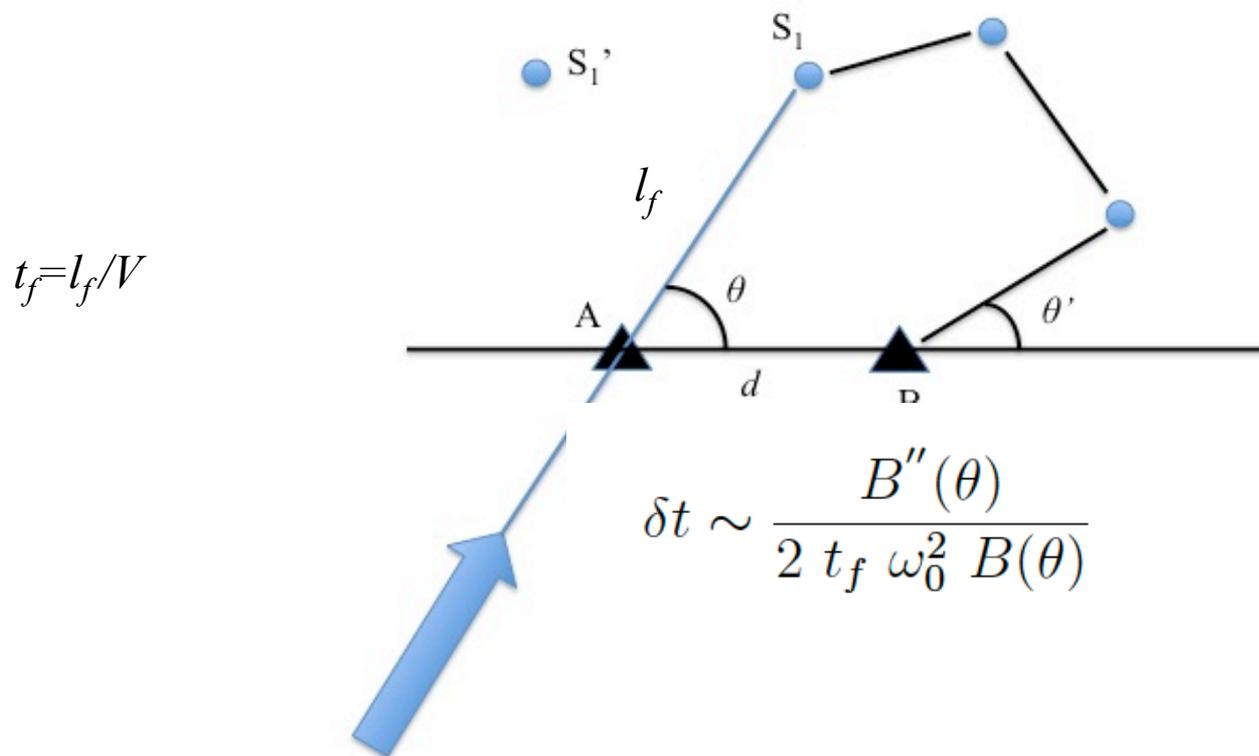
Red: relative delay



Fluctuations of  $dt/t$  of the order of  $10^{-4}$

## Representation of coda waves as the sum of contributions of paths

For a single path:



We have to compute the contributions of paths with first scatterers at all distances  $l_f$  and all azimuths  $\theta$

We have to consider that the distribution of distance between scattering events is exponential:

$$P(l_f) = \frac{1}{l} e^{-\frac{l_f}{l}}$$

where  $l$  is the mean free path

$$\langle l_f \rangle = l \quad t_f = l_f / V$$

We make use of

$$\delta t \sim \frac{B''(\theta)}{2 t_f \omega_0^2 B(\theta)}$$

valid for  $l_f > \lambda$

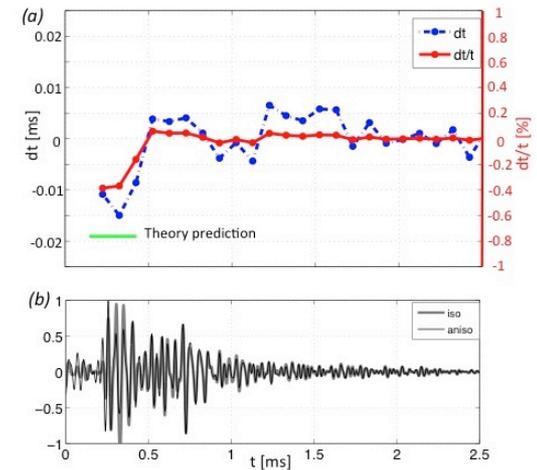
## Applications

### Numerical simulations

$$l = 0.5m, c = 2000m/s,$$

$$f_0 = 30000Hz, B_2 = -0.6 \text{ and } \tau_m = 0.002s$$

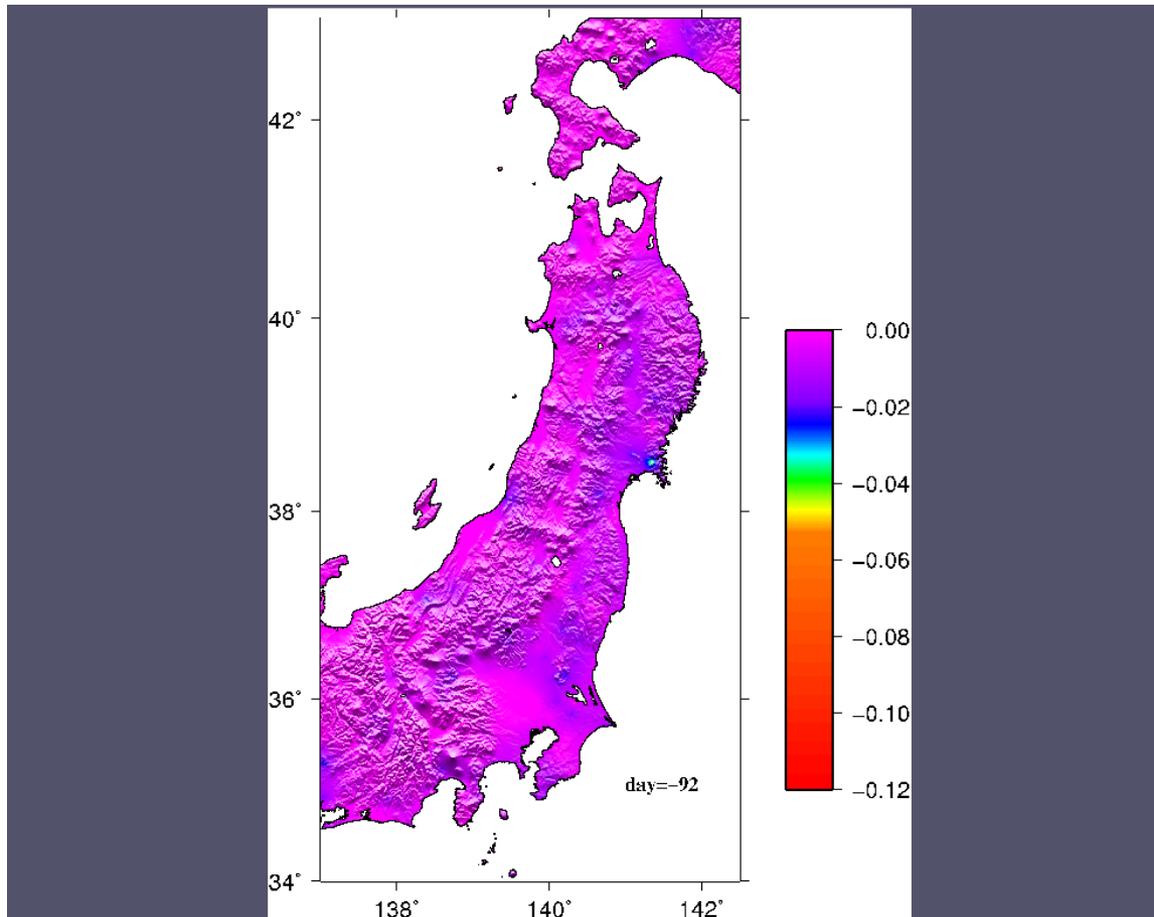
→ fractional error  $\frac{\delta t(\tau_m)}{\tau_m}$  of  $10^{-4}$



## Seismic sensors in Japan....



Relative velocity change ( in %) measured in the band 0.1-0.9 Hz

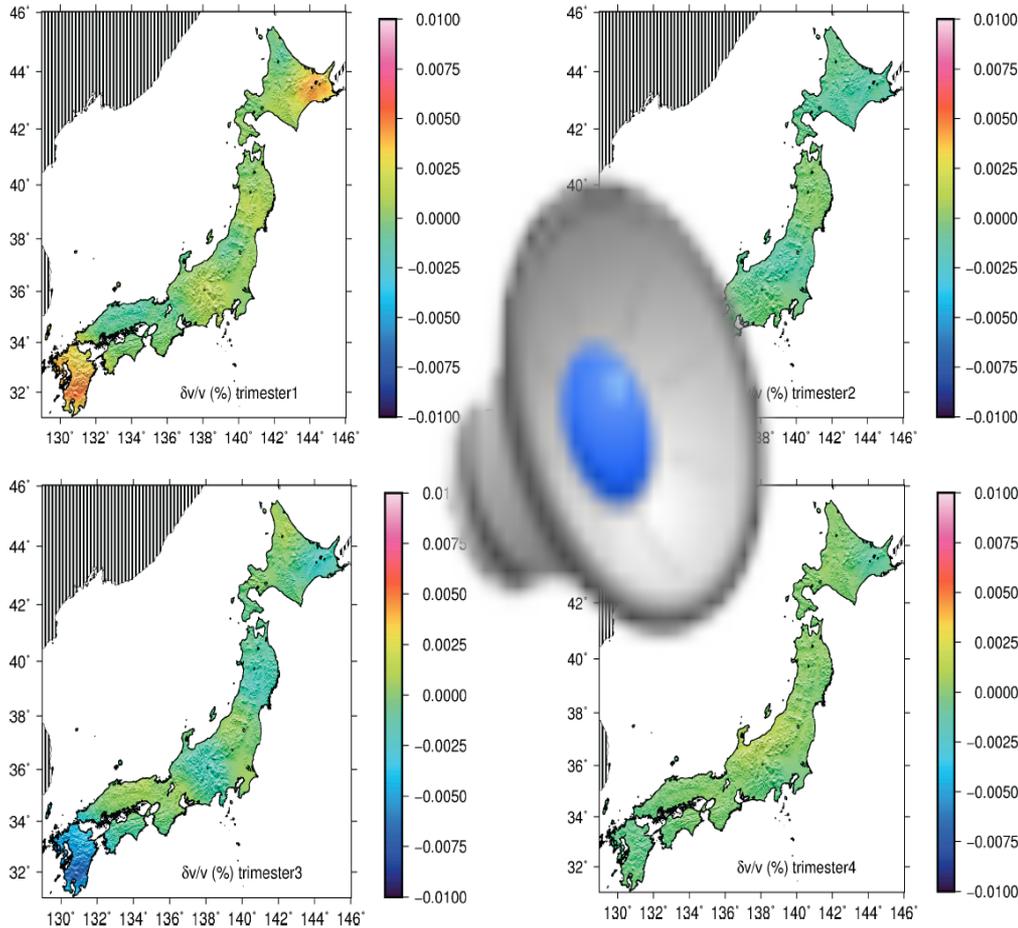


Calendar time measured in days with respect to March 11 (M9 Tohoku EQ)

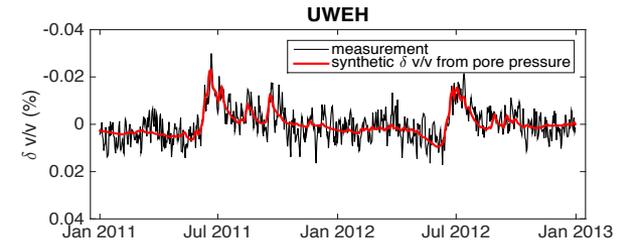
Velocity change also observed for slow slip event (Rivet et al. 2011, 2014)

From Brenguier, Campillo, Takeda, Aoki, Shapiro, Briand, Emoto and Miyake, Science 2014

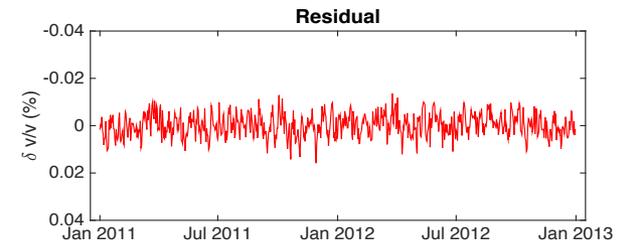
# Seasonal changes of velocities and correction from external forcings



b)



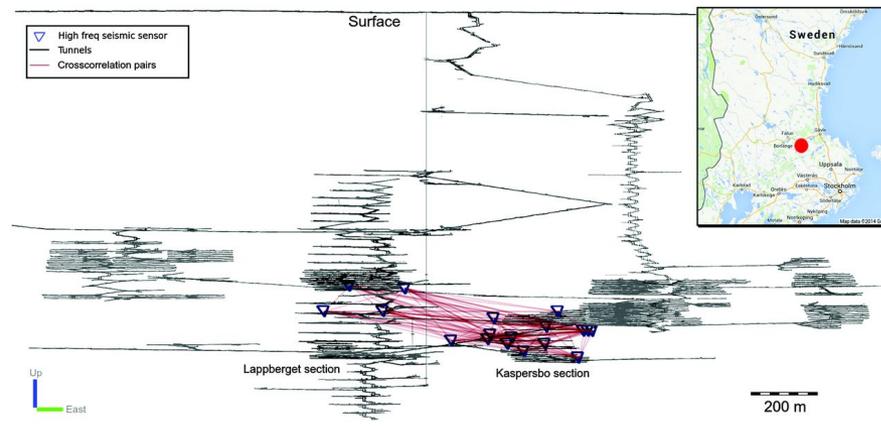
c)



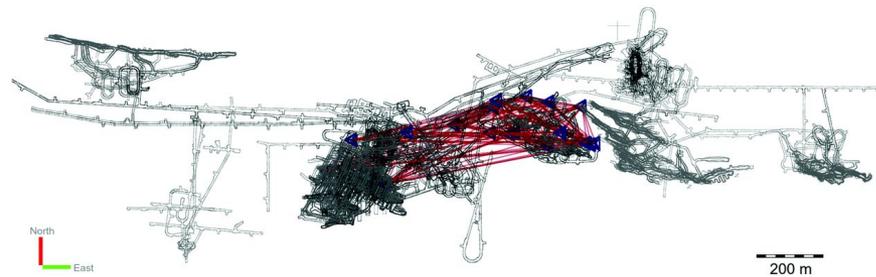
## Smaller scale, industrial environment

Active mine: various sources of noise  
tunnels (scattering)

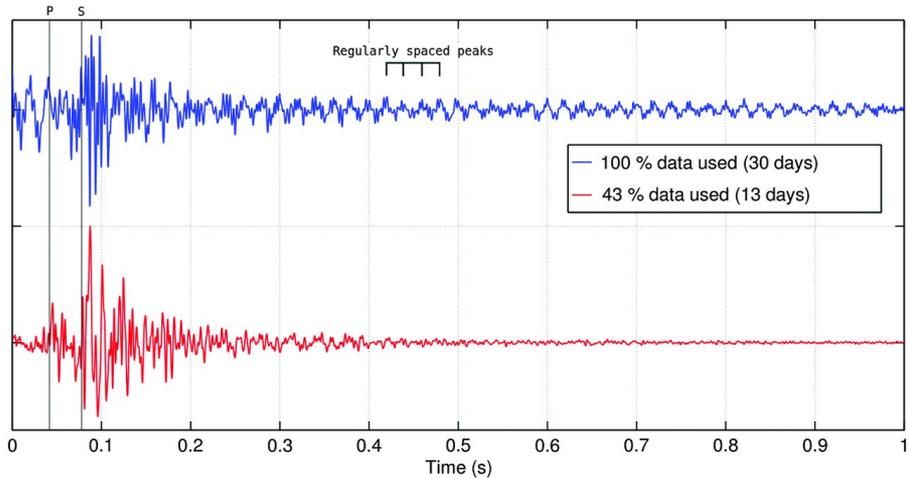
Section view



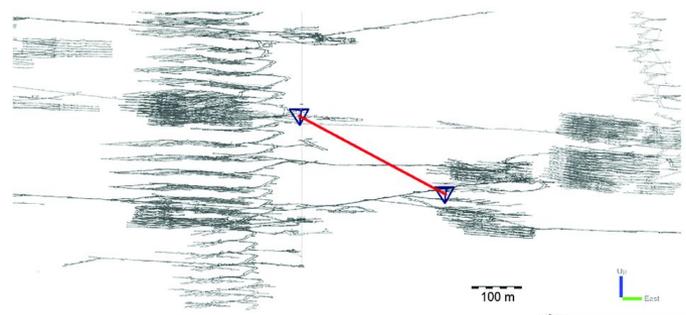
Plan view



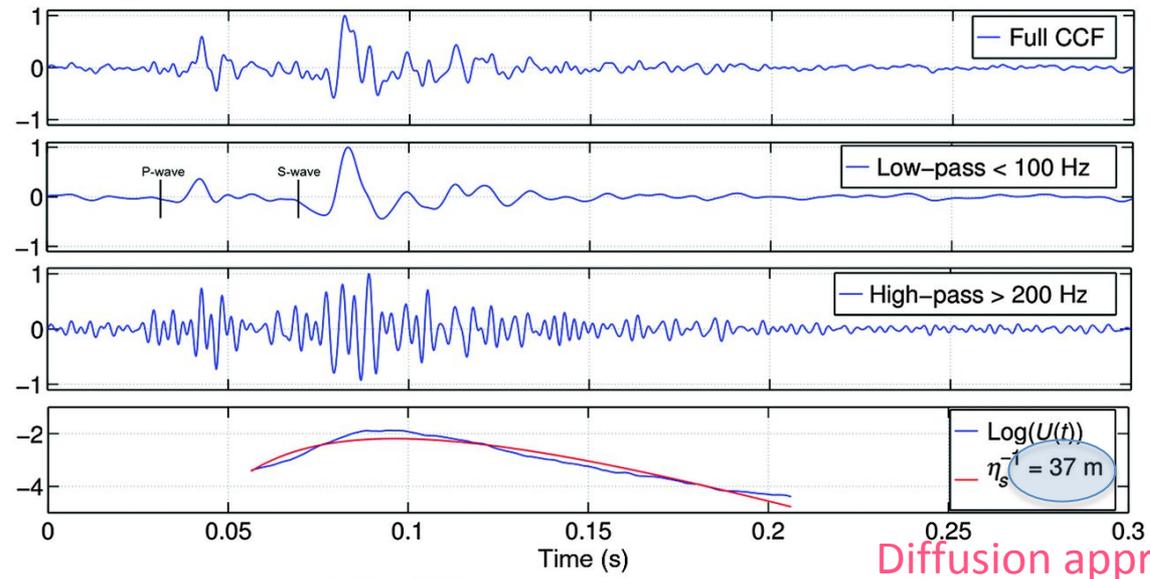
Results from Olivier, Brenguier, Campillo, Lynch and Roux, 2015  
GEOPHYSICS, VOL. 80, NO. 3 (MAY-JUNE 2015); P. KS11–KS25



## Correlation functions (ZZ)



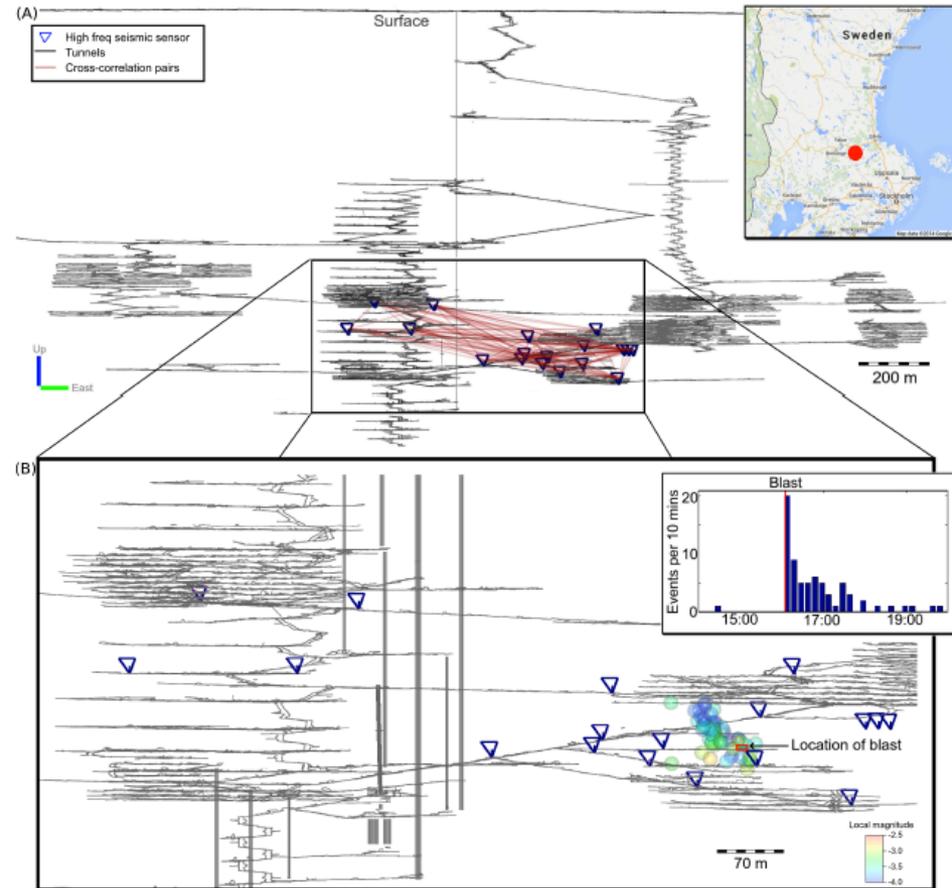
## Scattering properties from noise correlations



Diffusion approx.

# Local scale

## Velocity change due to blast and excavation in a mine

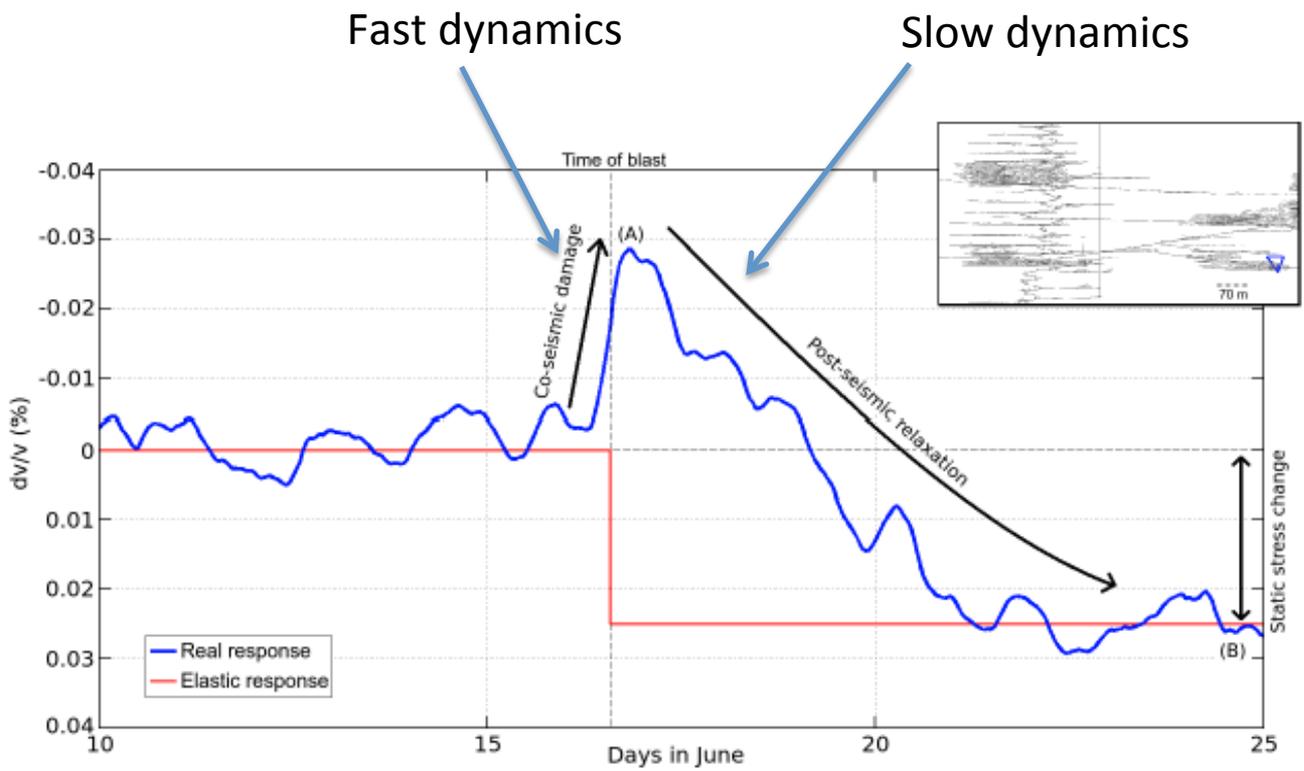


Use of the strong industrial noise in the mine.

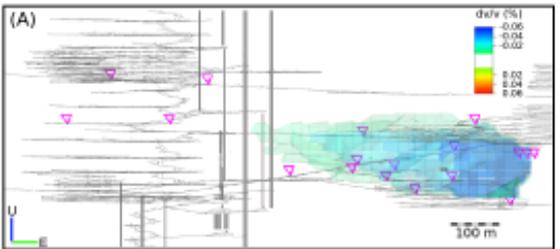
Note the intense scattering associated with the tunnels.

*Olivier et al., 2014*

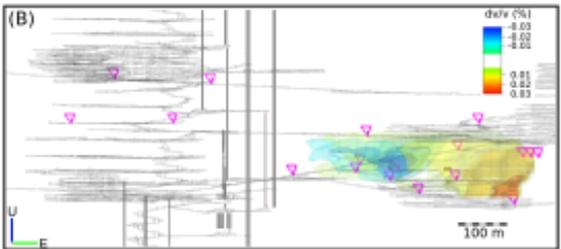
# Velocity change due to blast and excavation in a mine



Change of baseline due to static stress change



Instantaneous change



Static change