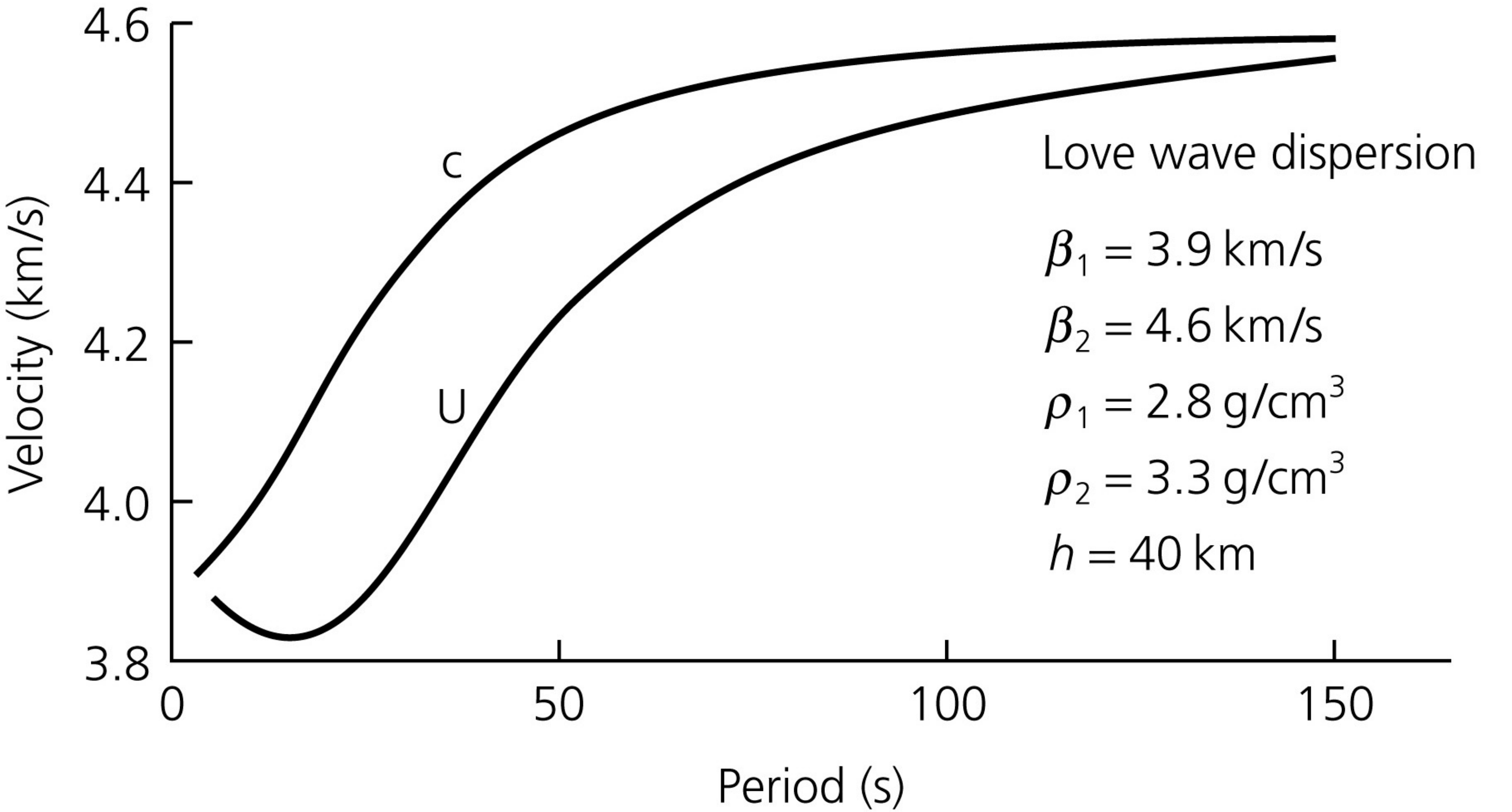
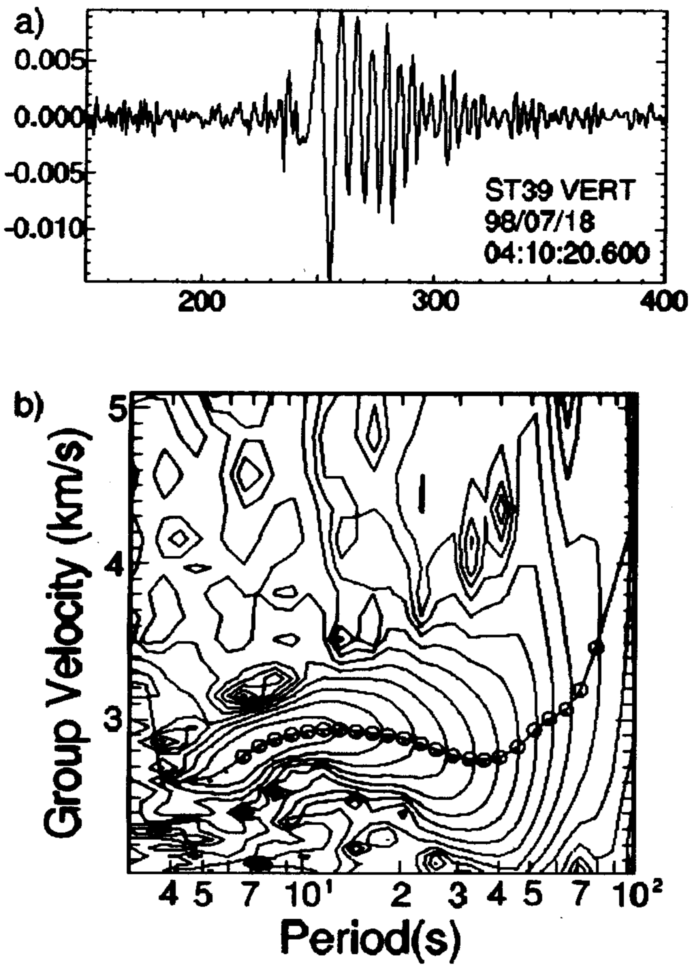
MEEES and M2R STU

Seismology (Michel Campillo)

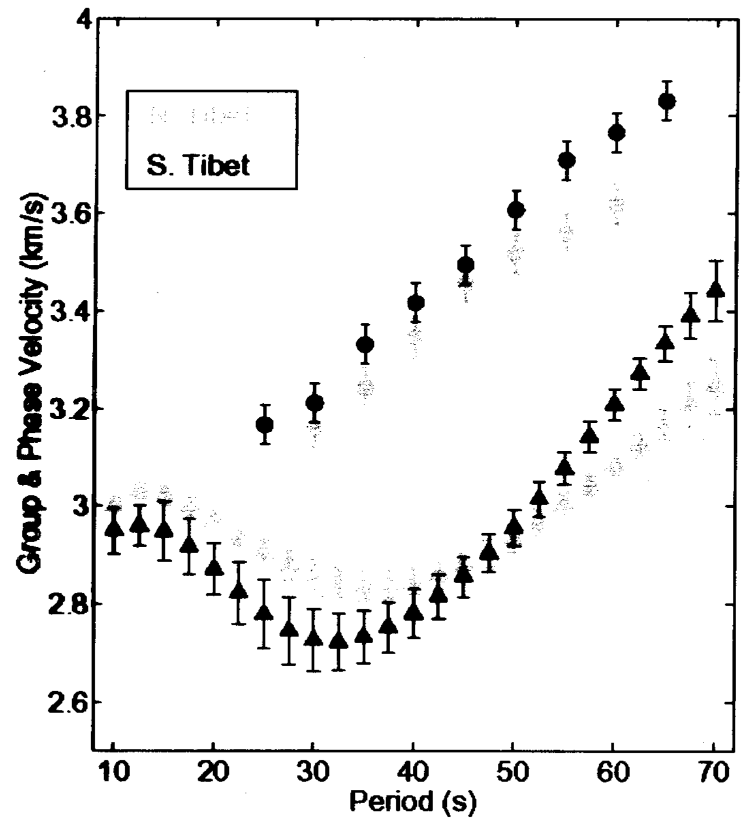
Figure 2.8-2: Fundamental mode Love wave group and phase velocities.



$$U = \frac{d\omega}{dk} = \frac{d(ck)}{dk} = c + k\,\frac{dc}{dk} = c - \lambda\,\frac{dc}{d\lambda}$$



a)
0.005
0.000
-0.005
-0.010
ST39 VERT
98/07/18
04:10:20.600
200
300
400
b)
Group Velocity (km/s)
5
4
3
4 5 7 10¹ 2 3 4 5 7 10²
Period(s)



S. Tibet
Group & Phase Velocity (km/s)
4
3.8
3.6
3.4
3.2
3
2.8
2.6
10 20 30 40 50 60 70
Period (s)

Fitting observations with a layered model…..

Sensitivity of phase velocity to the velocity of the medium at depth.

Relation with eigenfunctions. The case of Love waves

h is the eigenfunction of a mode in the reference medium : $\rho(z)$,$\mu(z)$,$\beta(z)$

$$h = h(\rho, \mu, k, \omega)$$

In a perturbed medium $(\rho + \delta\rho, \mu + \delta\mu)$:

$$h' = h + \delta h = h(\rho + \delta\rho, \mu + \delta\mu, k + \delta k, \omega)$$

We use stationarity properties of energy quantities to find the change of phase velocity associated with a small change of the properties of the medium.

The Lagrangian L = *kinetic energy* - *potential energy.*

For elastic motions:

$$L = \frac{\rho}{2}(\dot{u}_i)^2 - [\frac{\lambda}{2}(e_{kk})^2 + \mu e_{ij}e_{ij}]$$

Love wave:

$$u_2 = h(k, z, \omega)exp(i(kx - \omega t))$$

After integration over one cycle:

$$\langle L \rangle = \frac{1}{4}\rho\omega^2 h^2 - \frac{1}{4}\mu[k^2h^2 + (\frac{\partial h}{\partial z})^2]$$

Integrating L over z, we introduce the energy integrals I_i:

$$H = \int_0^\infty \langle L \rangle dz = \frac{1}{2}\{\omega^2 I_1 - k^2 I_2 - I_3\}$$

with

$$I_1 = \frac{1}{2}\int_0^\infty \rho(z) h^2(z) dz$$

$$I_2 = \frac{1}{2}\int_0^\infty \mu(z) h^2(z) dz$$

$$I_3 = \frac{1}{2}\int_0^\infty \mu(z) (\frac{\partial h}{\partial z})^2 dz$$

Properties of H

Equation of motion

$$\omega^2 \rho h + \frac{\partial}{\partial z}(\mu \frac{\partial h}{\partial z}) - k^2 \mu h = 0$$

$\times h$ et integration over z :

$$\int_0^\infty [\omega^2 \rho h^2 + h \frac{\partial}{\partial z}(\frac{\mu \partial h}{\partial z}) - k^2 \mu h^2] dz = 0$$

$$\int_0^\infty [\omega^2 \rho h^2 - k^2 \mu h^2 - \mu (\frac{\partial h}{\partial z})^2] dz + [h \mu \frac{\partial h}{\partial z}]_0^\infty = 0$$

$h \longrightarrow 0$ for $z \longrightarrow \infty$

$\frac{\partial h}{\partial z} = 0$ at the free surface

$$I_1 = \frac{1}{2} \int_0^\infty \rho(z) h^2(z) dz$$

$$I_2 = \frac{1}{2} \int_0^\infty \mu(z) h^2(z) dz$$

$$I_3 = \frac{1}{2} \int_0^\infty \mu(z) (\frac{\partial h}{\partial z})^2 dz$$

$$\omega^2 I_1 - k^2 I_2 - I_3 = 0$$

$H = \int_0^\infty \langle L \rangle dz = 0$ for an eigenfunction.

Properties of H

perturbation of h:

$$\delta H = \frac{1}{2}\{\omega^2 \delta I_1 - k^2 \delta I_2 - \delta I_3\}$$

$$\delta H = \int_0^\infty \omega^2 \rho(z) h(z) \delta h(z) dz \quad - \quad \int_0^\infty k^2 \mu(z) h(z) \delta h(z) dz$$
$$- \quad \int_0^\infty \mu(z) \frac{\partial h}{\partial z} \frac{\partial \delta h}{\partial z} dz$$

Integration by part and conditions for $z = 0$ and $z \to \infty$

$$\delta H = \int_0^\infty [\omega^2 \rho(z) h(z) - k^2 \mu(z) h(z) + \frac{\partial}{\partial z}(\mu(z) \frac{\partial h}{\partial z})] \delta h(z) dz$$

equation of motion $\rightarrow = 0$

$$\delta H = 0$$

$$\omega^2 \delta I_1 - k^2 \delta I_2 - \delta I_3 = 0$$

H is *stationary.*

$$I_1 = \frac{1}{2} \int_0^\infty \rho(z) h^2(z) dz$$

$$I_2 = \frac{1}{2} \int_0^\infty \mu(z) h^2(z) dz$$

$$I_3 = \frac{1}{2} \int_0^\infty \mu(z) (\frac{\partial h}{\partial z})^2 dz$$

Perturbation ~~of~~ h.

$$H = \tfrac{1}{2}(\omega^2 I_1 - k^2 I_2 - I_3) = 0$$

$$\delta H = \frac{1}{2}(\omega^2 \delta I_1 - k^2 \delta I_2 - \delta I_3)$$

$$I_1 = \frac{1}{2}\int \rho h^2 dz \Rightarrow \frac{\delta I}{\delta} = \frac{\partial I}{\partial h} \cdot \delta h(z) \text{ for } z.$$

$$\Rightarrow \delta I_1 = \frac{1}{2}\int \rho(z) 2 h(z) \delta h(z) dz$$

$$I_3 = \frac{1}{2}\int \mu \left(\frac{\partial h}{\partial z}\right)^2 dz$$

for z fixed: $\delta\left[\left(\frac{\partial h}{\partial z}\right)^2\right] = \delta f$

$$= \left(\frac{\partial(h+\delta h)}{\partial z}\right)^2 - \left(\frac{\partial h}{\partial z}\right)^2 = \left(\frac{\partial h}{\partial z} + \frac{\partial \delta h}{\partial z}\right)^2 - \left(\frac{\partial h}{\partial z}\right)^2$$

$$= \left(\frac{\partial h}{\partial z}\right)^2 + \left(\frac{\partial \delta h}{\partial z}\right)^2 + 2\frac{\partial h}{\partial z}\frac{\partial \delta h}{\partial z} - \left(\frac{\partial h}{\partial z}\right)^2.$$

2nd order. $\Rightarrow \sim 2\frac{\partial h}{\partial z}\frac{\partial \delta h}{\partial z}$

Perturbations: $\rho(z), \mu(z) \rightarrow \rho(z) + \delta\rho(z), \mu(z) + \delta\mu(z)$

$$h + \delta h = h(\rho + \delta\rho, \mu + \delta\mu, k + \delta k, \omega)$$

$H(h + \delta h) = 0$:

$$\begin{aligned} \omega^2 \int_0^\infty (\rho + \delta\rho)(h + \delta h)^2 dz &= (k + \delta k)^2 \int_0^\infty (\mu + \delta\mu)(h + \delta h)^2 dz \\ &+ \int_0^\infty (\mu + \delta\mu)(\frac{\partial(h + \delta h)}{\partial z})^2 dz \end{aligned} \tag{1}$$

Noting $H(h) = 0$ and neglecting the terms of second order as $\delta\mu\delta h$ or $(\delta h)^2$:

$$\begin{aligned} \omega^2 (\int_0^\infty (\delta\rho h^2 + 2\rho h \delta h) dz &= k^2 \int_0^\infty (\delta\mu h^2 + 2\mu h \delta h) dz \\ &+ 2k\delta k \int_0^\infty \mu h^2 dz + \int_0^\infty \delta\mu (\frac{\partial(h)}{\partial z})^2 dz \\ &+ \int_0^\infty 2\mu \frac{\partial h}{\partial z} \frac{\partial \delta h}{\partial z} dz \end{aligned}$$

Considering the stationarity of H:

$$\int_0^\infty \omega^2 \delta\rho h^2 dz = k^2 \int_0^\infty (\delta\mu h^2) dz + 2k\delta k \int_0^\infty \mu h^2 dz + \int_0^\infty \delta\mu (\frac{\partial(h)}{\partial z})^2 dz$$

$$\frac{\delta C}{C} = -\frac{\delta k}{k} = \frac{\int_0^\infty \delta\mu (k^2 h^2 + (\frac{\partial(h)}{\partial z})^2) dz - \int_0^\infty \delta\rho \omega^2 h^2 dz}{2k^2 \int_0^\infty \mu h^2 dz}$$

→ a relation between variation of phase velocity and perturbation of the medium: the base of linear inversion of dispersion curves.

Perturbation of ω -

$$\omega^2 I_1 - k^2 I_2 - I_3 = 0$$

$\omega \to \omega + \delta\omega$

$$2\omega I_1 + \omega^2 \frac{\delta I}{\delta\omega} - 2k \frac{dk}{d\omega} I_2$$

$$- k^2 \frac{\delta I_2}{\partial\omega} - \frac{\delta I_3}{\delta\omega} = 0$$

perturbation ω for I_1 = pertn in k.

$\to$ Stationarity*: $\omega^2 \delta I_1 - k^2 \delta I_2 - \delta I_3 = 0$

same k

$$\omega^2 \frac{\partial I_1}{\delta\omega} \delta\omega - k^2 \frac{\partial I_2}{\partial\omega} \delta\omega - \frac{\partial I_3}{\partial\omega} \delta\omega = 0$$

$$\Rightarrow \quad 2\omega I_1 - 2k \frac{\partial k}{\partial\omega} I_2 = 0$$

$$2\omega I_1 = 2k \frac{\partial k}{\partial\omega} I_2$$

$$\frac{\partial\omega}{\partial k} = \frac{k}{\omega} \frac{I_2}{I_1}$$

$$u = \frac{1}{c} \frac{I_2}{I_1}$$

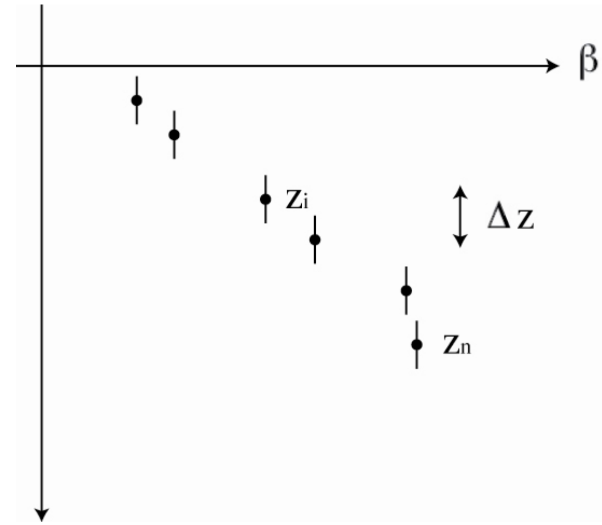
Note I_i depends only on h

Imaging (lithosphere $\rightarrow$ alluvium)

Measured phase velocity: $C_{obs}(T_j)$

Starting model $M_0(\beta\ (z_i))$

parametrization $z - \Delta z$..



$$\Rightarrow\ C_0\ (T) \quad , \quad \frac{\partial C_0(T)}{\partial \beta(z_i)}$$

$\rightarrow$ new model M_1

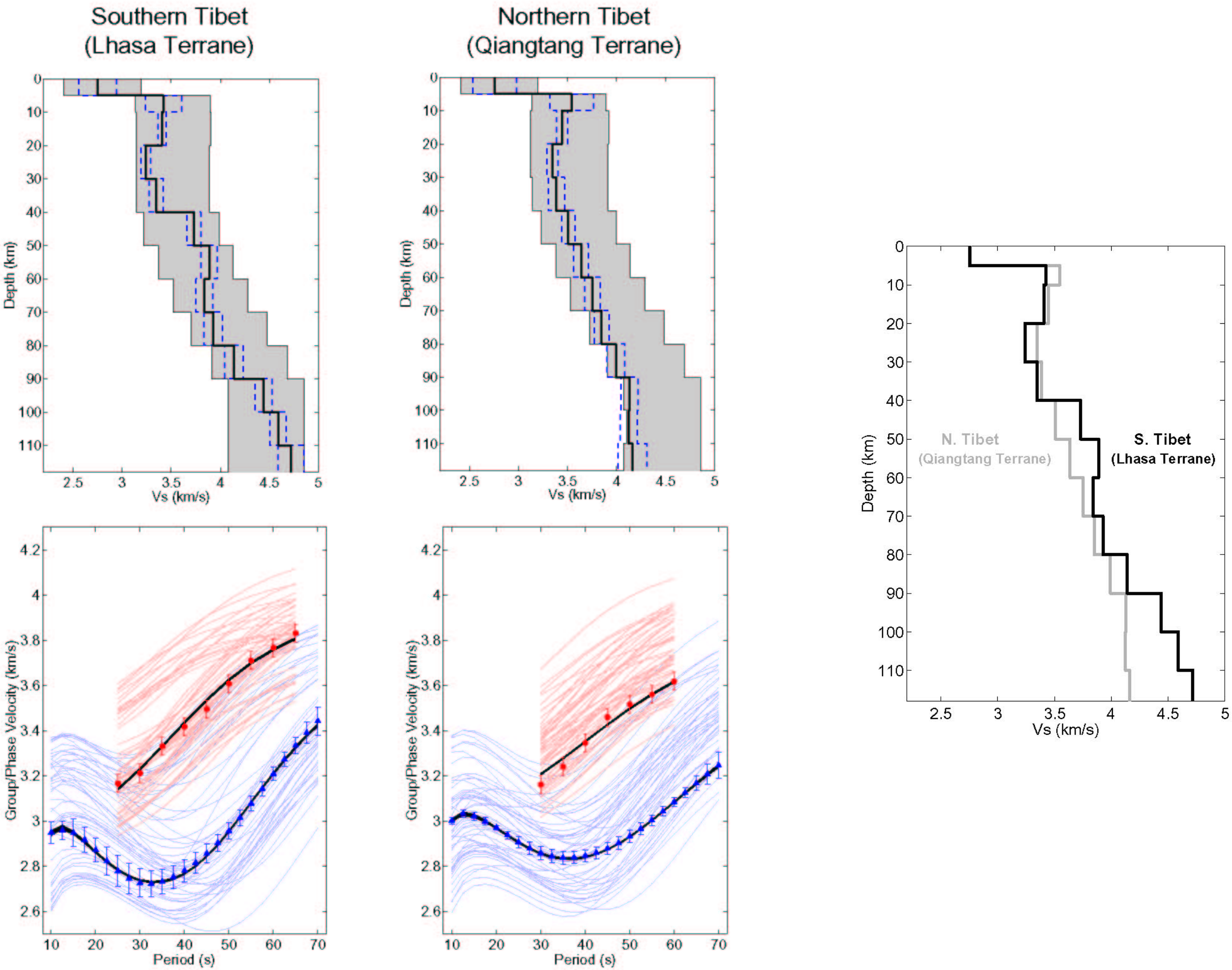
$$C_1(T) \ = \ C_0(T) \ + \ \sum_i \frac{\partial C_0(T)}{\partial \beta(z_i)} \qquad \delta\beta(z_i)$$

- Find $\delta\beta(z_i)$ such as $\sum_j \ \|C_1(T_j) - C_{obs}(T_j)\|^2$ is minimal

$\Rightarrow$ iteration $\Rightarrow$ $C_k(T)$, $\frac{\partial C_k}{\partial \beta(z_i)}$

- Find $\partial\beta(z_i)^k$ / $\sum_j \ \|C_k(T_j) - C_{obs}(T_j)\|^2$ minimal

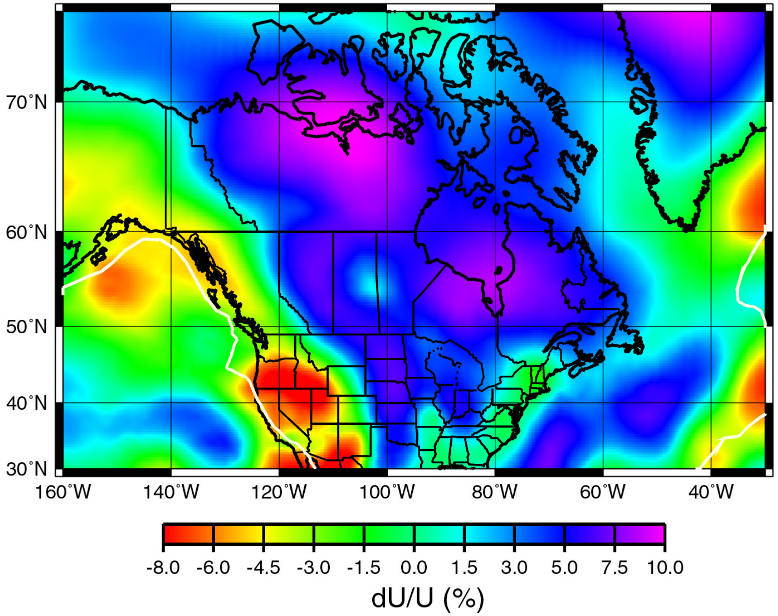
$\rightarrow$ model M_{k+1} $\rightarrow$ convergence



Southern Tibet
(Lhasa Terrane)
Northern Tibet
(Qiangtang Terrane)
Depth (km)
Vs (km/s)
Group/Phase Velocity (km/s)
Period (s)
N. Tibet
(Qiangtang Terrane)
S. Tibet
(Lhasa Terrane)

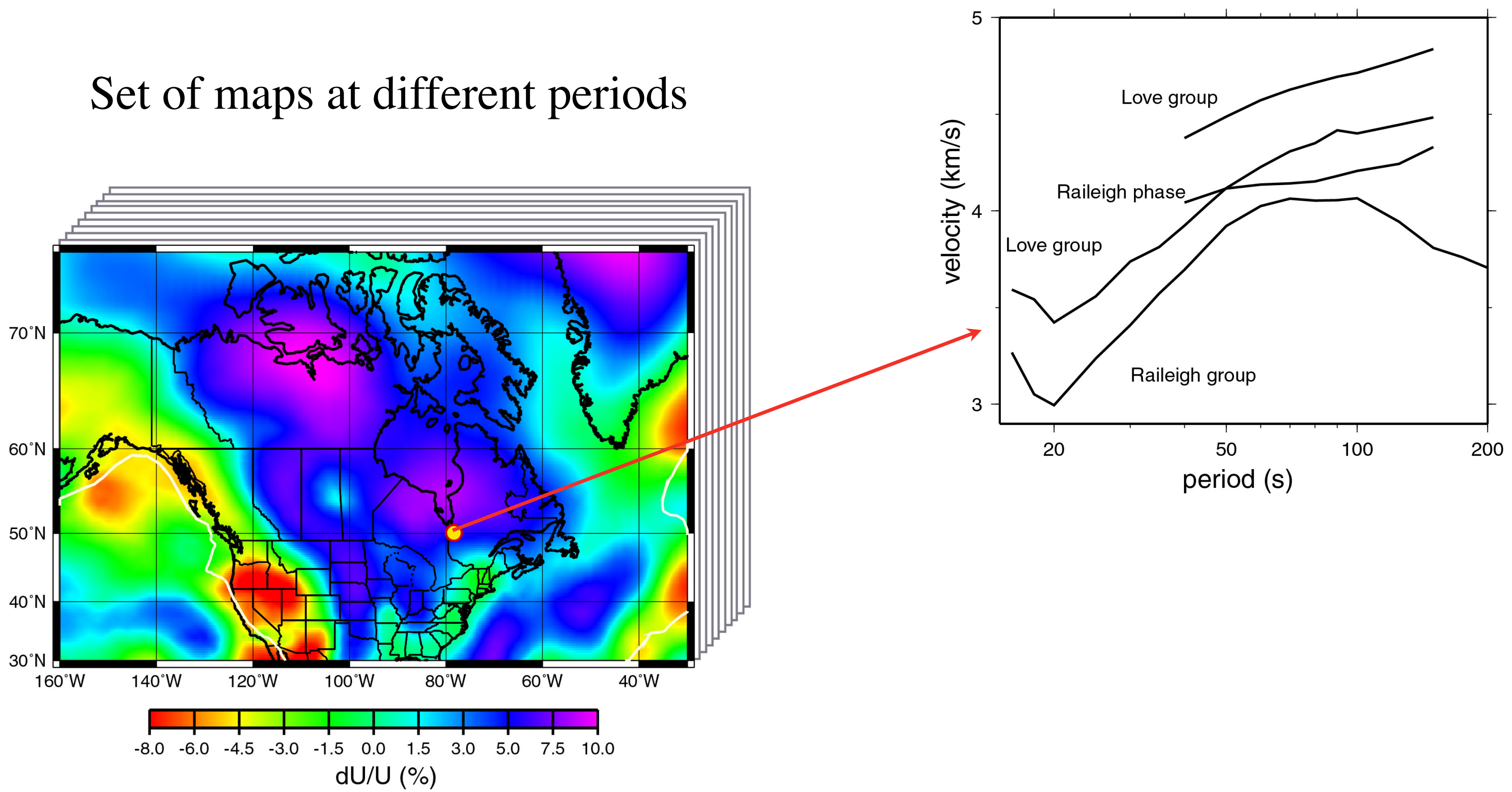
Dispersion map

Rayleigh group velocity (100 s)



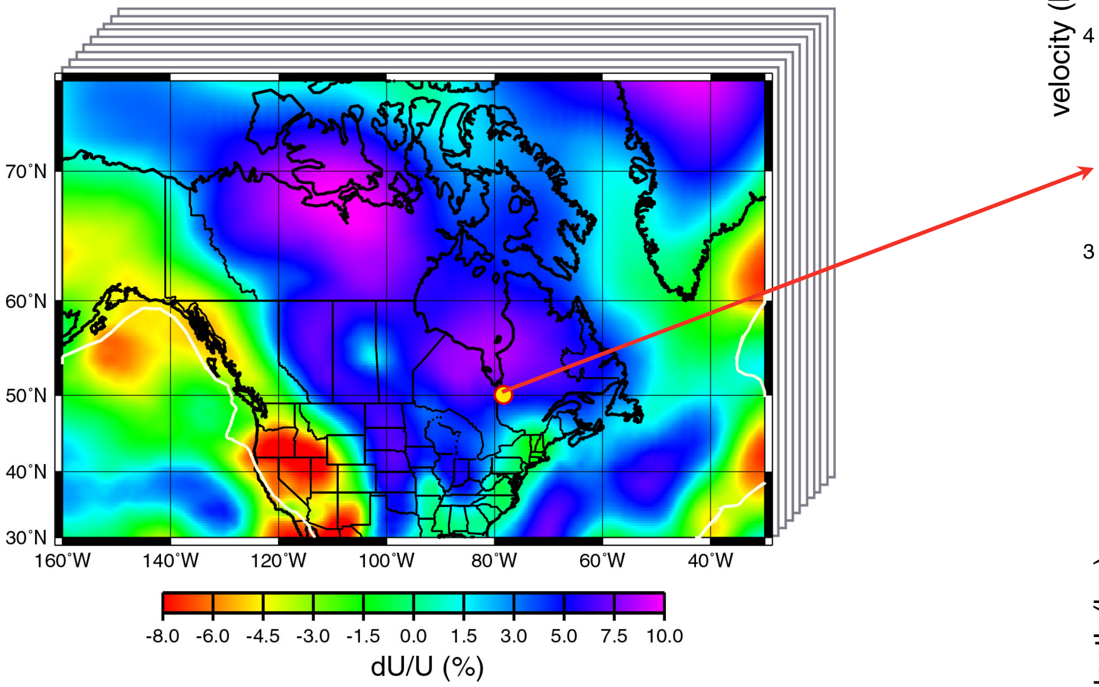
Local dispersion curves

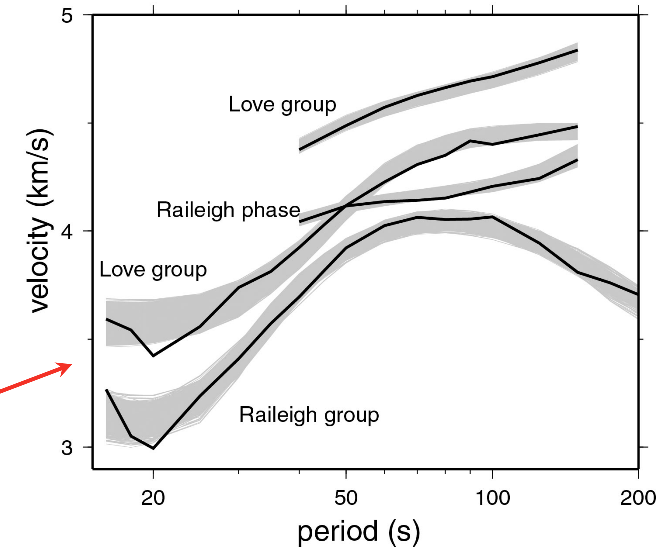
Set of maps at different periods



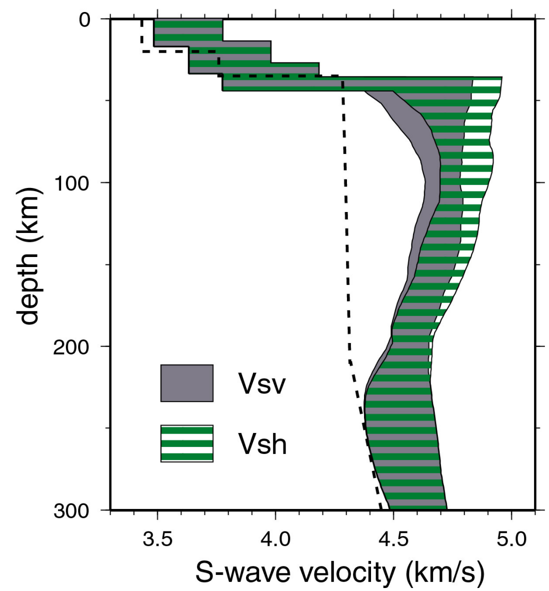
Inversion of dispersion curves

Set of maps at different periods

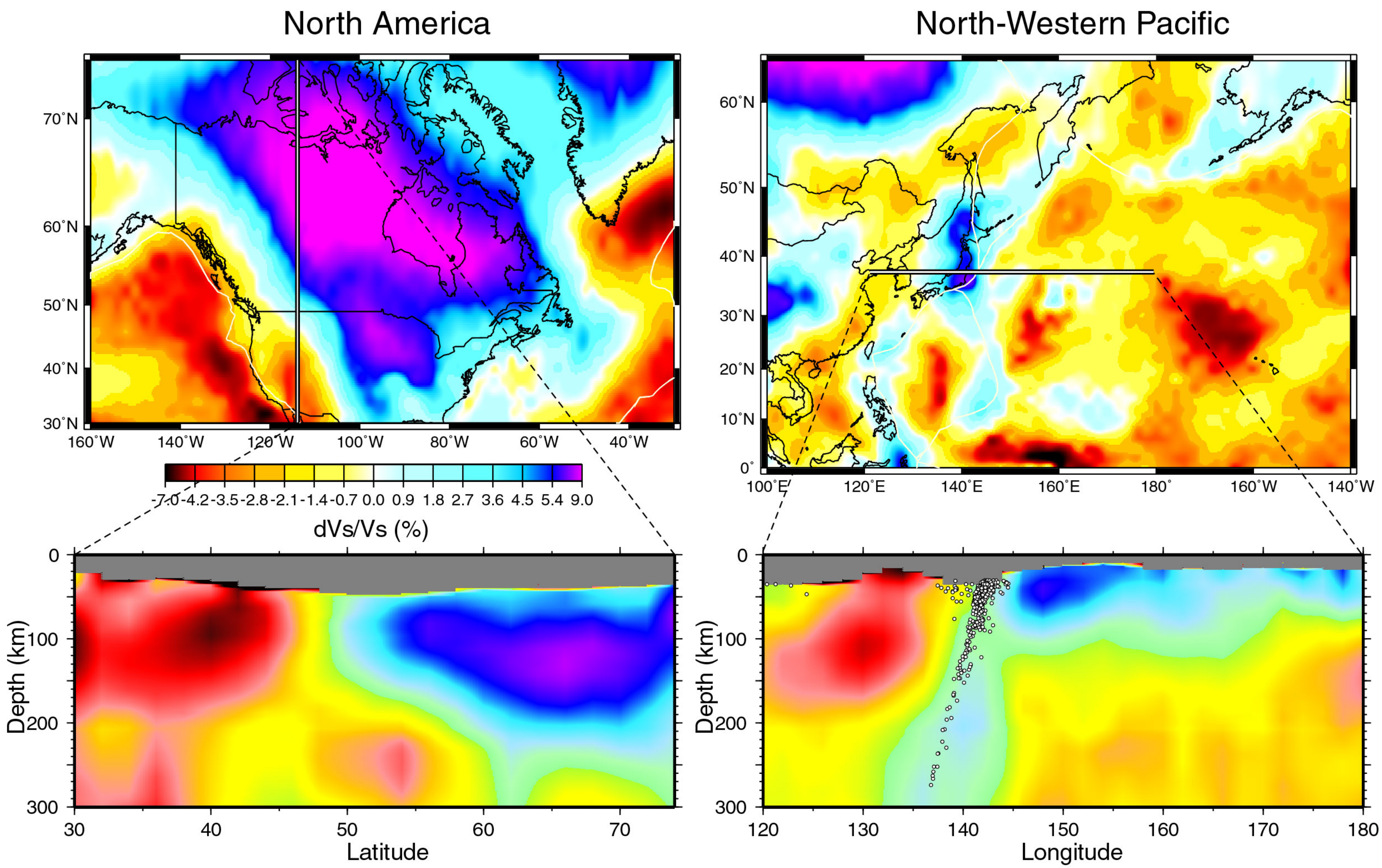




Local model



3D Vs Model


North America
North-Western Pacific
70°N
60°N
50°N
40°N
30°N
160°W
140°W
120°W
100°W
80°W
60°W
40°W
-7.0 -4.2 -3.5 -2.8 -2.1 -1.4 -0.7 0.0 0.9 1.8 2.7 3.6 4.5 5.4 9.0
dVs/Vs (%)
60°N
50°N
40°N
30°N
20°N
10°N
0°
100°E
120°E
140°E
160°E
180°
160°W
140°W
Depth (km)
0
100
200
300
30
40
50
60
70
Latitude
120
130
140
150
160
170
180
Longitude

Propagation in a weakly heterogeneous medium ($Ref.\ Aki\ et\ Richards$ 1980)

Reference medium $(\lambda_0, \mu_0, \rho_0)$
$\vec{u}$ displacement

$$\rho_0\ \ddot{u}_i = (\lambda_0 \vec{\nabla}.\vec{u})_{,i} + [\mu_0\ (u_{i,j} + u_{j,i})\ ]_{,j}$$

(same as: $\rho_0 \frac{\partial^2 \vec{u}}{\partial t^2} = (\lambda_0 + \mu_0)\ \vec{grad}\ (div\ \vec{u})\ + \mu_0 \vec{\Delta}.\vec{u})$

Perturbed medium
$\rho = \rho_0 + \delta\rho\ (\vec{r});\ \lambda = \lambda_0 + \delta\lambda\ (\vec{r});\ \mu = \mu_0 + \delta\mu\ (\vec{r})$
$\delta\rho\ ,\ \delta\lambda\ ,\ \delta\mu\ \ << \rho\ ,\ \lambda\ ,\ \mu$

Equation of motion:

$$\rho\ \ddot{u}_i = (\lambda \vec{\nabla}.\vec{u})_{,i} + [\mu\ (u_{i,j} + u_{j,i})\ ]_{,j}$$

$$(\rho_0 + \delta\rho)\ddot{u}_i = ((\lambda_0 + \delta\lambda)\vec{\nabla}.\vec{u})_{,i} + [(\mu_0 + \delta\mu)(u_{i,j} + u_{j,i})]_{,j}$$

$$\Rightarrow \;\rho_0\; \ddot{u}_i - \lambda_0(\vec{\nabla}.\vec{u})_{,i} - \mu_0(u_{i,j} + u_{j,i})_{,j} = \; - \;\delta\rho\; \ddot{u}_i \;+\; \delta\lambda(\vec{\nabla}.\vec{u})_{,i}$$

$$+\; \delta\lambda_{,i}\; \vec{\nabla}.\vec{u} \;+\; \delta\mu(u_{i,j} + u_{j,i})_{,j} \;+\; (\delta\mu)_{,j}(u_{i,j} + u_{j,i})$$

with $(u_{i,j} + u_{j,i})]_{,j} = \nabla^2 u_i + (\vec{\nabla}.\vec{u})_i$

$$\Rightarrow \;\rho_0\; \ddot{u}_i - (\lambda_0 \;+\; \mu_0)(\vec{\nabla}.\vec{u})_{,i} \;-\; \mu_0\, \nabla^2\, u_i \;=\; -\delta\rho\; \ddot{u}_i \;+\; (\delta\lambda + \delta\mu)(\vec{\nabla}.\vec{u})_{,i}$$

$$+\; \delta\mu\, \nabla^2\, u_i \;+\; (\delta\lambda)_i \vec{\nabla}.\vec{u} \;+\; (\delta\mu)_{,j}\; (u_{i,j} + u_{j,i})$$

$$\Rightarrow \; \rho_0 \, \ddot{u}_i - (\lambda_0 \; + \; \mu_0)(\vec{\nabla}.\vec{u})_{,i} \; - \; \mu_0 \, \nabla^2 \, u_i \; = \; -\delta\rho \, \ddot{u}_i \; + \; (\delta\lambda + \delta\mu)(\vec{\nabla}.\vec{u})_{,i}$$
$$+ \; \delta\mu \, \nabla^2 \, u_i \; + \; (\delta\lambda)_i \vec{\nabla}.\vec{u} \; + \; (\delta\mu)_{,j} \, (u_{i,j} + u_{j,i})$$

$$u \; = \; u^0 \; + \; u^d$$

$\rightarrow$ u^0 satisfies elastodynamic equation for $(\rho_0, \lambda_0, \mu_0)$

Hypothesis: weak perturbations

$\rightarrow$ neglect terms like $\delta\mu \times (u^d)'$ ($\delta\mu << \mu_0$; $u^d << u_0$)

(First order Born approximation)

$$\rho_0 \, \ddot{u}_i^d - (\lambda_0 \; + \; \mu_0)(\nabla u^d)_{,i} \; - \; \mu_0 \, \nabla^2 \, u_i^d \; = \; Q_i$$

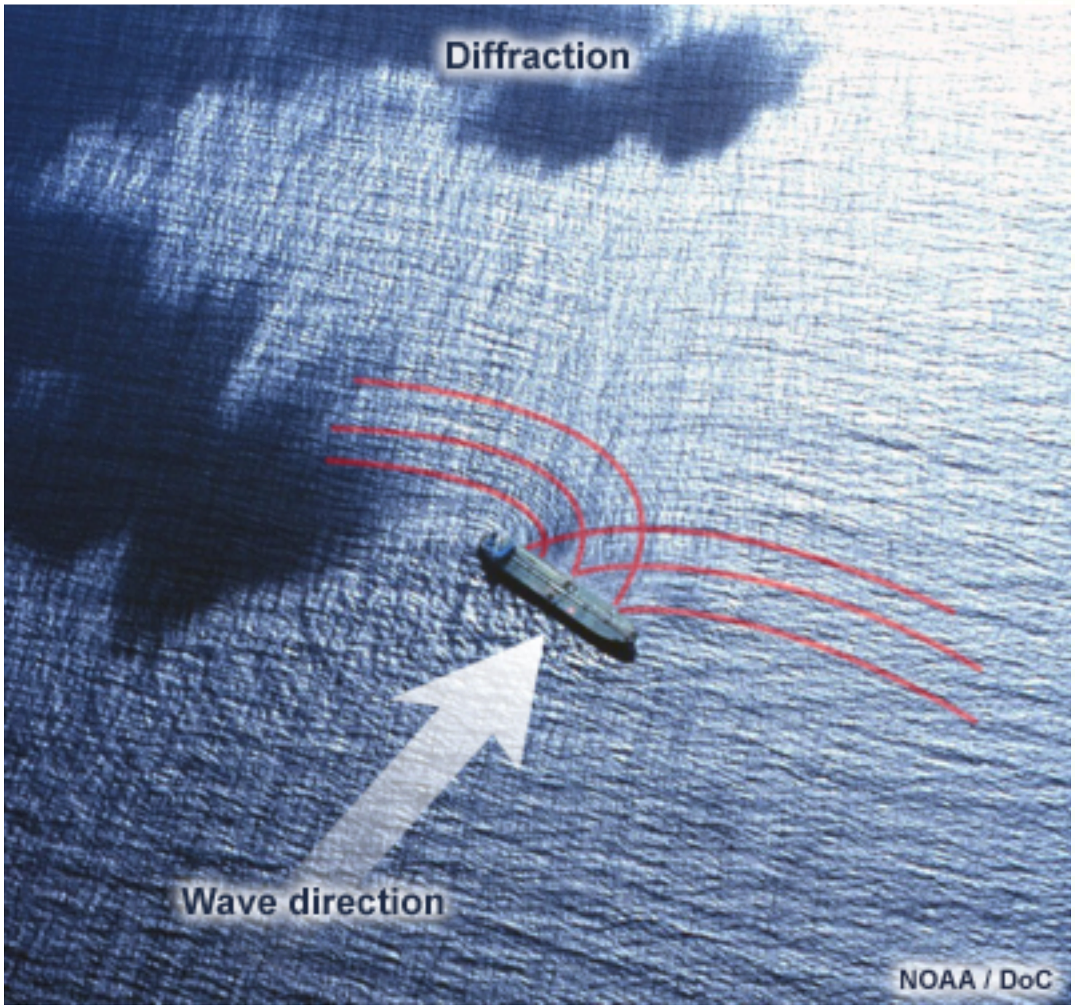
with :

$$Q_i = \; -\delta\rho \, \ddot{u}_i^o + (\delta\lambda + \delta\mu)(\vec{\nabla}.\vec{u}^o)_{,i} + \delta\mu \nabla^2 u_i^o + (\delta\lambda)_i \vec{\nabla}.\vec{u}^o + (\delta\mu)_{,j} \, (u_{i,j}^o + u_{j,i}^o)$$

$\Rightarrow$ extra source terms in the reference model

$\rightarrow$ diffraction = virtual sources

$\rightarrow$ base formula for linearized inverse problem



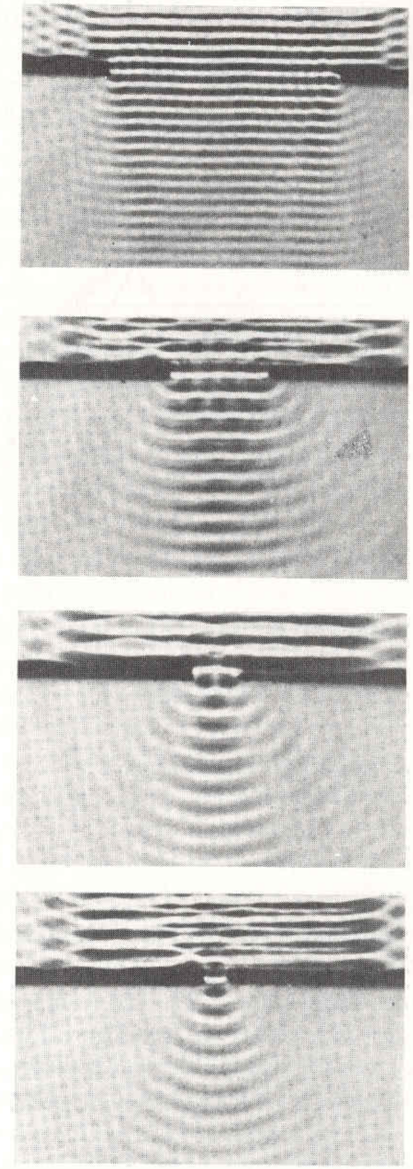


Figure 8.1 Ripples diffracted into quiet water by an opening.

Smooth medium

Exo SH Source term $f(\delta\mu, \delta\rho)$ and $f\left(\frac{\delta c}{c_0}\right)$

M_0

$$\rho_0 \frac{\partial^2 u_0}{\partial t^2} = \mu_0 \Delta u_0$$

$M = M_0 + \delta M$ $\quad \rho = \rho_0 + \delta\rho \,;\ \mu = \mu_0 + \delta\mu \,,\ u = u_0 + \delta u$

$$\rho \frac{\partial^2 u}{\partial t^2} = \mu \Delta u_0$$

$$(\rho_0 + \delta\rho) \frac{\partial^2 (u_0 + \delta u)}{\partial t^2} = (\mu_0 + \delta\mu) \Delta (u_0 + \delta u)$$

1st order! $(\delta .. \times \delta .. \to 0)$

$$\rho_0 \frac{\partial^2 u_0}{\partial t^2} + \rho_0 \frac{\partial^2 \delta u}{\partial t^2} + \delta\rho \frac{\partial^2 u_0}{\partial t^2} = \mu_0 \Delta u_0 + \mu_0 \Delta(\delta u) + \delta\mu \Delta u_0$$

$$\rho_0 \frac{\partial^2 \delta u}{\partial t^2} = \mu_0 \Delta \delta u + \underbrace{\delta\mu \Delta u_0 - \delta\rho \frac{\partial^2 u_0}{\partial t^2}}_{\text{source terme } = F}$$

$$F = \delta\mu \Delta u_0 - \delta\rho \frac{\partial^2 u_0}{\partial t^2} = (\delta\mu - \delta\rho\, c_0^2) \Delta u_0$$

$$\delta c = \frac{\partial c}{\partial \mu} \delta\mu + \frac{\partial c}{\partial \rho} \delta\rho = \frac{1}{2} \left(\frac{1}{\sqrt{\rho_0 \mu_0}} \delta\mu - \frac{\sqrt{\mu_0}}{\rho_0^{3/2}} \delta\rho \right)$$

$$= \frac{1}{2\rho_0} \left(\frac{1}{c_0} \delta\mu - c_0 \delta\rho \right) = \frac{1}{2\rho c_0} (\delta\mu - \delta\rho\, c_0^2)$$

$$\Rightarrow F = 2\rho_0 c_0\, \delta c\, \Delta u_0 = 2\mu_0 \frac{\delta c}{c_0} \Delta u_0$$

$$\Rightarrow \; \rho_0 \, \ddot{u}_i - (\lambda_0 \, + \, \mu_0)(\vec{\nabla}.\vec{u})_{,i} \; - \; \mu_0 \, \nabla^2 \, u_i \; = \; -\delta\rho \, \ddot{u}_i \; + \; (\delta\lambda + \delta\mu)(\vec{\nabla}.\vec{u})_{,i}$$
$$+ \, \delta\mu \, \nabla^2 \, u_i \; + \; (\delta\lambda)_i \vec{\nabla}.\vec{u} \; + \; (\delta\mu)_{,j} \; (u_{i,j} + u_{j,i})$$

$$u \; = \; u^0 \; + \; u^d$$

$\rightarrow$ u^0 satisfies elastodynamic equation for $(\rho_0, \lambda_0, \mu_0)$

Hypothesis: weak perturbations

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(First order Born approximation)

$$\rho_0 \, \ddot{u}_i^d - (\lambda_0 \, + \, \mu_0)(\nabla u^d)_{,i} \; - \; \mu_0 \, \nabla^2 \, u_i^d \; = \; Q_i$$

with :

$$Q_i = \; -\delta\rho \, \ddot{u}_i^o \, + \, (\delta\lambda{+}\delta\mu)(\vec{\nabla}.\vec{u}^o)_{,i} \, + \, \delta\mu \nabla^2 u_i^o \, + \, (\delta\lambda)_i \vec{\nabla}.\vec{u}^o \, + \, (\delta\mu)_{,j} \, (u_{i,j}^o {+} u_{j,i}^o)$$

$\Rightarrow$ extra source terms in the reference model

$\rightarrow$ diffraction = virtual sources

$\rightarrow$ base formula for linearized inverse problem

DIFFRACTION OF P WAVES :

Primary wave propagating in direction x_1 :

$$u_i^o = \delta_{1i} \;\exp\left(-i\;\omega\;(t - \frac{x_1}{\alpha_0})\right)$$

$$\alpha_0 = \left(\frac{\lambda_0 + 2\mu_0}{\rho_0}\right)^{1/2}$$

$$\Rightarrow$$

$$Q_1 = \left\{\delta\rho\;\omega^2 \;-\; \frac{(\delta\lambda + 2\delta\mu)\omega^2}{\alpha_0^2} \;+\; i\;\frac{\omega}{\alpha_0}(\delta\lambda)_{,1} \;+\; 2i\;\frac{\omega}{\alpha_0}(\delta\mu)_{,1}\right\}\exp(-i\omega(t-\frac{x_1}{\alpha_0})$$

$$Q_2 \;=\; i\;\frac{\omega}{\alpha_0}(\delta\lambda)_{,2} \;\;\exp(-i\omega(t - \frac{x_1}{\alpha_0})$$

$$Q_3 \;=\; i\;\frac{\omega}{\alpha_0}(\delta\lambda)_{,3} \;\;\exp(-i\omega(t - \frac{x_1}{\alpha_0})$$

Identification :

- velocity fluctuation :

$$\alpha = \frac{(\lambda + 2\mu)^{1/2}}{\rho^{1/2}} \qquad \Rightarrow \delta\alpha = \frac{\partial\alpha}{\partial\lambda}\partial\lambda \ + \ \frac{\partial\alpha}{\partial\mu}\partial\mu \ + \ \frac{\partial\alpha}{\partial\rho}\partial\rho$$

$$\delta\alpha = \frac{1}{2} \ \frac{1}{\rho} \ \frac{1}{\alpha}\delta\lambda \ + \ \frac{1}{2} \ \frac{2}{\rho} \ \frac{1}{\alpha}\delta\mu \ - \ \frac{(\lambda + 2\mu)}{\rho^2} \ \frac{1}{2} \ \frac{1}{\alpha}\delta\rho$$

$$\frac{\delta\alpha}{\alpha} = \frac{1}{2}\left(\frac{\delta\lambda + 2\delta\mu}{\lambda + 2\mu} - \frac{\delta\rho}{\rho}\right)$$

$\longrightarrow$ For Q_1 :

$$\delta \ \rho \ \omega^2 \ - \ \frac{(\delta\lambda + 2\delta\mu)}{\alpha_0}\omega^2 \ = \ -\omega^2 \ \rho_0 \ \left(2 \ \frac{\delta\alpha}{\delta_0}\right)$$

$\longrightarrow$ force q_1 proportional to the velocity perturbation:

$$q_1 \ = \ -2 \ \omega^2 \ \rho_0 \ \frac{\delta\alpha}{\alpha_0} \ \exp\left(-\omega \ (t \ - \frac{x_1}{\alpha_0})\right)$$

(simple force dans la direction de propagation)

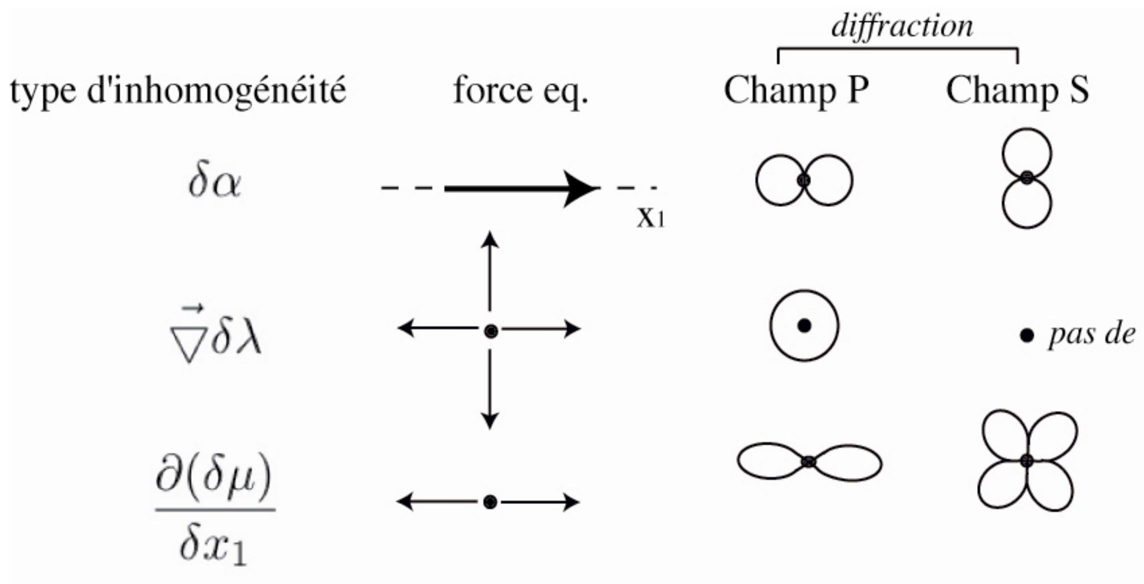
• term proportional to the gradient of λ

$$q' = i \frac{\omega}{\alpha_0} \vec{\nabla} . \delta\lambda$$

• terme proportional to the spatial variation of μ

$$q''_1 = 2i \frac{\omega}{\alpha_0} (\delta\mu)_{,1}$$

For a point perturbation :



• No scattered S wave in the forward direction
• scattered P wave maximum along $x_1 \rightarrow$ backscattering

DIFFRACTION OF S WAVE: $u_i^o = \delta_{2i} \ \exp\left(-i\omega\ (t - \frac{x_1}{\beta_o})\right)$

$\Rightarrow$

- term proportional to the velocity perturbation:

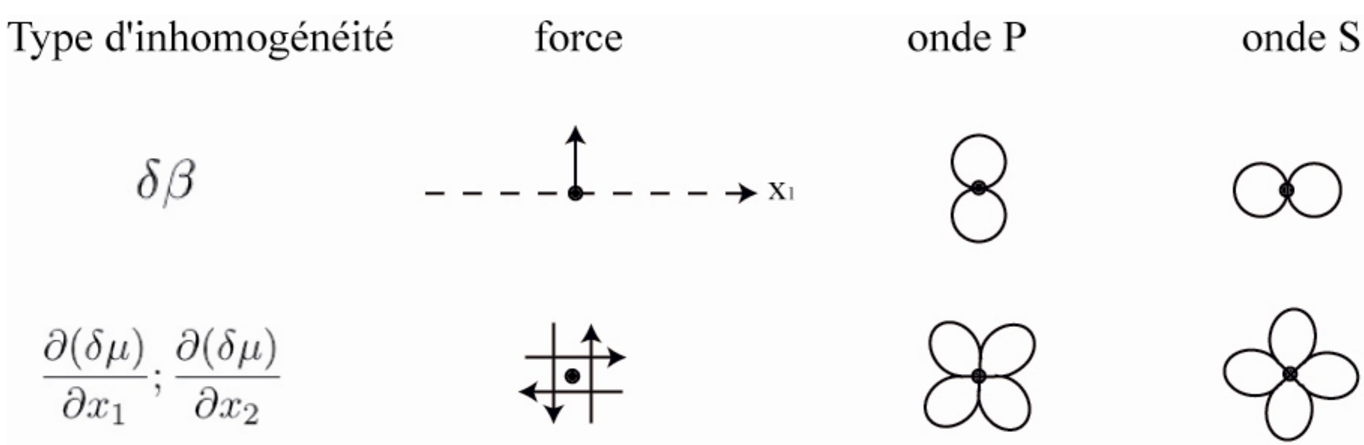
$$q_2 = -2\omega^2 \rho_o \frac{\delta\beta}{\beta_o} \exp\left(-i\omega\ (t - \frac{x_1}{\beta})\right)$$

- term proportional to the variation of $\delta\mu$:

$$q_1' = i\ \frac{\omega}{\beta_o}\ (\delta\mu)_{,2}\ \exp\left(-i\omega\ (t - \frac{x_1}{\beta})\right)$$

$$q_2' = i\ \frac{\omega}{\beta_o}\ (\delta\mu)_{,1}\ \exp\left(-i\omega\ (t - \frac{x_1}{\beta})\right)$$

Point perturbation:



- No scattered P along x_1

- P to S coupling via multiple scattering