Topographic coupling in a stratified layer at the top of the core

GreZuMarBru Workshop 2021

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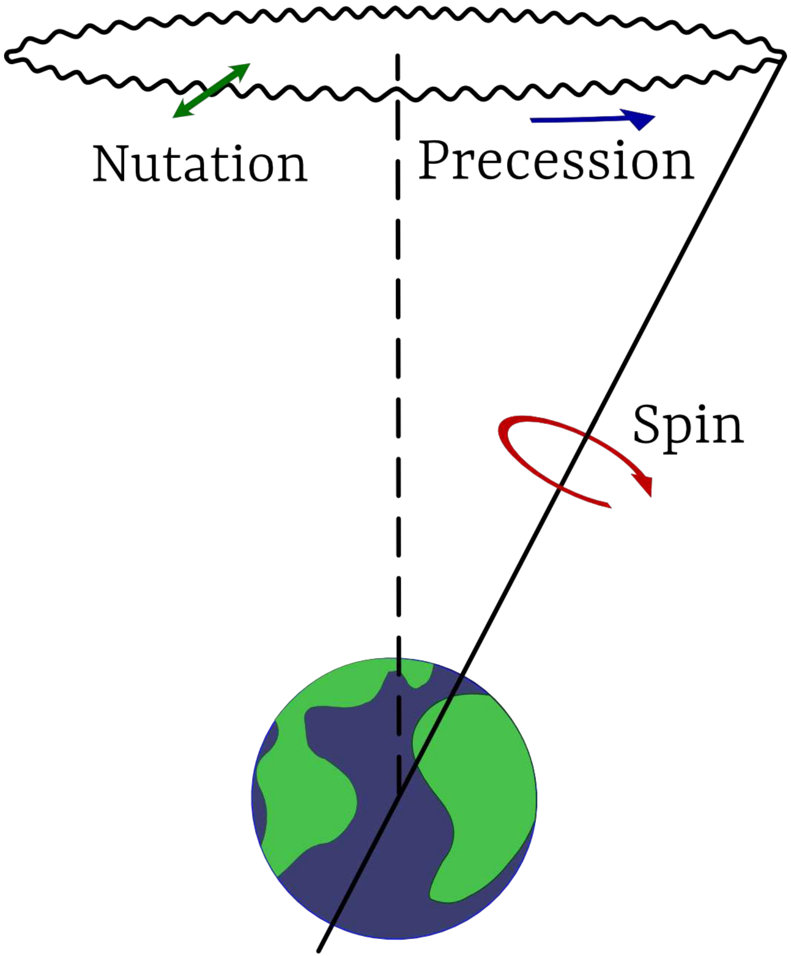
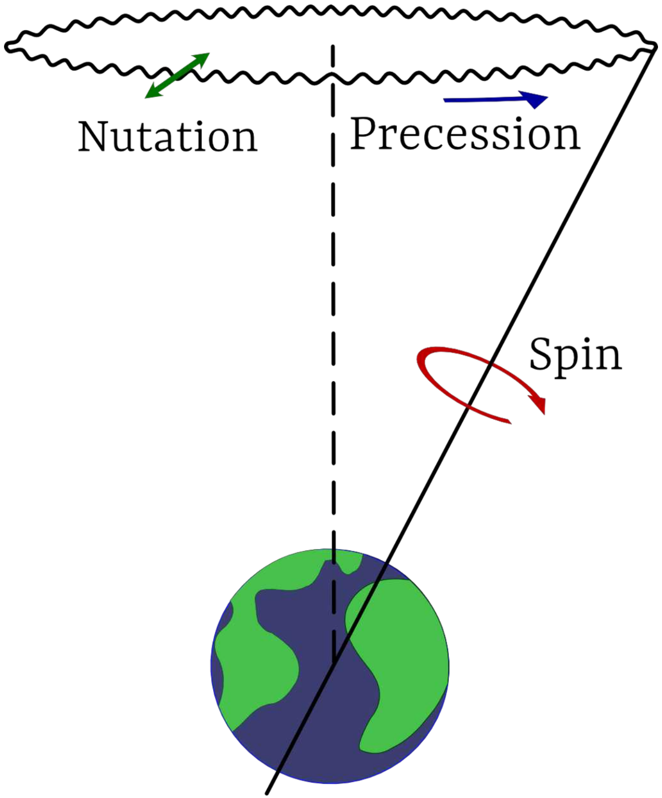
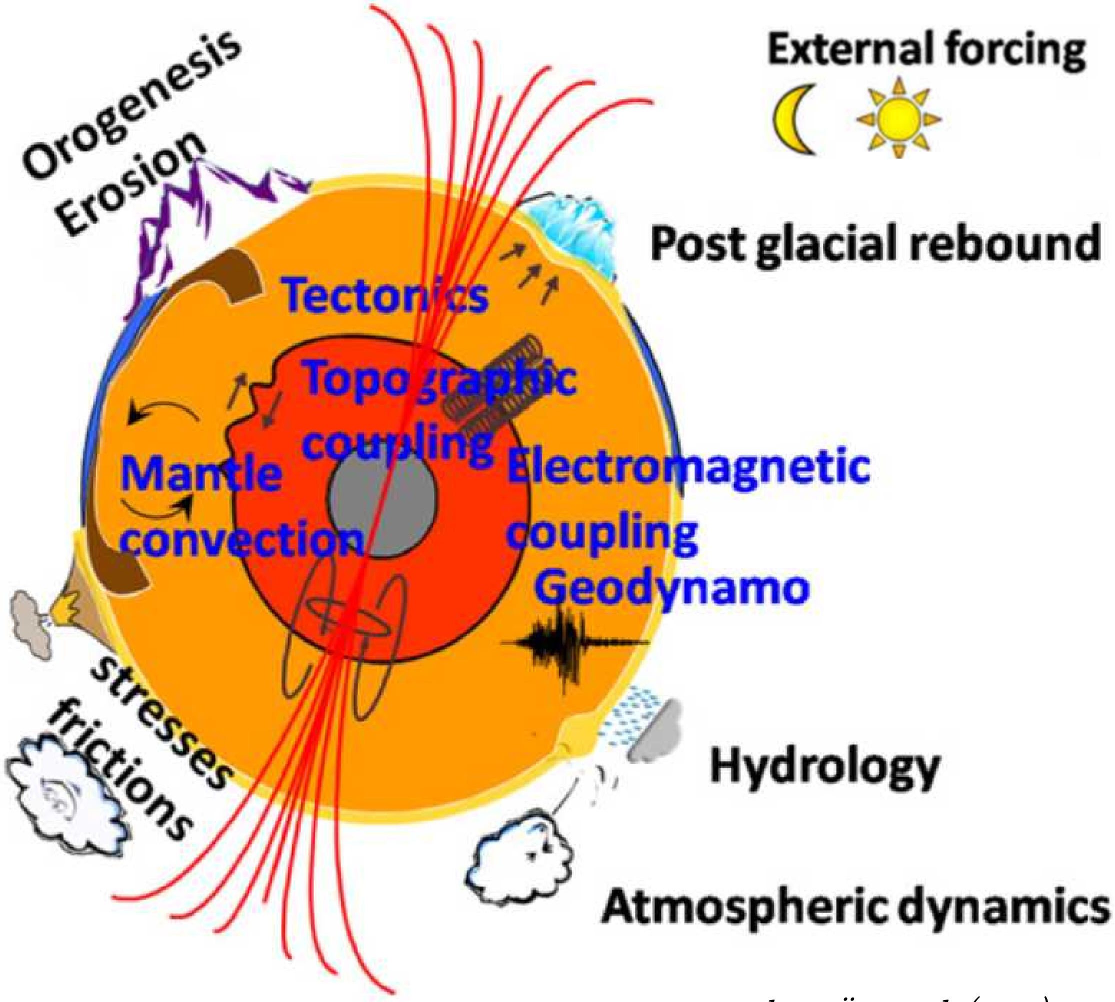
Variations of the Earth’s rotation



Variations of the Earth's rotation



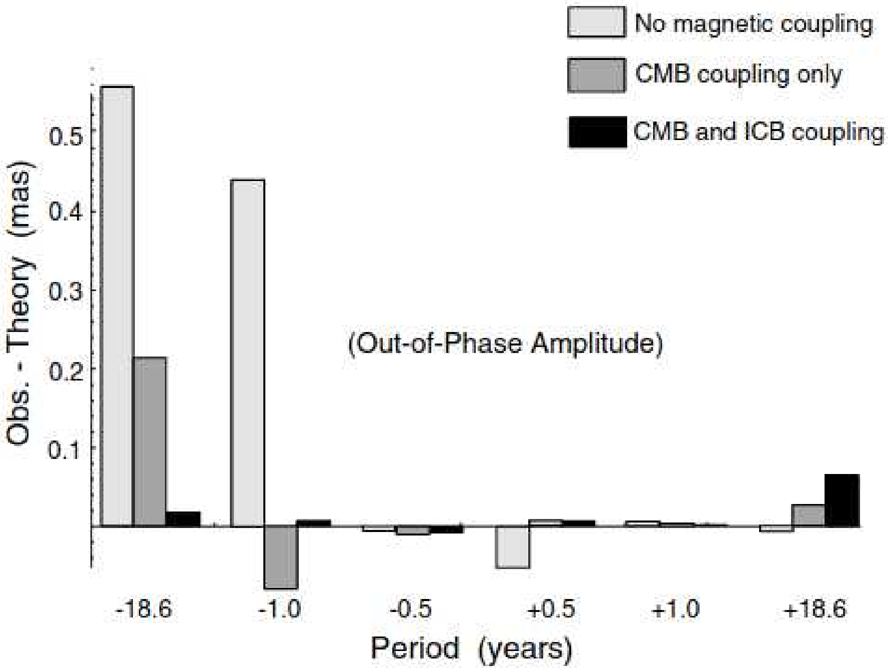


Karatekin, Ö. et al. (2011)

Exchanges of angular momentum

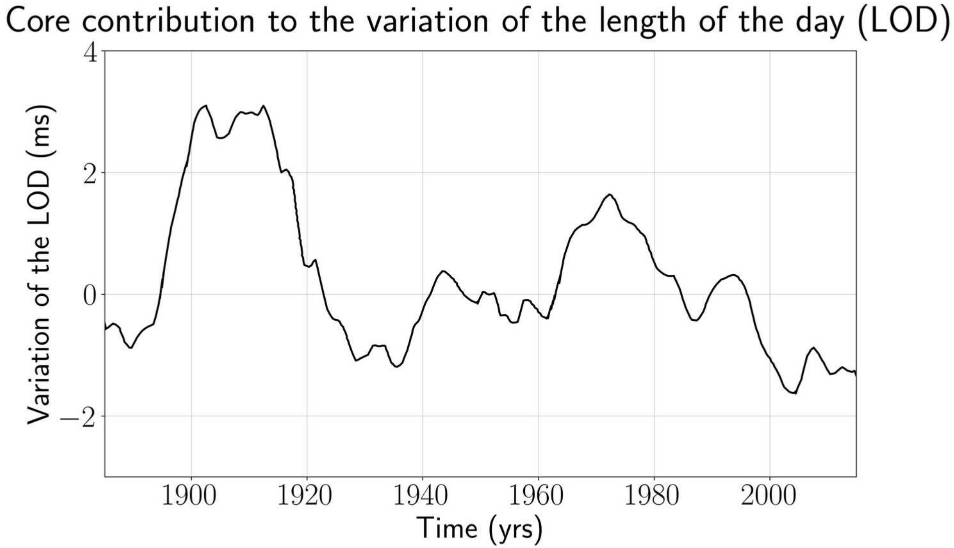
Residuals in :

Nutation



Residual of Earth's nutation, and correction by Buffett (2002)

Lenght of the day



Gillet, et al. (2019).

Exchanges of angular momentum

Possible core-mantle coupling mechanisms:

- **Viscous**
- **Gravitational**
- **Electromagnetic**
- **Pressure torque on small scale topography**

Exchanges of angular momentum

Possible core-mantle coupling mechanisms:

- ~~Viscous~~ → Too weak in the Earth's core : min. 100x smaller than other torques
- Gravitational
- Electromagnetic
- Pressure torque on small scale topography

Exchanges of angular momentum

Possible core-mantle coupling:

- ~~Viscous~~ → Too weak in the Earth core : min. 100x smaller than other torques
- **Gravitational**
- **Electromagnetic**
- **Pressure torque on small scale topography**

Topographic couplings

$$\Gamma_B = \frac{1}{\mu} \int (\boldsymbol{n} \cdot \boldsymbol{B})(\boldsymbol{r} \times \boldsymbol{B}) dS$$

$$\Gamma_p = \int (\boldsymbol{r} \times \boldsymbol{n}) p dS$$

Motivations

Can the **small scale** topographic coupling explain:

- The decadal changes in the **Length-of-Day** *(Glane and Buffett 2018, Jault 2020)* ?
- The out of phase component of the retrograde annual **nutation** of the Earth's rotation axis *(Buffett 2002,2010)* ?

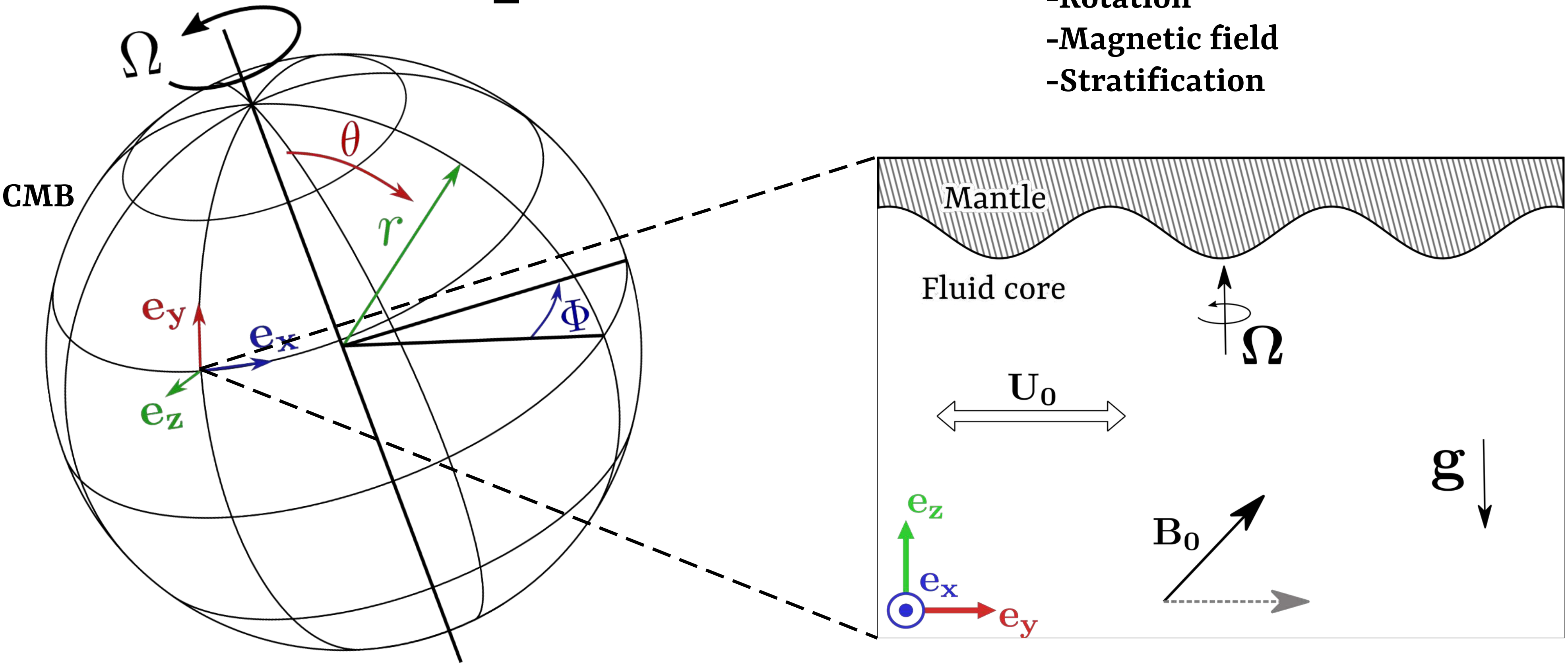
How well can a **local perturbative model** help us to understand these measurements, and what are its limitations ?

Geometry of the problem

Spherical shell → Cartesian frame

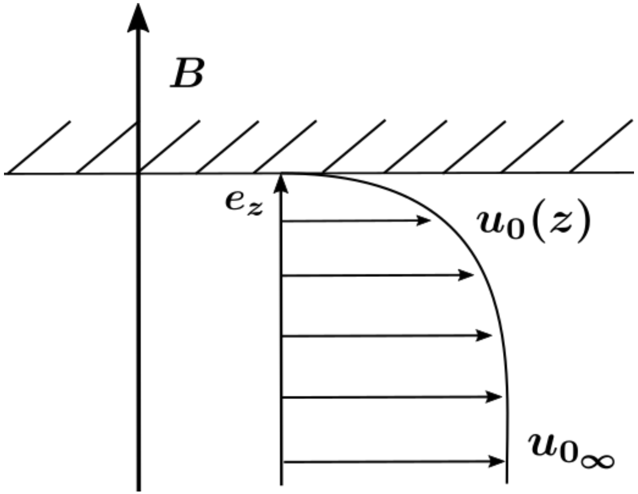
Key effects:

-Rotation
-Magnetic field
-Stratification



Base State

Solution of topography unperturbed base flow is an Hartmann flow :



An Hartmann layer

$$u_0 = u_{0\infty}(1 - e^{\frac{z}{\delta}})$$

Depends on position (θ, Φ)

In first approximation we considered a uniform flow with stress-free condition $u_0 = u_{0\infty}$

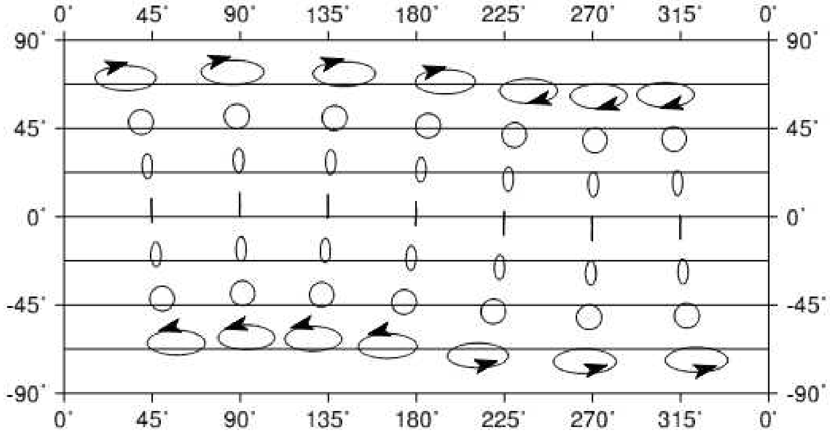
Base State

Nutation/Precession differential velocity between core and mantle:

$\boldsymbol{\omega} \times \boldsymbol{r}$ → global to local velocity

Equatorial rotation

$$\boldsymbol{u_{0\infty}} = \Re(\Omega \tilde{m} r [I \boldsymbol{e_y} - \cos\theta \boldsymbol{e_x}] e^{I\Phi} e^{I\Omega t})$$



Trajectories of particles at the CMB (from Pais & Le Mouël (2001))

Base State

Magnetic field:

Dipolar magnetic
magnetic field of equation

$$\mathbf{B_0} = b_0 \left(\cos\left(\Phi\right)\mathbf{e_y} - 2\sin\left(\Phi\right)\mathbf{e_z}\right)$$

Stratification:

Stable and linear
density profile

$$\rho = \rho_r(1 - \alpha z)$$

Equations of motion

$$D_t\mathbf{U} = \frac{2}{R_o}(\sin\theta\boldsymbol{e_y} + \cos\theta\boldsymbol{e_z}) \times \mathbf{U} - \nabla p + \frac{1}{R_e}\nabla^2\mathbf{U} - \frac{\rho}{F_r^2}\boldsymbol{e_z} + (\mathbf{B}\cdot\nabla)\mathbf{B},$$

$$\partial_t\rho + (\mathbf{U}\cdot\nabla)\rho = 0,$$

$$\partial_t\mathbf{B} = \frac{1}{Rm}\nabla^2\mathbf{B} + \nabla\times(\mathbf{U}\times\mathbf{B}),$$

$$\nabla\cdot\mathbf{U} = 0, \nabla\cdot\mathbf{B} = 0,$$

Magneto-hydro-dynamic equations (MHD), in Boussinesq approximation

Methods

Solving equations with perturbation approach

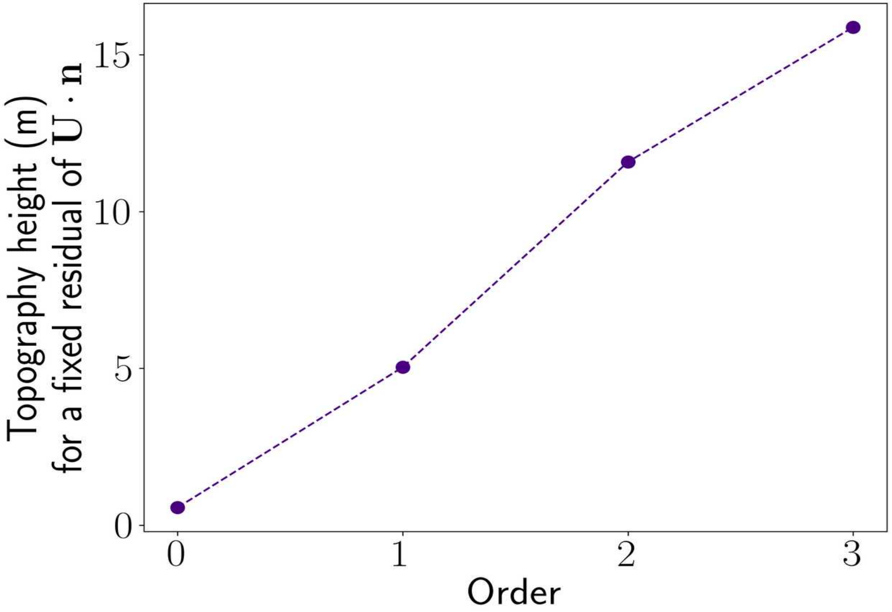
Limited by a small parameter

→ Topography height

Glane and Buffett (2018) : ~30-50m

Buffett (2010) : ~100m

Higher orders of perturbation



Quasi linear variation when the serie is convergent

Methods

Improvement provided by weakly non linear approach

- Increase of the topography height / decrease error for a given topography
- Determine the limits of the linear approach and study the convergence of the perturbative model
- Compute the stresses on the topography in a consistent way –> stresses were previously computed at order 2 with order 1 flows

Equations of motion

example of induction

$$\mathbf{U} = \mathcal{U}\mathbf{u_0} + \sum_{k=1}^{\infty} \epsilon_t^k \mathbf{u}_k,$$

$$\mathbf{B} = \frac{1}{A_l}\mathbf{b_0} + \sum_{k=1}^{\infty} \epsilon_t^k \mathbf{b}_k,$$

Order 1 : $$\partial_t \mathbf{b_1} = \frac{1}{Rm}\nabla^2 \mathbf{b_1} + \frac{1}{A_l}(\nabla \times (\mathbf{u_1} \times \mathbf{b_0})) + \mathcal{U}(\nabla \times (\mathbf{u_0} \times \mathbf{b_1}))$$

Equations of motion

example of induction

$$\mathbf{U} = \mathcal{U}\mathbf{u_0} + \textstyle\sum_{k=1}^{\infty} \epsilon_t^k \mathbf{u}_k,$$

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Order 1 :

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Order 2 :

$$\partial_t \mathbf{b_2} = \frac{1}{Rm}\nabla^2 \mathbf{b_2} + \frac{1}{A_l}(\nabla \times (\mathbf{u_2} \times \mathbf{b_0})) + \mathcal{U}(\nabla \times (\mathbf{u_0} \times \mathbf{b_2})) + (\nabla \times (\mathbf{u_1} \times \mathbf{b_1}))$$

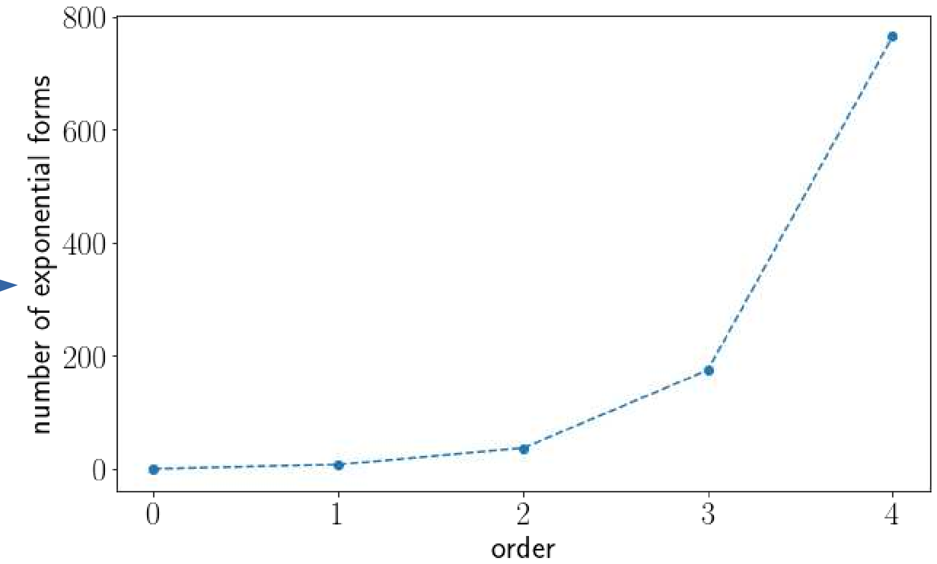
Non linear terms

Equations of motion

Equations and boundary conditions → spatial form of variables :

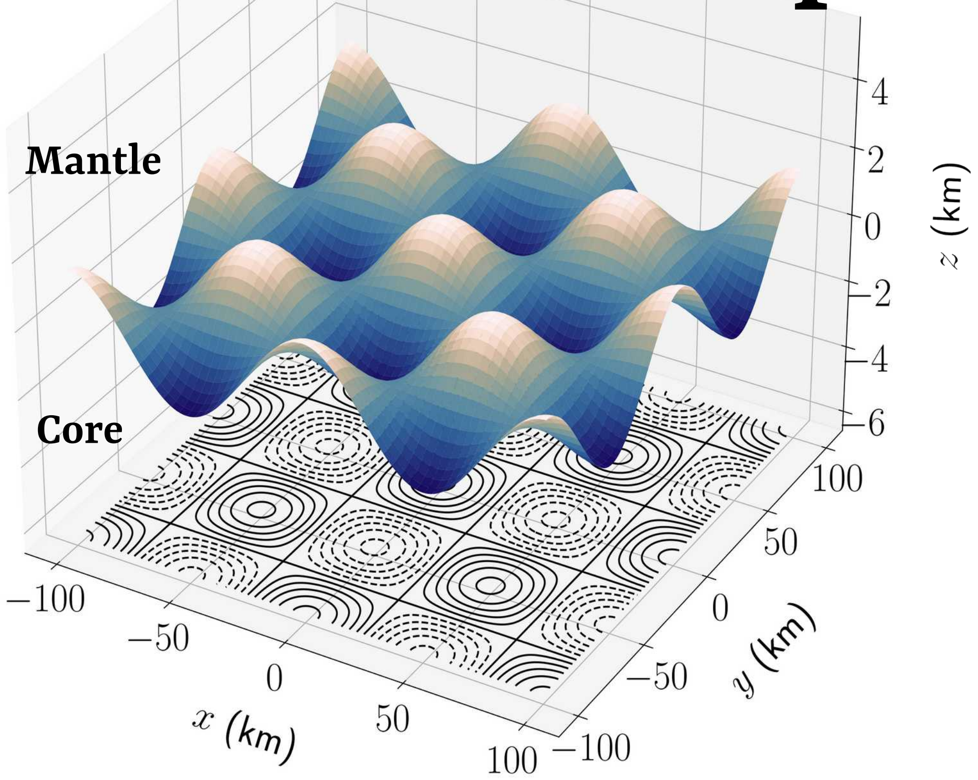
$$u = \sum_{0}^{n} u_n e^{k_x^n x + k_y^n y + k_z^n z}$$

Non linear terms



exponential growth of numbers of exponential forms

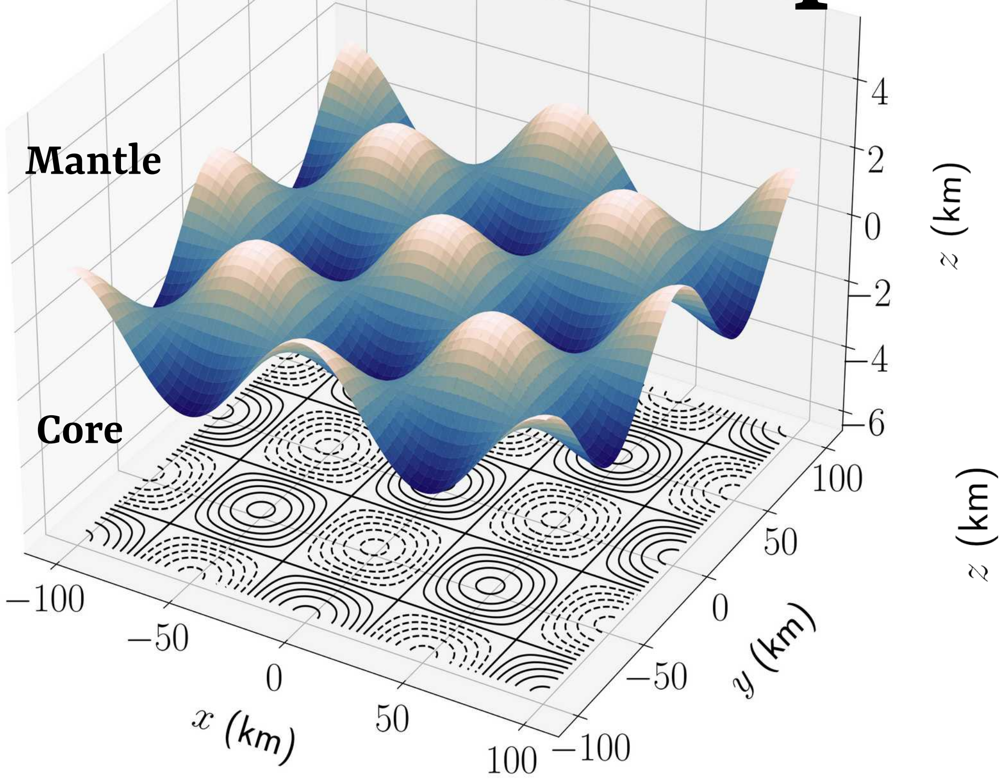
Topography shape



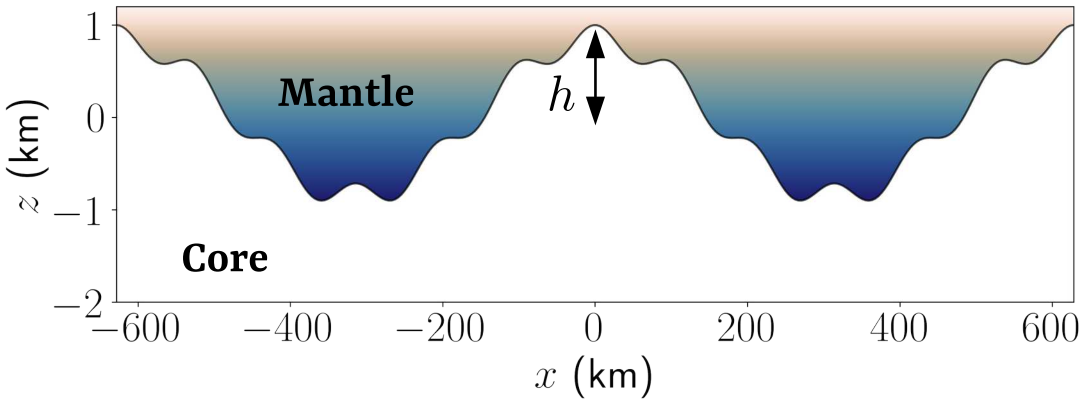
$$h(x, y) = \epsilon_t \sum_{j=0}^{n} \Re \left(\exp \left(\mathbf{i} \mathbf{k_j} \cdot \mathbf{r} \right) \right) / n,$$

$\epsilon_t \ll 1$ is the topography height divided by a typical length scale

Topography shape



$$h(x,y) = \epsilon_t \sum_{j=0}^{n} \Re\left(\exp\left(\mathbf{i}\mathbf{k_j}\cdot\mathbf{r}\right)\right)/n,$$



Methods

2 steps

Derive the equations

Solve systems

Methods

2 steps

Derive the equations

Solve systems

Sympy : symbolic mathematics

- Differentiation
- Taylor series
- Determinant of symbolic matrix

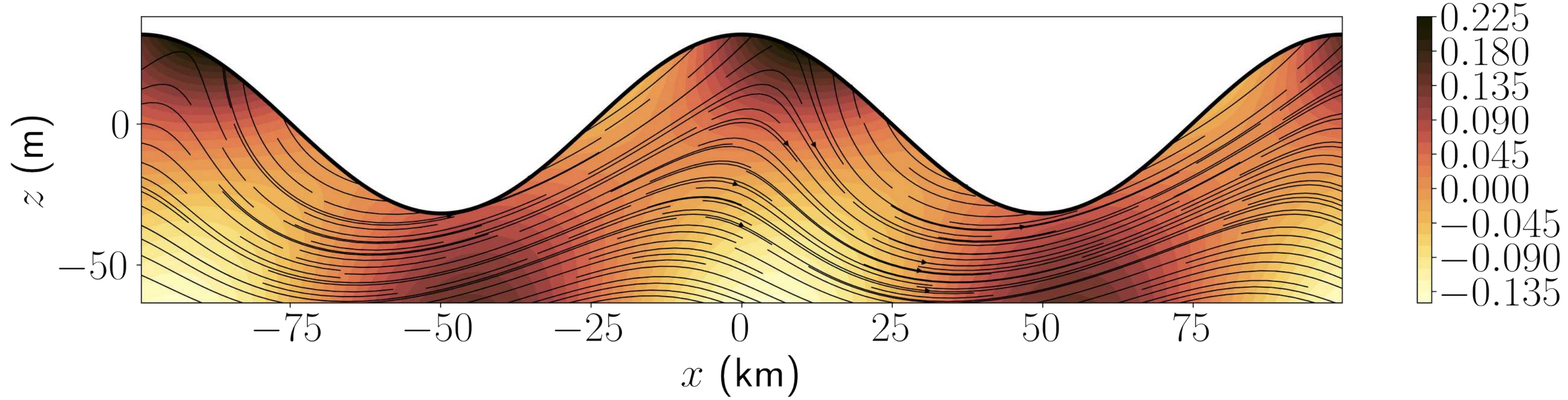
python

arithmetic with arbitrary precision

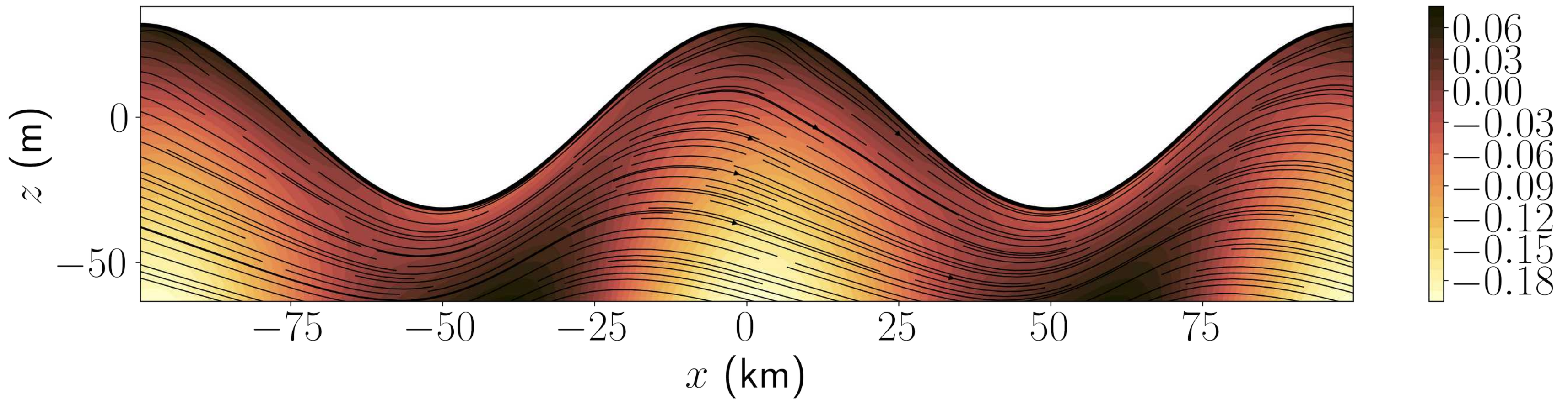
- Singular value decomposition
- Roots of polynomials

My code : **ToCCo**

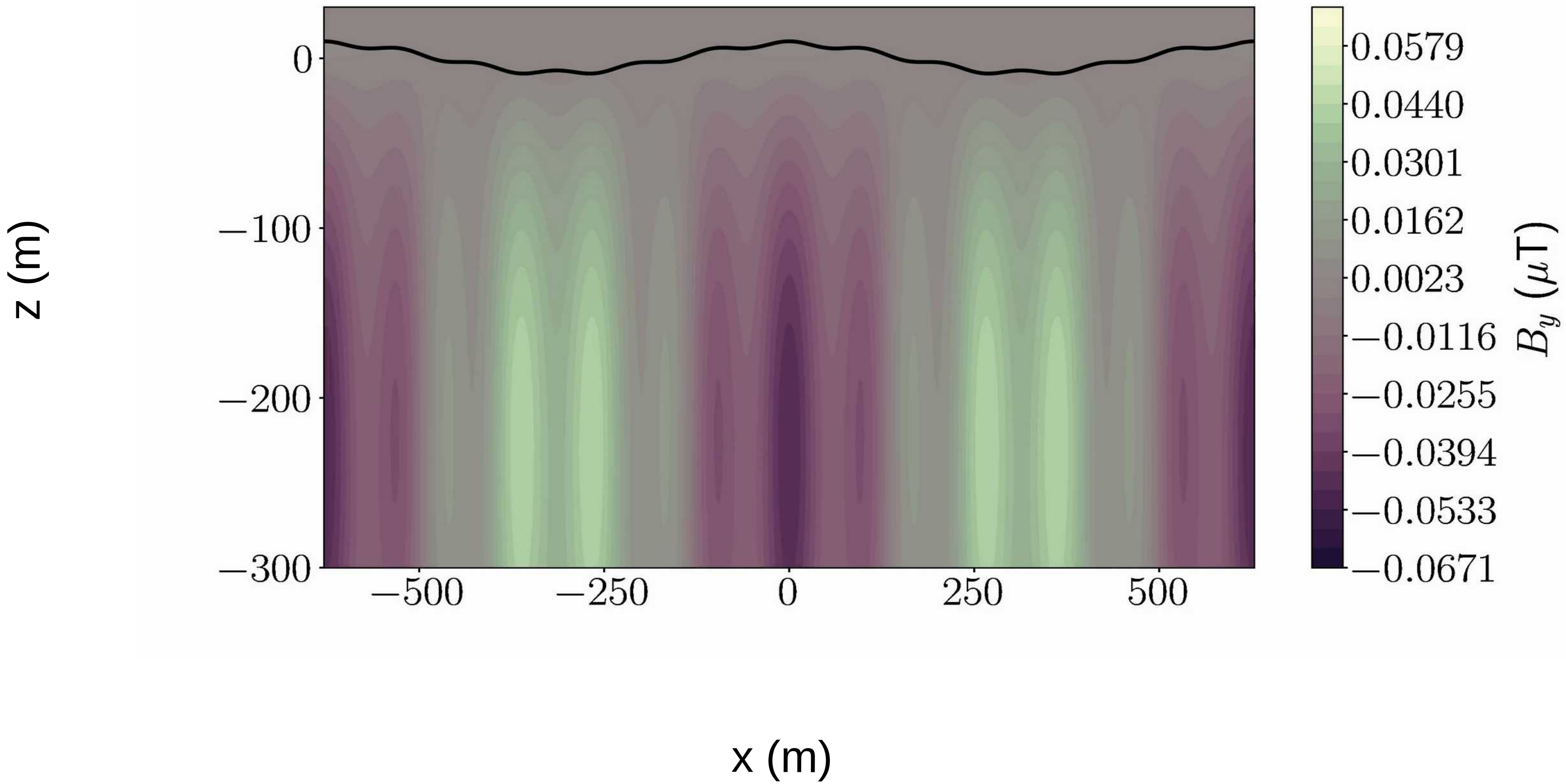
Methods



Order 1



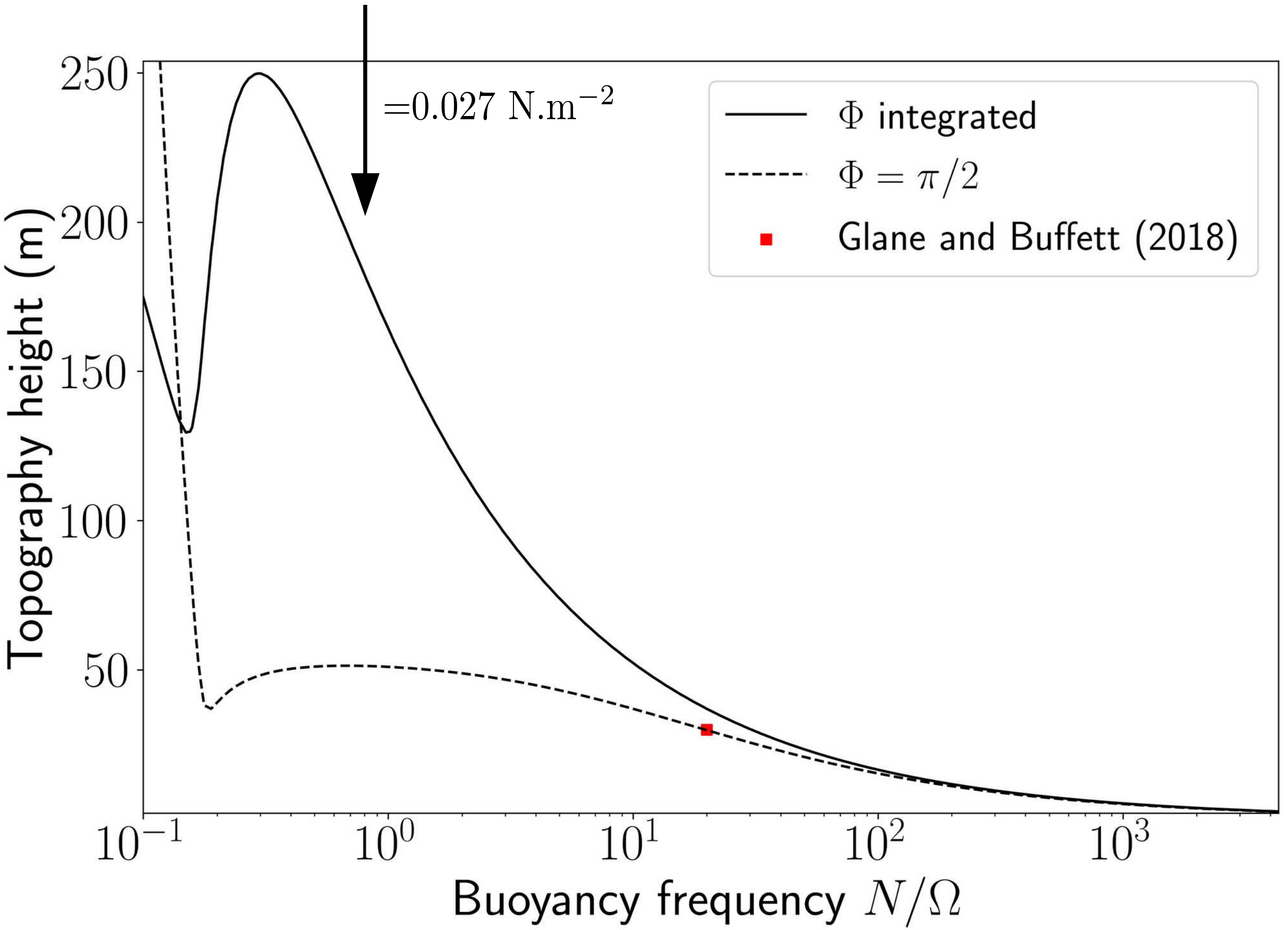
Order 4


0
−100
−200
−300
−500
−250
0
250
500
0.0579
0.0440
0.0301
0.0162
0.0023
−0.0116
−0.0255
−0.0394
−0.0533
−0.0671
B_y (μT)
z (m)
x (m)

What is the value for (h,N) required to explain the observed variation of the length of the day ?

- Steady and uniform flow

- Insulating mantle



integrated = integration with latitude, taking into account the variation of $\boldsymbol{\Omega}$ and $\mathbf{B_0}$

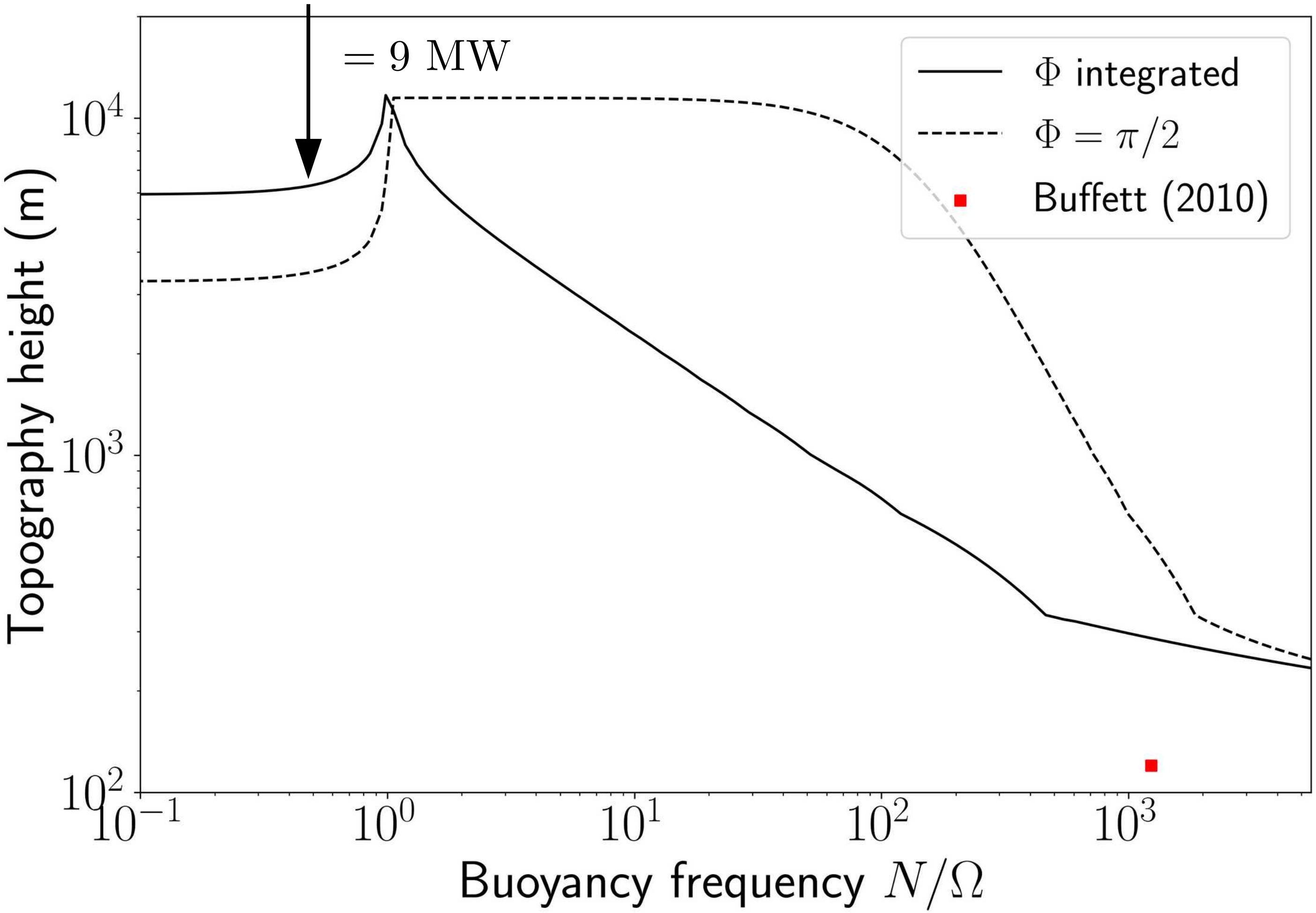
What is the value for (h,N) required to explain the observed dissipative coupling ?

- Oscillating flow with diurnal period

- Conducting mantle
  → electrical conductivity ratio : $\frac{\sigma_{core}}{\sigma_{mantle}} = 500$

- At the pole : $\mathrm{B}_0 = 0.5$ mT

integrated = integration with latitude, taking into account the variation of $\boldsymbol{\Omega}$ and $\mathbf{B_0}$



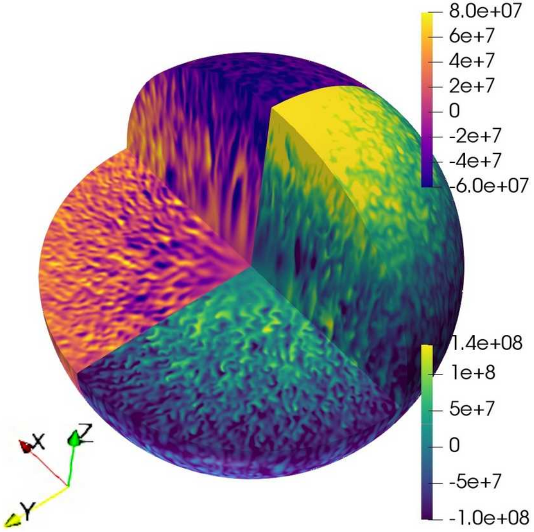
Conclusion

- We can, with our model, explore a wide panel of parameters in a consistent manner
- developed at a higher order of perturbation
- We can compare measurement with our model of coupling

Perspectives

From plane to full sphere:

Implement a boundary perturbation in the XSHELLS code of Nathanael Shaeffer. → global approach



XSHELLS output, from Monville et al. 2019

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S. Glane and B. Buffett, “Enhanced Core-Mantle Coupling Due to Stratification at the Top of the Core,” Frontiers in Earth Science, vol. 6, 2018, doi: 10.3389/feart.2018.00171.

D. Jault, “Tangential stress at the core–mantle interface,” Geophysical Journal International, vol. 221, no. 2, Art. no. 2, Jan. 2020, doi: 10.1093/gji/ggaa048.

Ö. Karatekin et al., “Atmospheric angular momentum variations of Earth, Mars and Venus at seasonal time scales,” Planetary and Space Science, vol. 59, no. 10, Art. no. 10, 2011.

Equations of motion

Inviscid fluid

Inertia | Coriolis force (with Beta-plane approximation) | Reduced pressure gradient | Buoyancy | Lorentz force

$$\boxed{\rho_r D_t \mathbf{U}} = -\boxed{\rho_r \chi \mathbf{e_z} \times \mathbf{U}} - \boxed{\nabla p} + \mu \cancel{\nabla^2 \mathbf{U}} + \boxed{\rho \mathbf{g}} + \boxed{\mathbf{J} \times \mathbf{B}},$$

$$\partial_t \rho + (\mathbf{U} \cdot \nabla) \rho = 0,$$

$$\partial_t \mathbf{B} = \boxed{\eta \nabla^2 \mathbf{B}} + \boxed{\nabla \times (\mathbf{U} \times \mathbf{B})},$$

Joule Dissipation | Coupling term

$$\nabla \cdot \mathbf{U} = 0,$$

Magneto-hydro-dynamic equations (MHD)