Propagation of a Scalar Wavelet through Random Media Having a Power-Law Spectrum

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Envelope broadening and coda excitation

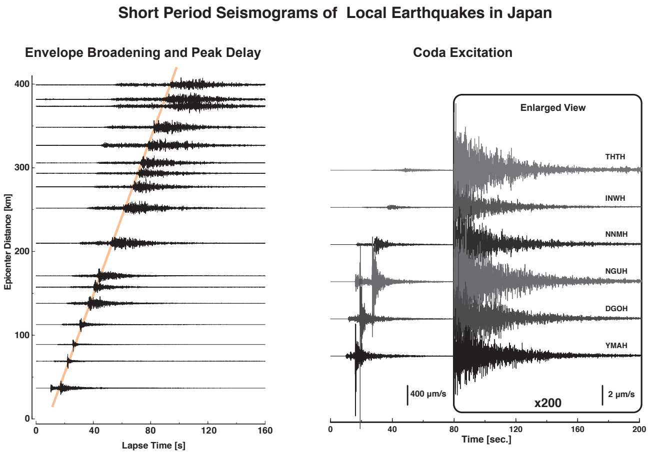


Figure 1 : Left: Normalized by the max. amplitude. Right: Same gain.

Results of scattering by distributed random heterogeneities.

Spatial distribution of energy density of an earthquake

Flat distribution of coda energy near the source.

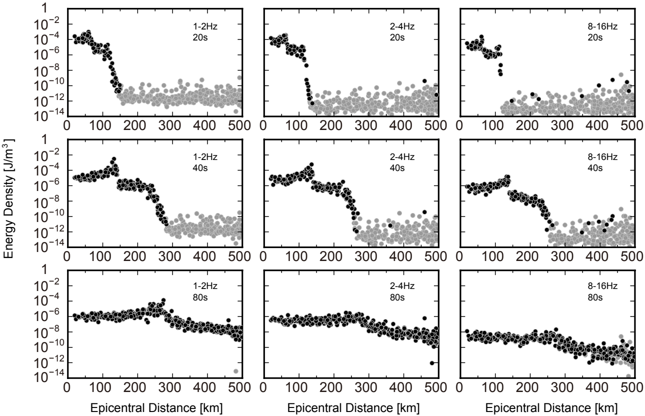
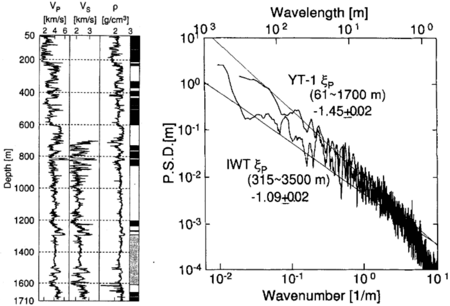


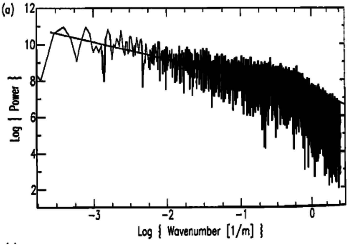
Figure 2 : Energy density (3-compo. 2.6 g cm^{-3}) of N. Hiroshima, Japan earthquake (2011/11/21, M5.4, D=12 km, Hi-net)

Power-law decay of the power spectrum of acoustic well-log data

$$P_{1D}(m) \propto m^{-2\kappa-1}, \text{ where } \kappa = 0 \sim 0.45.$$

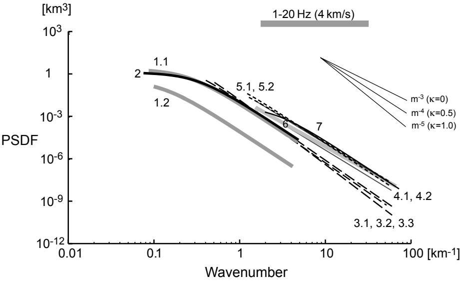


Japan (Shiomi et al. 1997)

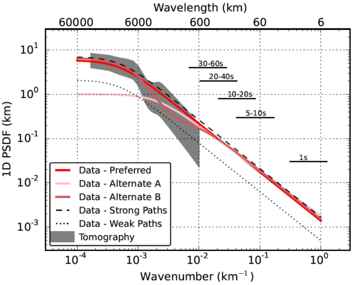


KTB, Germany (Wu et al. 1994)

$P_{3D}(m) \propto m^{-2\kappa-3}$, where $\kappa : 0 \sim 0.5$. $P_{1D}(m) \propto m^{-2\kappa-1}$, where $\kappa \sim 0$.



Reported 3D-PSDF (Sato et al. 2012)



Reported 1D-PSDF (Mancineli et al. 2016).

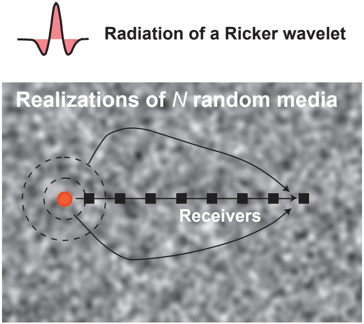
Observed facts:

Seismograms . . . Coda excitation and envelope broadening

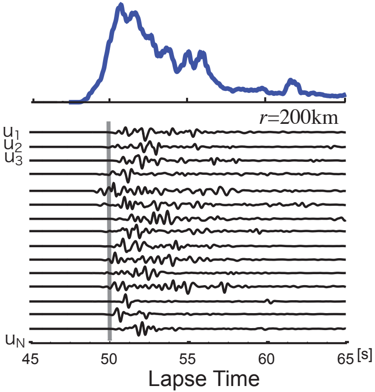
Earth medium . . . Random heterogeneity having a power-law spectrum

Objective

- Stochastic synthesis of the intensity of a scalar wavelet propagating through von Kármán-type random media.
- Comparison with the ensemble averaged intensity $\langle |u(r,t)|^2 \rangle$ with FD simulations.



$$I(r,t) = \frac{1}{N}\sum_{i=1}^{N} |u_i(r,t)|^2$$



Outline

- Scalar wave eq. for $V_0(1+\xi(\boldsymbol{x}))$:

$$\Delta u - \frac{1}{V_0^2}\partial_t^2 u + \frac{2}{V_0^2}\xi\partial_t^2 u = \delta(\boldsymbol{x})s(t).$$

- Ensemble of von Kármán-type random media $\{\xi(\boldsymbol{x})\}$ characterized by ε, a, and κ.
- Intensity $\langle |u|^2 \rangle$ for a spherical radiation of a Ricker wavelet with the center freq. f_c.

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- Ensemble of von Kármán-type random media $\{\xi(\boldsymbol{x})\}$ characterized by ε, a, and κ.
- Intensity $\langle |u|^2 \rangle$ for a spherical radiation of a Ricker wavelet with the center freq. f_c.
- Born ap. for $\varepsilon^2 a^2 k_c^2 \ll 1$, where $k_c = 2\pi f_c / V_0$.
- Spectrum division method for $ak_c > 1$ and $(\varepsilon^2 a^2 k_c^2 \not\ll 1)$.
  Divide the random medium spectrum into two.
  Apply the Born and Markov ap. to the short and long-scale compo., respectively.
  Convolve those in the time domain.
- Comparison with FD simulations
- Summary

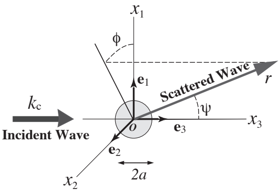
Scattering by a localized velocity anomaly

Incidence of a monochromatic plane wave to an obstacle:

$$u = e^{ik_c z} + \frac{f(\psi,\phi)}{r} e^{ik_c r}, \text{ where } k_c = 2\pi f_c / V_0.$$

Scattering cross-section (Scattering power of the obstacle):

$$\frac{d\sigma}{d\Omega} \equiv |f(\psi,\phi)|^2, \quad \sigma_0 = \oint \frac{d\sigma}{d\Omega} d\Omega$$



Born ap. when $|\xi| \ll 1$: $\dfrac{d\sigma}{d\Omega} = |\dfrac{k_c^2}{2\pi}\tilde{\xi}(\boldsymbol{q})|^2$,

where the tilde means the Fourier T., $\boldsymbol{q} = k_c \boldsymbol{e}_r - k_c \boldsymbol{e}_z$ and $|\boldsymbol{q}| = 2k_c \sin\frac{\psi}{2}$.

Scattering by a high velocity sphere of radius a=5 km:
$V(r) = V_0(1+\varepsilon)$ for $r < a$ (V_0=4 km/s, ε=0.05)

Phase change increases as ak_c increases.
Born ap. is applicable when $\varepsilon^2 a^2 k_c^2 \ll 1$.

Distorted wave Born ap. (Eikonal ap.) is necessary for $ak_c > 1$ and $\varepsilon^2 a^2 k_c^2 \not\ll 1$
$\sigma_0 \approx 2\pi a^2$ as $k_c \to \infty$ (Shadow scattering)

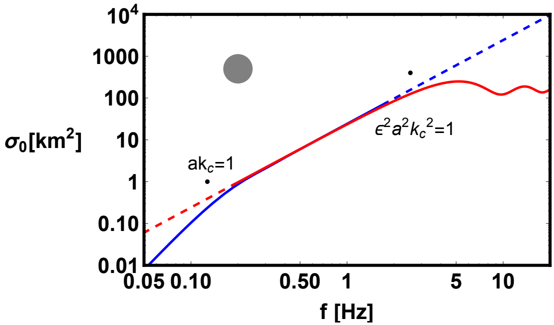
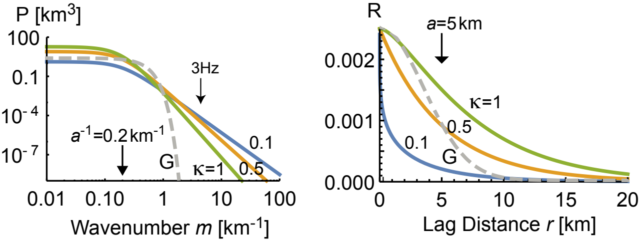


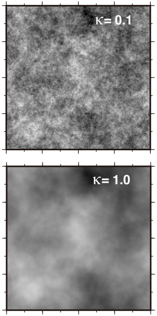
Figure 3 : Total scattering cross-section σ_0 vs. frequency f_c

Uniform and isotropic 3-D random media characterized by ε, a, and κ

$$P(m) = \frac{8\pi^{\frac{3}{2}}\Gamma(\kappa+\frac{3}{2})\varepsilon^2 a^3}{\Gamma(\kappa)\,(1+a^2m^2)^{\kappa+\frac{3}{2}}}, \qquad R(r) = \varepsilon^2 \frac{2^{1-\kappa}}{\Gamma(\kappa)}\left(\frac{r}{a}\right)^{\kappa} K_{\kappa}\left(\frac{r}{a}\right).$$

von Karman-type random media (ε=0.05, a=5km, V_0=4km/s)





Power law decay at $m \gg a^{-1}$

Comparison of Intensity traces by the RTE with the Born ap. and FD

Born ap. is applicable when $\varepsilon^2 a^2 k_c^2 \ll 1$.

Scat. coef. (scat. power per unit volume of random media): $g^B(k_c, \psi) = \frac{k_c^4}{\pi} P(2k_c \sin \frac{\psi}{2})$

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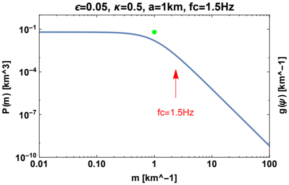

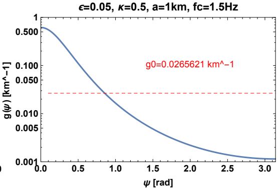

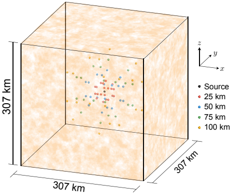

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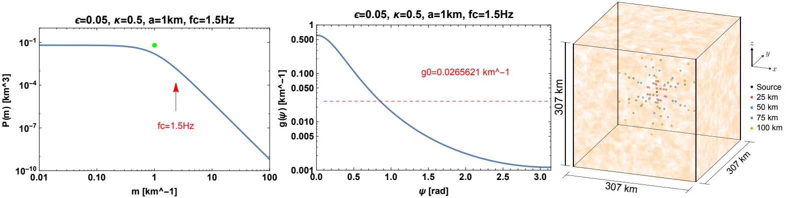
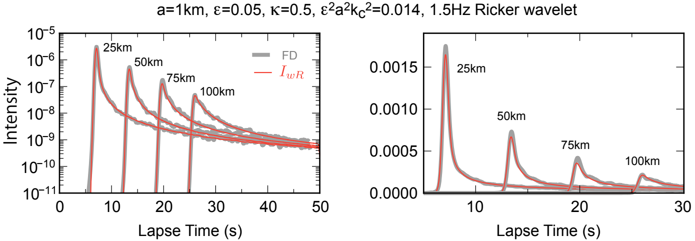


RTE with the Born ap. I_{wR} (red) and FD simulation (gray) in a large 3-D model space.



Good fit from the onset until late coda

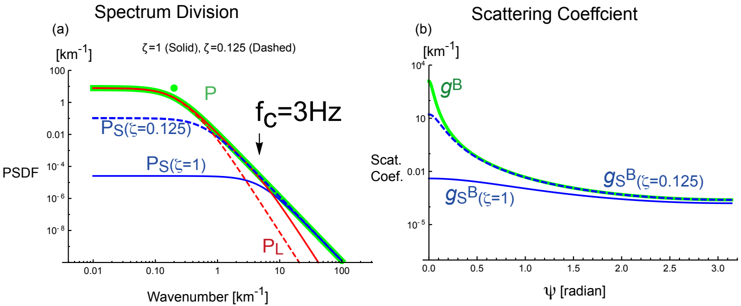
Spectrum division method (four steps)

When $ak_c > 1$ and $\varepsilon^2 a^2 k_c^2 \not\ll 1$, Born is inapplicable to P.

(1) Taking ζk_c as a reference, we decompose P into two components: $P = P_S + P_L$.
We first define the PSDF of the short-scale compo. (tuning parameter $\zeta : \frac{1}{ak_c} \sim 1$):

$$P_S(k_c, \zeta, m) \equiv \frac{8\pi^{\frac{3}{2}}\Gamma(\kappa + \frac{3}{2})\varepsilon_S^2 {a_S}^3}{\Gamma(\kappa)(1 + {a_S}^2 m^2)^{\kappa+\frac{3}{2}}}, \text{ where } a_S^{-1} = \zeta k_c, \text{ and } \varepsilon_S = \frac{\varepsilon}{(\zeta a k_c)^\kappa}.$$

If $\varepsilon_S^2 a_S^2 k_c^2 \ll 1$, Born ap. is applicable to P_S: $g_S^B(k_c, \zeta, \psi) = \frac{k_c^4}{\pi} P_S(k_c, \zeta, 2k_c \sin\frac{\psi}{2})$.



$\varepsilon^2 a^2 k_c^2 \approx 1.4$ at $f_c = 3$ Hz. ε=0.05, a=5km, κ=0.5

$\varepsilon_S^2 a_S^2 k_c^2 \approx 0.054$ (ζ=0.125), 10^{-4} (ζ=1.0)

(2) **Radiative transfer equation (RTE)** using g_S^B for the directional distribution of intensity $F_S\left(k_c,\zeta,\boldsymbol{x},t;\boldsymbol{n}\right)$:

$$\partial_t F_S\left(k_c,\zeta,\boldsymbol{x},t;\boldsymbol{n}\right)+V_0\boldsymbol{n}\nabla F_S\left(k_c,\zeta,\boldsymbol{x},t;\boldsymbol{n}\right)=-g_{S0}^B(k_c,\zeta)V_0F_S\left(\boldsymbol{x},t;\boldsymbol{n}\right)$$
$$+\frac{V_0}{4\pi}\oint g_S^B\left(k_c,\zeta,\boldsymbol{n},\boldsymbol{n}'\right)F_S\left(k_c,\zeta,\boldsymbol{x},t;\boldsymbol{n}'\right)d\Omega_{n'}+\frac{1}{4\pi}\delta\left(\boldsymbol{x}\right)\delta\left(t\right),$$

where $g_{S0}^B=\frac{1}{4\pi}\oint g_S^B d\Omega$.

Intensity Green function: $G_{RS}\left(k_c,\zeta,\boldsymbol{x},t\right)=\oint F_S\left(k_c,\zeta,\boldsymbol{x},t;\boldsymbol{n}\right)d\Omega_n$

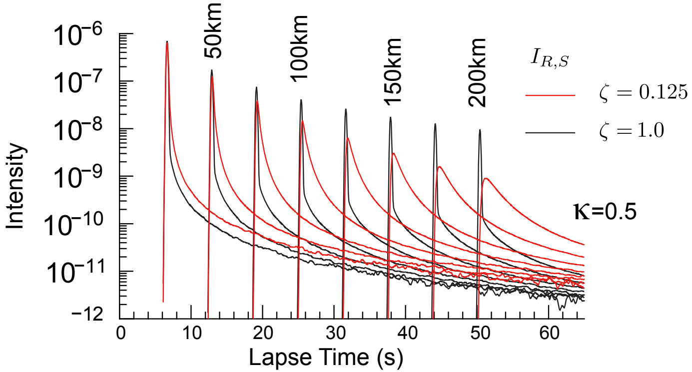
Advection =scattering loss +scattering gain +isotropic source radiation

Dynamic ray-bending process using a constant velocity

We practically solve this equation by using the **Monte Carlo simulation.**

Intensity: $I_{RS}(k_c, \zeta, \boldsymbol{x}, t) = G_{RS}(k_c, \zeta, \boldsymbol{x}, t) \otimes S(k_c, t)$

Monte Carlo ($N = 10^7$) for a 3 Hz Ricker wavelet source, where ε=0.05, a=5km.



Scattering contribution of the long-scale compo.

Spherical wavelet at a long distance r near around the z-axis:

$$u(r, \boldsymbol{x}_\perp, t) = \frac{1}{2\pi r}\int_{-\infty}^{\infty} U\left(k_0, r, \boldsymbol{x}_\perp\right) e^{ik_0 r - i\omega t} d\omega \quad \text{where } k_0 = \omega / V_0.$$

Parabolic ap. for the long scale compo. $\xi_L(\boldsymbol{x})$ since smooth variation of U:

$$2ik_0 \partial_r U + \Delta_\perp U - 2k_0^2 \xi_L U = 0.$$

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(3) Markov approximation

For a quasi-monochromatic wavelet, we define the two-freq. mutual coherence func. (TFMCF):

$$\Gamma_2 (k_c, k_d, r, \boldsymbol{x}_{\perp c}, \boldsymbol{x}_{\perp d}) \equiv \left\langle U\left(r, \boldsymbol{x}'_\perp, \omega'\right) U\left(r, \boldsymbol{x}''_\perp, \omega''\right)^* \right\rangle.$$

Intensity Green func.:

$$G_L(k_c, \zeta, r, t) = \frac{1}{r^2} \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega_d \; e^{-i\omega_d (t - r/V_0)} \Gamma_2 (k_c, k_d, r, \psi_{\perp d} = 0).$$

Master equation for the TFMCF:

$$\partial_r \Gamma_2 + i \frac{k_d}{2k_c^2} \Delta_{\perp d} \Gamma_2 + k_c^2 (A_L(0) - A_L(r_{\perp d})) \Gamma_2 + \frac{k_d^2}{2} A_L(0) \Gamma_2 = 0,$$

where the transverse distance $r_{\perp d} = r\psi_d$, and $\Delta_{\perp d} \approx \frac{1}{r^2} \left({\partial_{\psi d}}^2 + \frac{1}{\psi_d} \partial_{\psi_d} \right)$ for $\psi_d \ll 1$.

Initial condition: $\Gamma_2 (k_c, k_d, r = 0, \psi_{\perp d} = 0) = \frac{1}{4\pi V_0}$.

Taylor expansion of A_L near $r_\perp = 0$

$$\partial_r \Gamma_2 + i \frac{k_d}{2k_c^2} \Delta_{\perp d} \Gamma_2 + k_c^2 \left(A_L(0) - A_L(r_{\perp d})\right) \Gamma_2 + \frac{k_d^2}{2} A_L(0) \Gamma_2 = 0,$$

$$A_L(r_\perp) \equiv \int_{-\infty}^{\infty} R_L(\boldsymbol{x}_\perp, z)\, dz.$$

Strong scattering near around the forward direction $\Rightarrow$ Contribution from small $r_\perp$.

$$A_L(k_c, \zeta, r_\perp) \approx \frac{\varepsilon^2 a\, 2\pi^{1/2} \Gamma\left(\kappa + \frac{1}{2}\right)}{\Gamma(\kappa)} \left[\left(1 - \frac{1}{(\zeta a k_c)^{2\kappa+1}}\right) - \left(1 - \frac{1}{(\zeta a k_c)^{2\kappa-1}}\right) \frac{1}{2(2\kappa - 1)} \left(\frac{r_\perp}{a}\right)^2 + \cdots \right] \quad \text{for} \quad \kappa \neq \frac{1}{2}$$

$$A_L(k_c, \zeta, r_\perp) \approx 2\varepsilon^2 a \left(1 - \frac{1}{(\zeta a k_c)^2}\right) - \varepsilon^2 a (\ln \zeta a k_c) \left(\frac{r_\perp}{a}\right)^2 + \cdots \quad \text{for} \quad \kappa = \frac{1}{2}$$

Quadratic function of $r_\perp$ $\Rightarrow$ Analytic solution of $\Gamma_2 \to G_L$.

Wandering and broadening terms

Intensity Green function for the long-scale compo. as a convolution in time:

$$G_L\left(k_c,\zeta,r,t\right) = w_L(k_c,\zeta,r,t)\otimes b_L(k_c,\zeta,r,t)\otimes G_g(r,t)$$

$$w_L(r,t) = \frac{1}{\sqrt{\pi}t_{wL}}e^{-\frac{t^2}{{t_{wL}}^2}},\, b_L(r,t) = \frac{\pi^2}{16\,t_{ML}}\vartheta_4''\left(0, e^{-\frac{\pi^2}{4}\frac{t}{t_{ML}}}\right),\, G_g(r,t) = \frac{1}{4\pi r^2 V_0}\delta\left(t-\frac{r}{V_0}\right).$$

Gaussian — Elliptic theta func. (4-th) — Geometrical decay and causality

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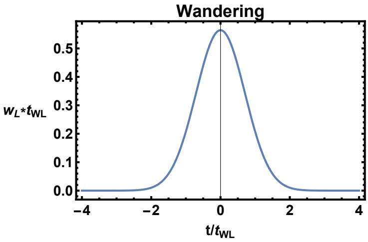
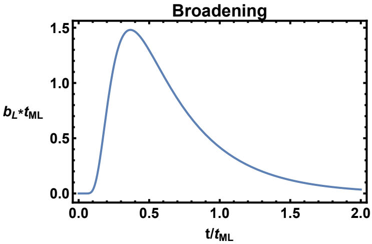




w_L: Travel-time fluctuation

Gaussian filter

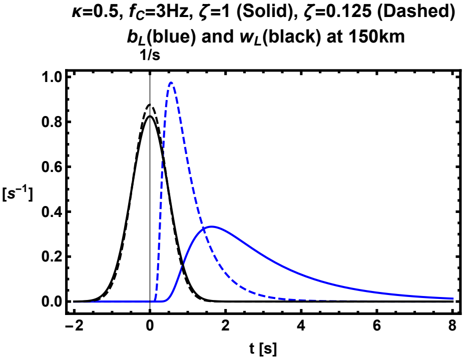
b_L: Multiple forward scattering

Lag filter

Wandering and broadening terms

$$t_{wL}(k_c,\zeta,r) = \frac{2}{V_0}\sqrt{\varepsilon^2 a \frac{\pi^{1/2}\Gamma\left(\kappa+\frac{1}{2}\right)}{\Gamma(\kappa)}\left(1-(\zeta a k_c)^{-2\kappa-1}\right)\sqrt{r}}.$$

$$t_{ML}(k_c,\zeta,r) = \frac{\varepsilon^2 r^2}{2V_0 a} \times \begin{cases} \frac{\pi^{1/2}\Gamma\left(\kappa+\frac{1}{2}\right)}{(2\kappa-1)\Gamma(\kappa)}\left(1-(\zeta a k_c)^{1-2\kappa}\right) & \text{for } \kappa \neq \frac{1}{2}, \\ \ln \zeta a k_c & \text{for } \kappa = \frac{1}{2}. \end{cases}$$



Synthesis of the intensity time trace $I_{L,S}$

$$G_L\,(k_c,\zeta,r,t) = w_L(k_c,\zeta,r,t) \otimes b_L(k_c,\zeta,r,t) \otimes G_g(r,t)$$

Let us propose the following as an approximation:

(4) Synthesis of the intensity Green function using the convolution in time ($G_g \rightarrow G_{RS}$)

$$G_{LS}\,(k_c,\zeta,r,t) = w_L(k_c,\zeta,r,t) \otimes b_L(k_c,\zeta,r,t) \otimes G_{RS}(k_c,\zeta,r,t)$$

$$G_L\,(k_c,\zeta,r,t) = w_L(k_c,\zeta,r,t)\otimes b_L(k_c,\zeta,r,t)\otimes G_g(r,t)$$

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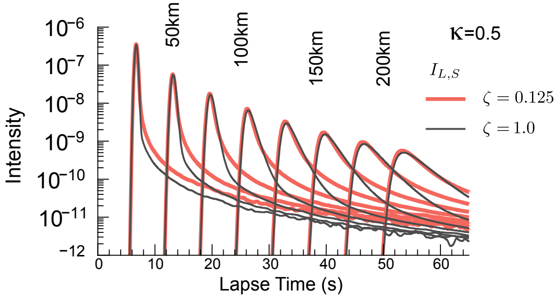
$$I_{LS} = G_{LS}\otimes S = w_L\otimes b_L\otimes I_{RS}$$



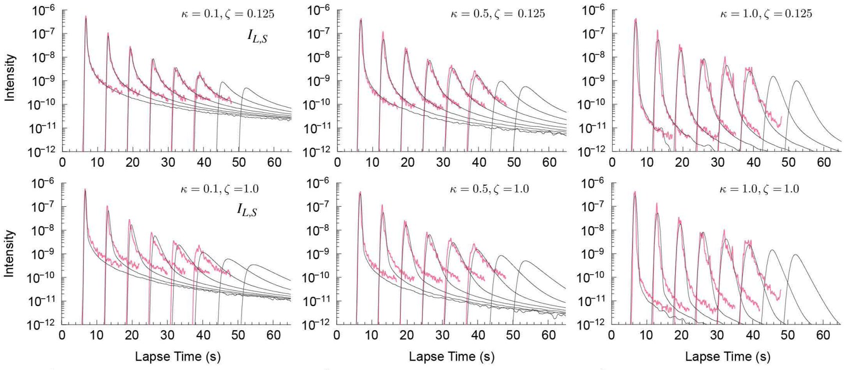
3 Hz Ricker wavelet source. ε=0.05, a=5km.

Traces are almost the same around the peak but different in coda for different ζ.

Comparison with FD: I_{LS} time traces

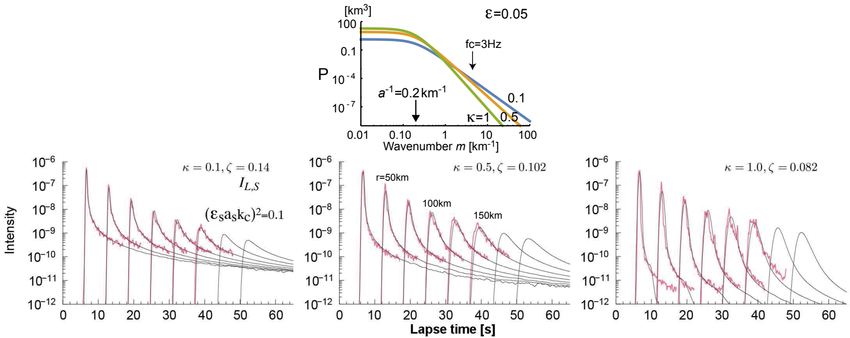
FD simulation of intensity traces for a 3 Hz Ricker wavelet source (r=25 km $\sim$ 150 km):

- FD intensity traces are calculated by using the Hilbert T. at each distance.
- Mean FD intensity trace (magenta) is the average over 6 realizations of random media $\times$ 9 receivers at each distance.



I_{LS} for $\zeta = 0.125$ (black, top) . . . Good fit from the onset through the peak until coda
I_{LS} for $\zeta = 1.0$ (black, bottom) . . . Good fit around the peak, but poor fit for coda

Best fit curves ... Maximum use of the Born ap. to P_S ($\varepsilon_S^2 a_S^2 k_c^2 = 0.1$)



Comparison with FD: Max. intensity vs. travel distance

Maximum peak is mostly composed of forward scattering waves:

$$G_L\left(k_c, \zeta, r, t\right) = w_L(k_c, \zeta, r, t) \otimes b_L(k_c, \zeta, r, t) \otimes G_g(r, t)$$

Component	w_L	b_L	G_g
Constraint	$\int_{-\infty}^{\infty} w_L dt = 1$	$\int_{-\infty}^{\infty} b_L dt = 1$	$4\pi r^2 V_0 \int_{-\infty}^{\infty} G_g dt = 1$
Width in time	$t_{wL} \propto r^{0.5}$	$t_{ML} \propto r^2$	$\delta(t) \sim 0$
Max. Intensity	$\propto r^{-0.5}$	$\propto r^{-2}$	$\propto r^{-2}$

(+ some scattering loss due to the short scale compo.)

Comparison with FD: Max. intensity vs. travel distance

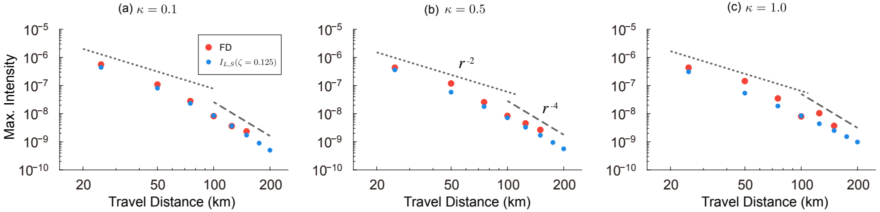
Maximum peak is mostly composed of forward scattering waves:

$$G_L\left(k_c,\zeta,r,t\right)=w_L(k_c,\zeta,r,t)\otimes b_L(k_c,\zeta,r,t)\otimes G_g(r,t)$$

Component	w_L	b_L	G_g
Constraint	$\int_{-\infty}^{\infty} w_L dt=1$	$\int_{-\infty}^{\infty} b_L dt=1$	$4\pi r^2 V_0 \int_{-\infty}^{\infty} G_g dt=1$
Width in time	$t_{wL}\propto r^{0.5}$	$t_{ML}\propto r^2$	$\delta(t)\sim 0$
Max. Intensity	$\propto r^{-0.5}$	$\propto r^{-2}$	$\propto r^{-2}$

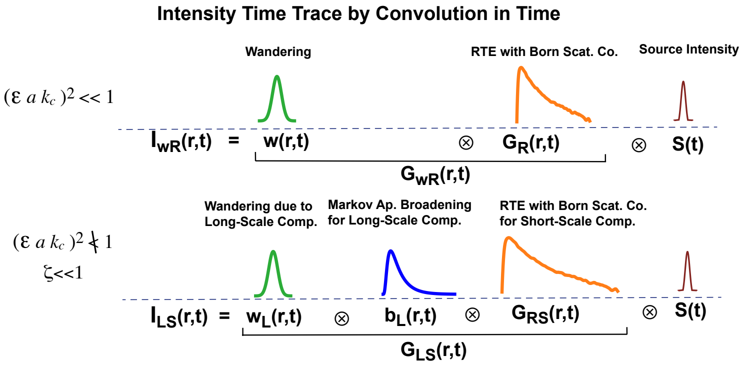
(+ some scattering loss due to the short scale compo.)

I_{max} at f_c=3 Hz (ε=0.05 and a=5 km)



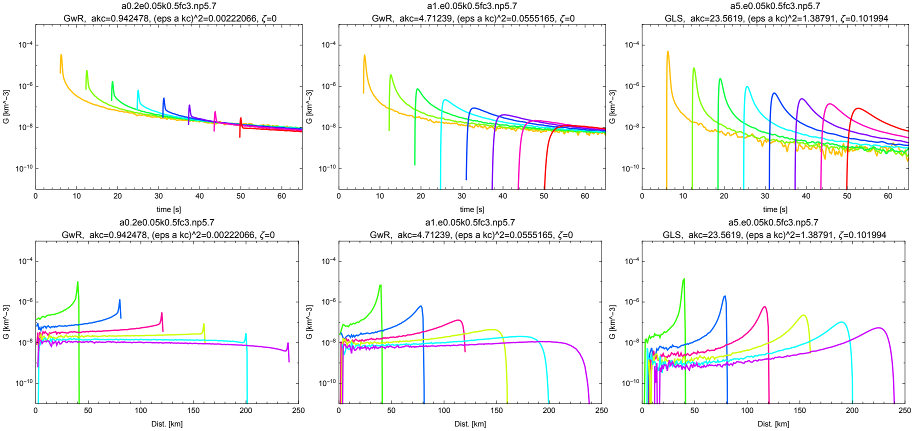
For the magnitude determination of small earthquakes:
Max vel. amp. $\propto r^{-1.73}$ → Max. vel. intensity $\propto r^{-3.5}$

Summary



- Weak velocity fluctuation: $\varepsilon \ll 1$.
- Choose ζ satisfying $\varepsilon_S^2 a_S^2 k_c^2 \approx 0.1$ when $\varepsilon^2 a^2 k_c^2 \not\ll 1$ (Max. use of the Born ap. for P_S).
- Small broadening compared with the travel time: $t_{ML}(r) \ll \frac{r}{2V_0}$

ε=0.05, κ=0.5, f_c=3 Hz, Monte Carlo $N = 10^{5.5}$



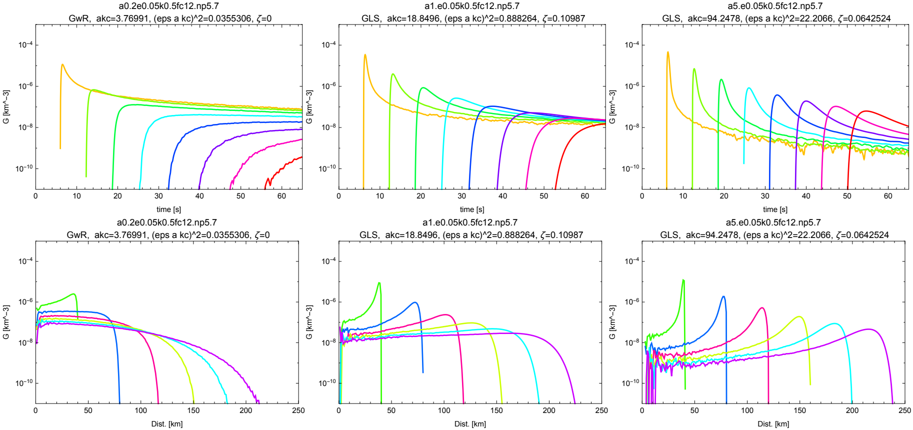
a=0.2 km
RTE+Born G_{wR}

1 km
RTE+Born G_{wR}

5 km
Spectrum division method G_{LS}

Time traces and space distributions of intensity Green functions at 12 Hz

ε=0.05, κ=0.5, f_c=12 Hz, Monte Carlo $N = 10^{5.5}$



a=0.2 km
RTE+Born G_{wR}

1 km
Spectrum division method G_{LS}

5 km
G_{LS}

Conclusion

Stochastic synthesis of a scalar wavelet intensity in von Kármán-type random media

- Spectrum division method leads to the broadening of a wavelet and the uniform distribution of coda intensity when $ak_c > 1$ and $\varepsilon^2 a^2 k_c^2 \not\ll 1$.
- Validity of this approximation is confirmed by comparisons with FD simulations.
- This approximation predicts the power-law decay of the max. amp. with distance.

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- This approximation predicts the power-law decay of the max. amp. with distance.

- Future problems and possible development:
  * Comparison with FD at a large ak_c (more CPU power)
  * Develop mathematical methods to solve the RTE (using discrete Hankel T.)
  * RTE with g_S^B using G_L as a propagator (convolution in space and time)
  * Statistical formulation of the distorted Born ap.
  * Energy conservation (parabolic ?)
  * Extension to the elastic waves
  * Realistic variation of the background velocity

References

- Sato, H., M. Fehler and T. Maeda, 2012. "Seismic wave propagation and scattering in the heterogenous earth: Second edition", Springer Verlag, Heidelberg.
- Sato, H., 2016. Envelope broadening and scattering attenuation of a scalar wavelet in random media having power-law spectra, Geophys. J. Int., 204(1), 386-398.
- Sato, H. and M. Fehler, 2016. Synthesis of wavelet envelope in 2-D random media having power-law spectra: comparison with FD simulations, Geophys. J. Int. 207, 333-342.
- Sato, H. and K. Emoto, 2017. Synthesis of a scalar wavelet intensity propagating through von Karman-type random media: Joint use of the radiative transfer equation with the Born approximation and the Markov approximation, Preprint.

End

Records of a blast at a volcano

Collapse of direct P and S phases **Diffusion-like peak near the source**

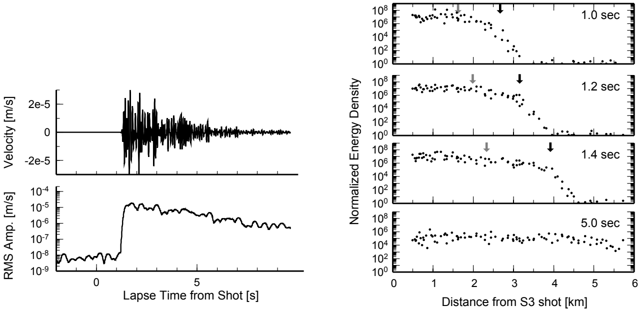
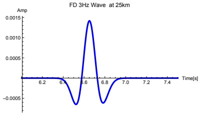


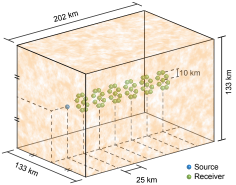
Figure 4 : Energy density of a downhall explosion (vertical compo. 8-16 Hz) at Asama volcano, Japan. (Yamamoto and Sato 2010)

- 3 Hz Ricker wavelet source.
- For given k_c and ζ, we calculate $g_S^B(k_c, \zeta, \psi)$ by the Born ap.
- Solving RTE using Monte Carlo simulations, we calculate G_{RS}.
- Each particle carries unit intensity.
- Particles are randomly shot into various directions from the origin.
- Particle trajectories are recorded until 70 s with Δt=0.01 s ($g_{S0}^B V_0 \Delta t \ll 1$).
- The total number of particles is 10^7.
- CPU time: 10 $\sim$ 15 min. by Java on a PC having 6 cores.

FD simulations of a wavelet in random media

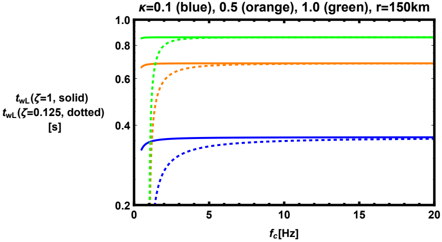
- FD scheme: 4th and 2nd order in space and time, respectively.
- 3 Hz Ricker wavelet source
- Grid space is 0.04 km (33 grids $/\lambda$) and time interval is 3 ms. Recorded up to 50 s.
- Model space size is $202 \times 133 \times 133\text{km}^3$ with absorbing boundaries.
- Random medium is composed of 96 small random media of 1024^3 grids.
- Each small random medium is synthesized for a given set of ε, κ and a.
- Receiver arrays are distributed from 25 to 150 km from the source with a 25 km separation. Each array is composed of 9 receivers with different offsets.
- Intensity time trace is calculated by using the Hilbert tr.
- Average of intensity over 54 traces at each travel distance (9 receivers and 6 realizations of random media).
- CPU time: 70 min. for each calculation by Fortran on Earth Simulator (super computer) of JAMSTEC, Japan having 140 nodes.

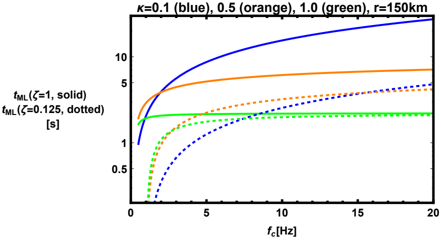


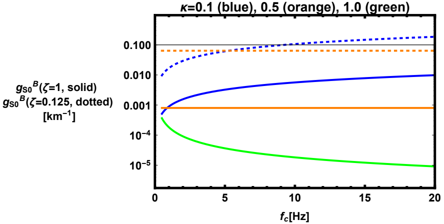


Frequency dependence of characteristic times and total scat. coef.

For the same κ and f_c, $t_{wL}(\zeta = 1) \approx t_{wL}(\zeta = 0.125)$,
$t_{ML}(\zeta = 1) > t_{ML}(\zeta = 0.125)$ and $g^B_{S0}(\zeta = 1) < g^B_{S0}(\zeta = 0.125)$



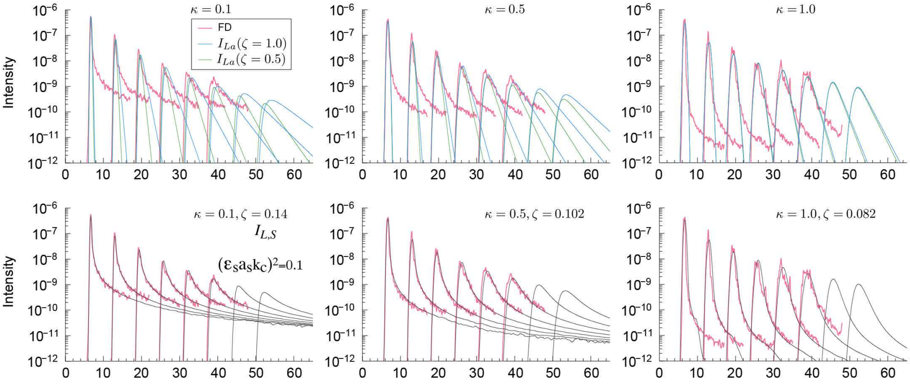




Comparison with FD for $f_c = 3$ Hz: I_{La} and I_{LS} time traces

Introducing scattering attenuation as the lowest correction (Sato, 2016, GJI):

$$I_{La} = e^{-g_{S0} V_0 t} G_L \otimes S$$



$$I_{LS} = G_{LS} \otimes S = w_L \otimes b_L \otimes I_{RS}$$

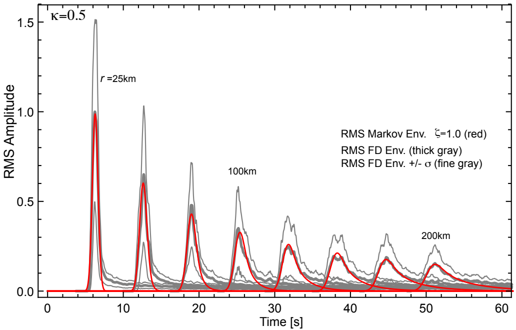
Comparison of Markov ap. and FD in 2-D

Parabolic ap. is applicable when $ak_c > 1$ and $\varepsilon^2 a^2 k_c^2 \ll 1$ (the "≪" is struck out)

2-D random media, von Karman ACF, κ=0.5, ε=0.05, a=5 km, V_0=4 km/s

f_c=2 Hz→ $k_c = 3.14$, $ak_c = 15.7$, $\varepsilon^2 a^2 k_c^2$=0.62

RMS Amplitude $\equiv \sqrt{I_{La}} = \sqrt{e^{-g_{S0} V_0 t} G_L \otimes S}$ (red)



Mean FD intensity trace (gray) is averaged over 50 realizations (Sato and Fehler, 2016)

Good fit from the onset through the peak until very early coda, but poor fit at late coda.