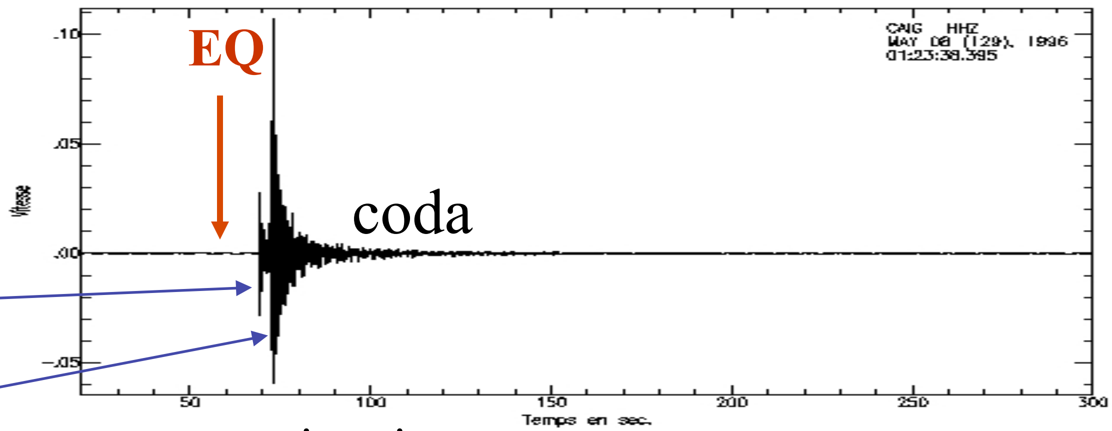
Scattering

(Michel Campillo)

Seismograms and regime of propagation/scaling
From field to intensity and energy
RTE
Diffusion
Consequences of scattering: equipartition and isotropization
Scattering in the mantle

Example of a record of a local earthquake in the band .5-20Hz

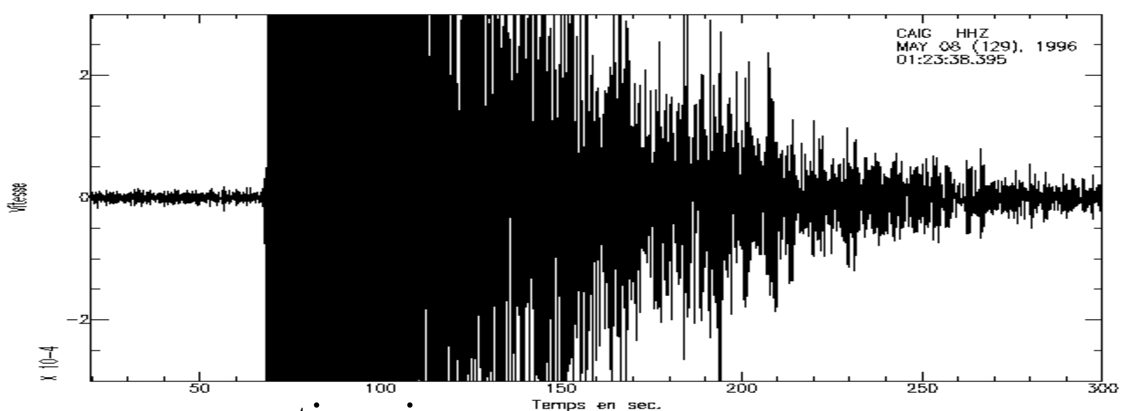


P wave

S wave

time in s

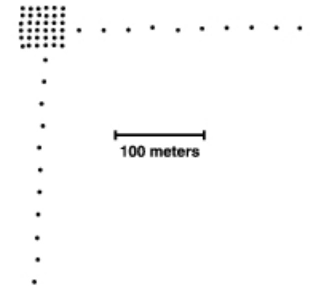
scale x 1000:



time in s

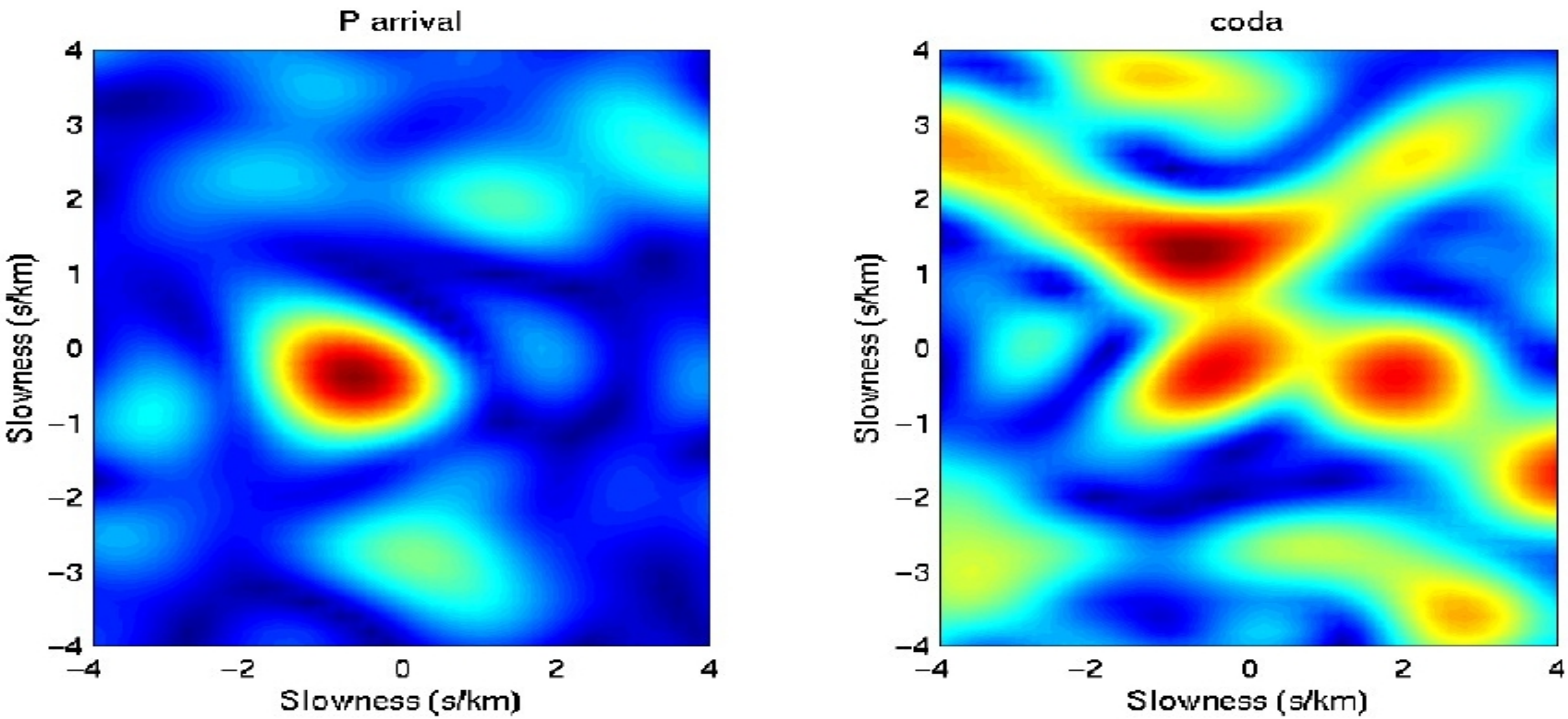
Coda: tail, end of a piece of music….

Frequency-wavenumber analysis

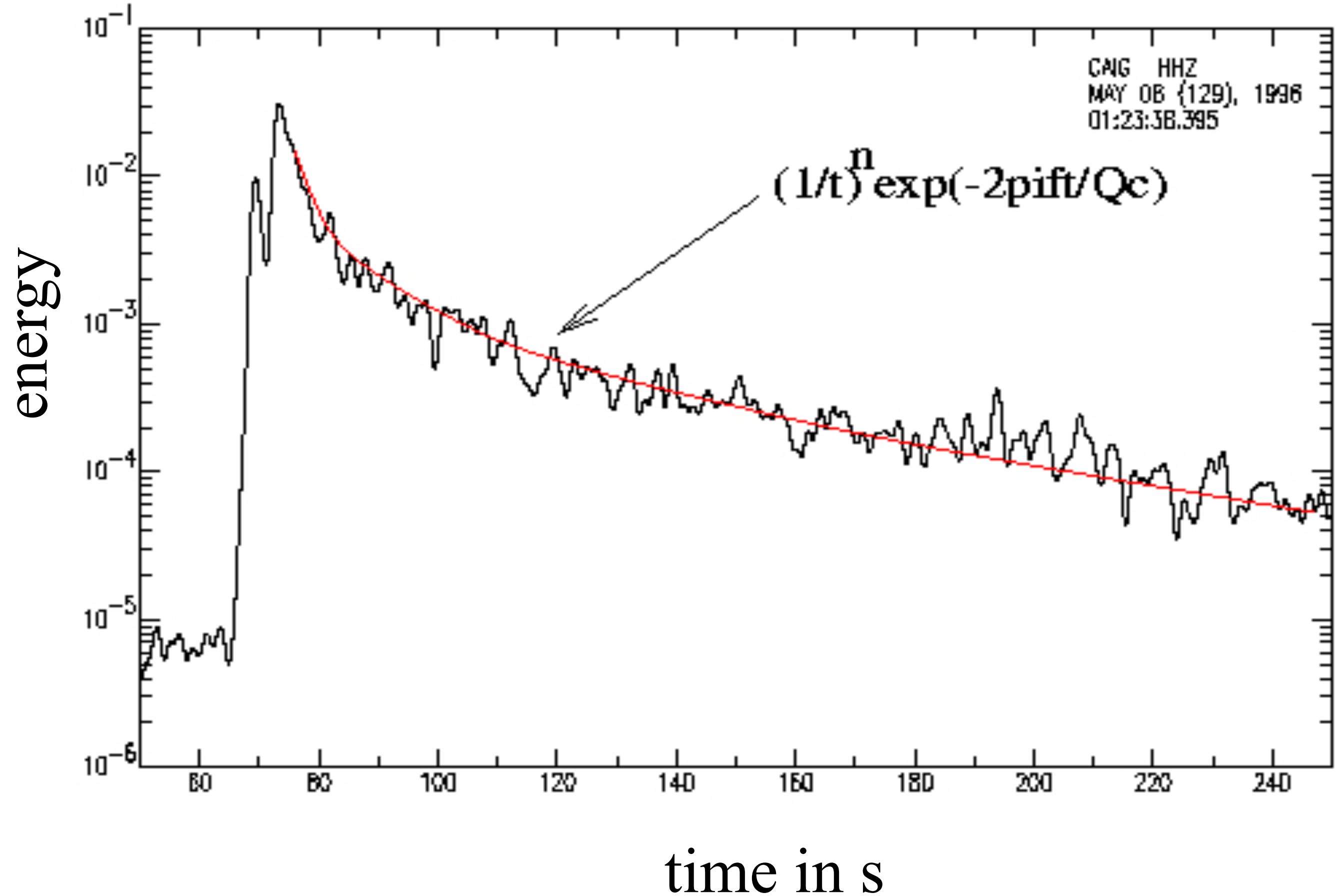


(Pinon Flat Seismometer Array)

$$u(x,y) \rightarrow u(k_x,k_y)$$

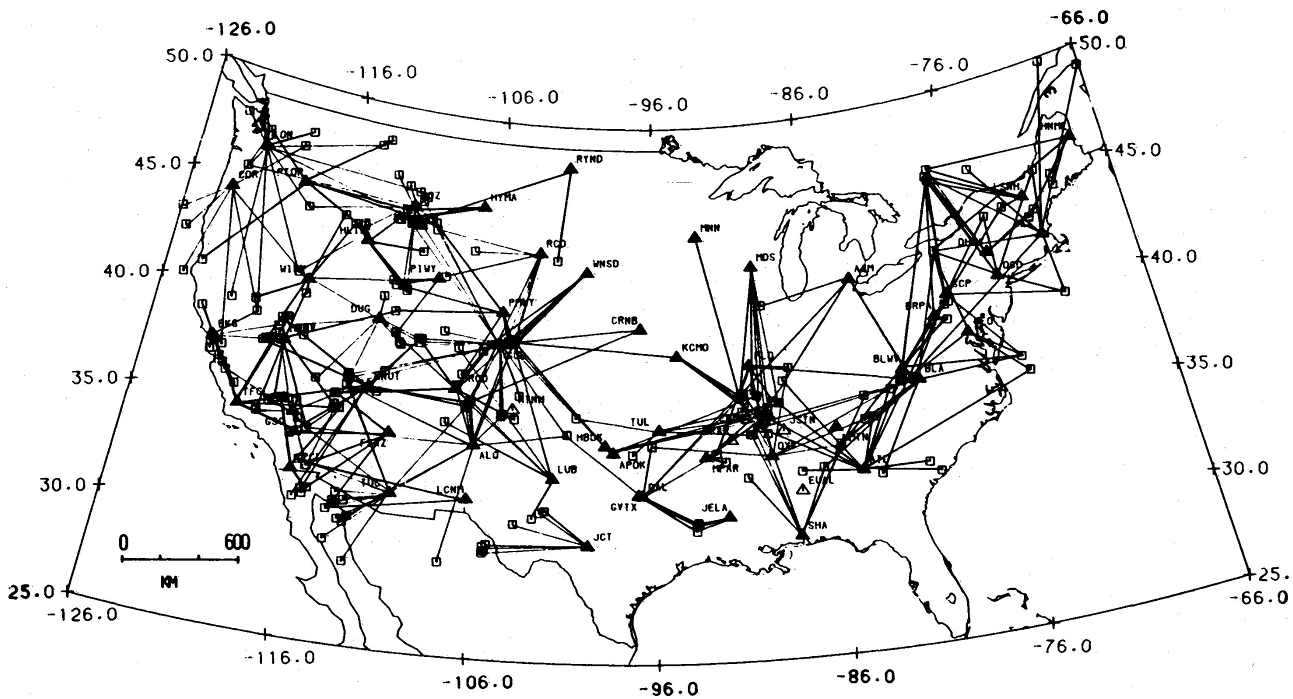


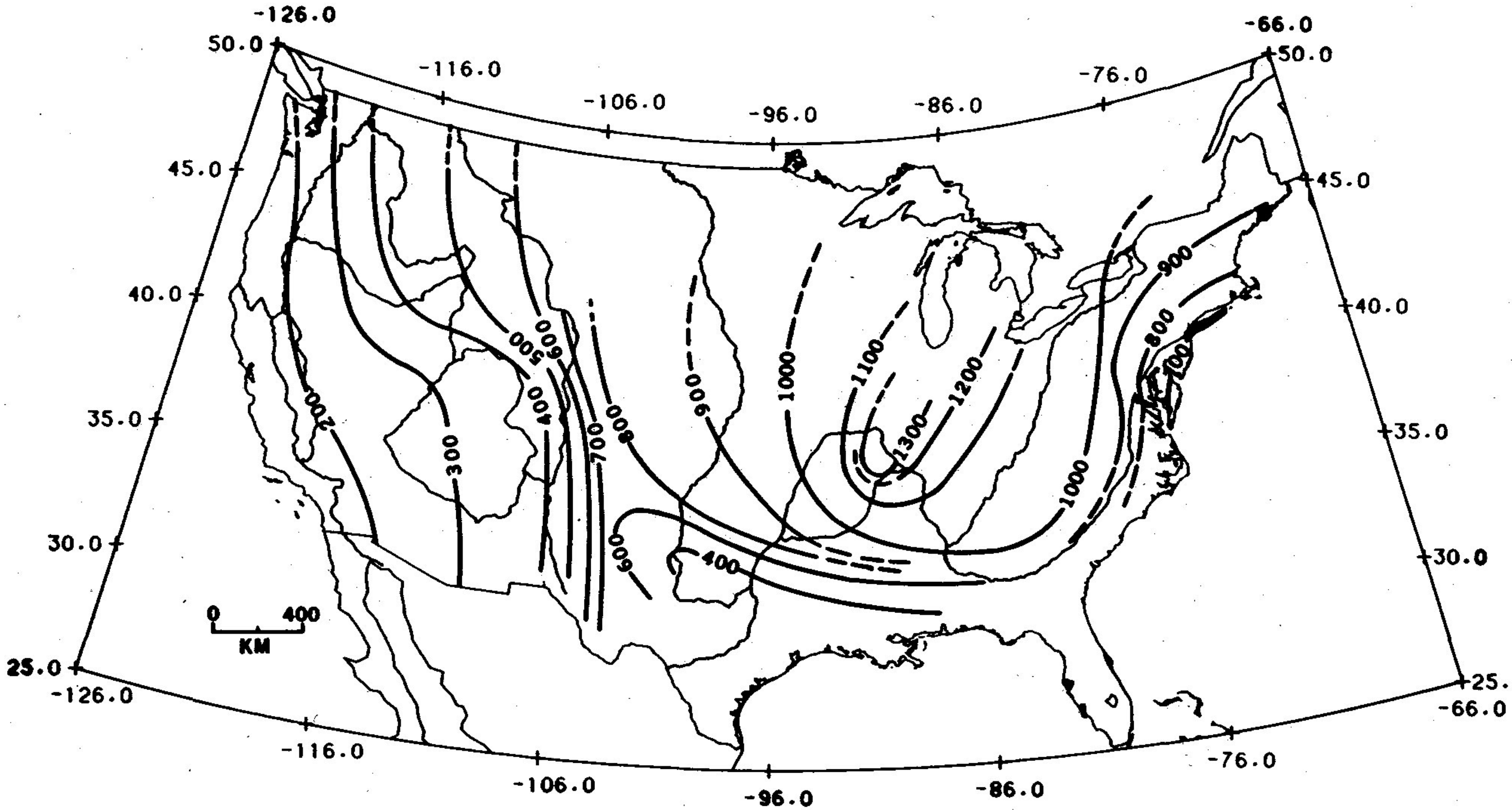
Energy decay in the coda (Aki and Chouet, 1975)



The decay is constant in a region, independently of source and receiver: Qcoda

Coda Q in US (Singh and Herrmann, 1983)




-126.0
-116.0
-106.0
-96.0
-86.0
-76.0
-66.0
50.0
45.0
40.0
35.0
30.0
25.0
200
300
400
500
600
700
800
900
1000
1100
1200
1300
0 400
KM

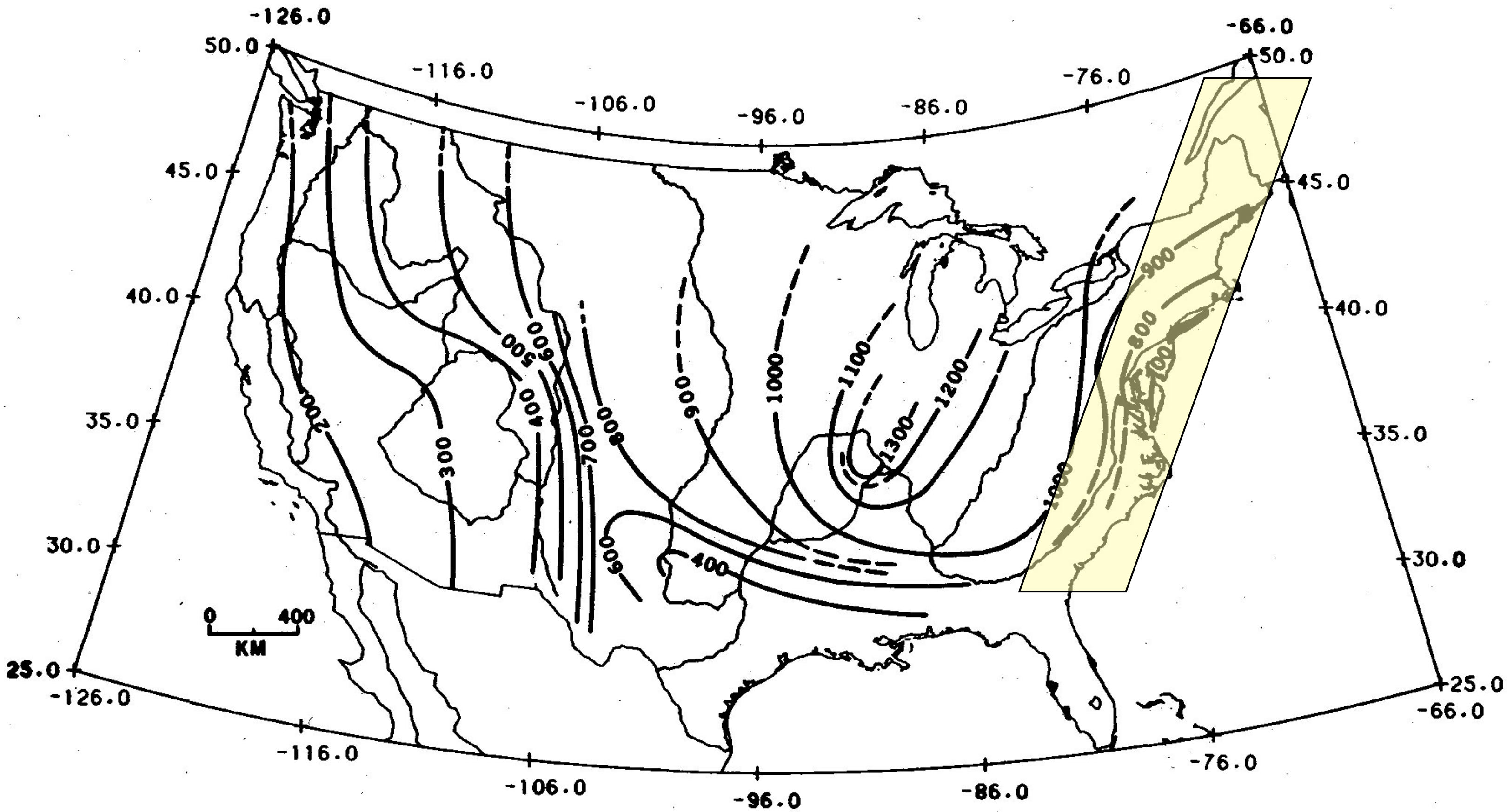


Fig. 15. Contour map of coda Q_0 for the entire continental United States.

Appalachian (Hercynian) belt : Qc ~ 600

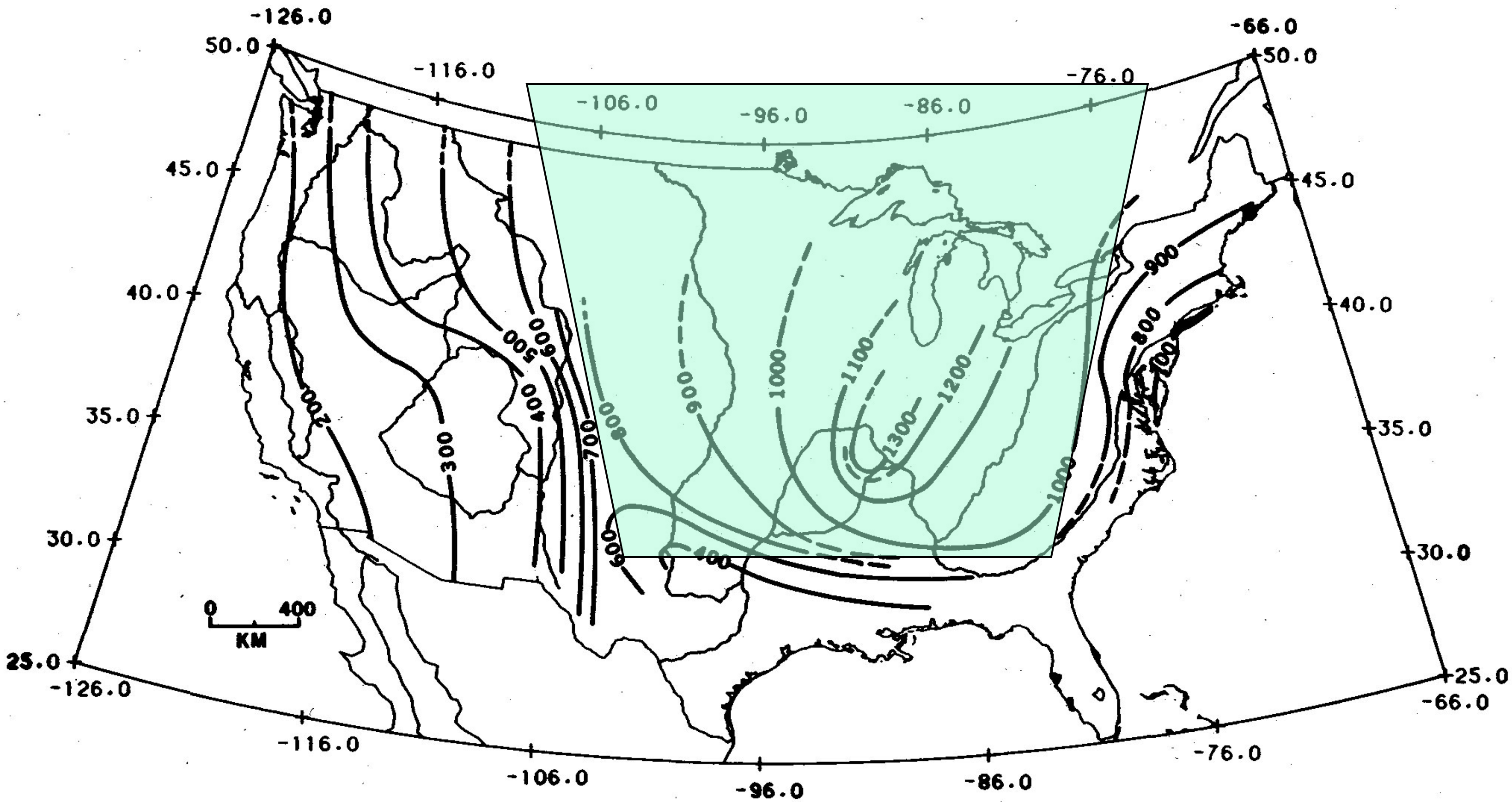


Fig. 15. Contour map of coda Q_0 for the entire continental United States.

Central shield : Qc ~ 1000

536 SINGH AND HERRMANN: CRUSTAL CODA Q REGIONALIZATION IN THE U.S.

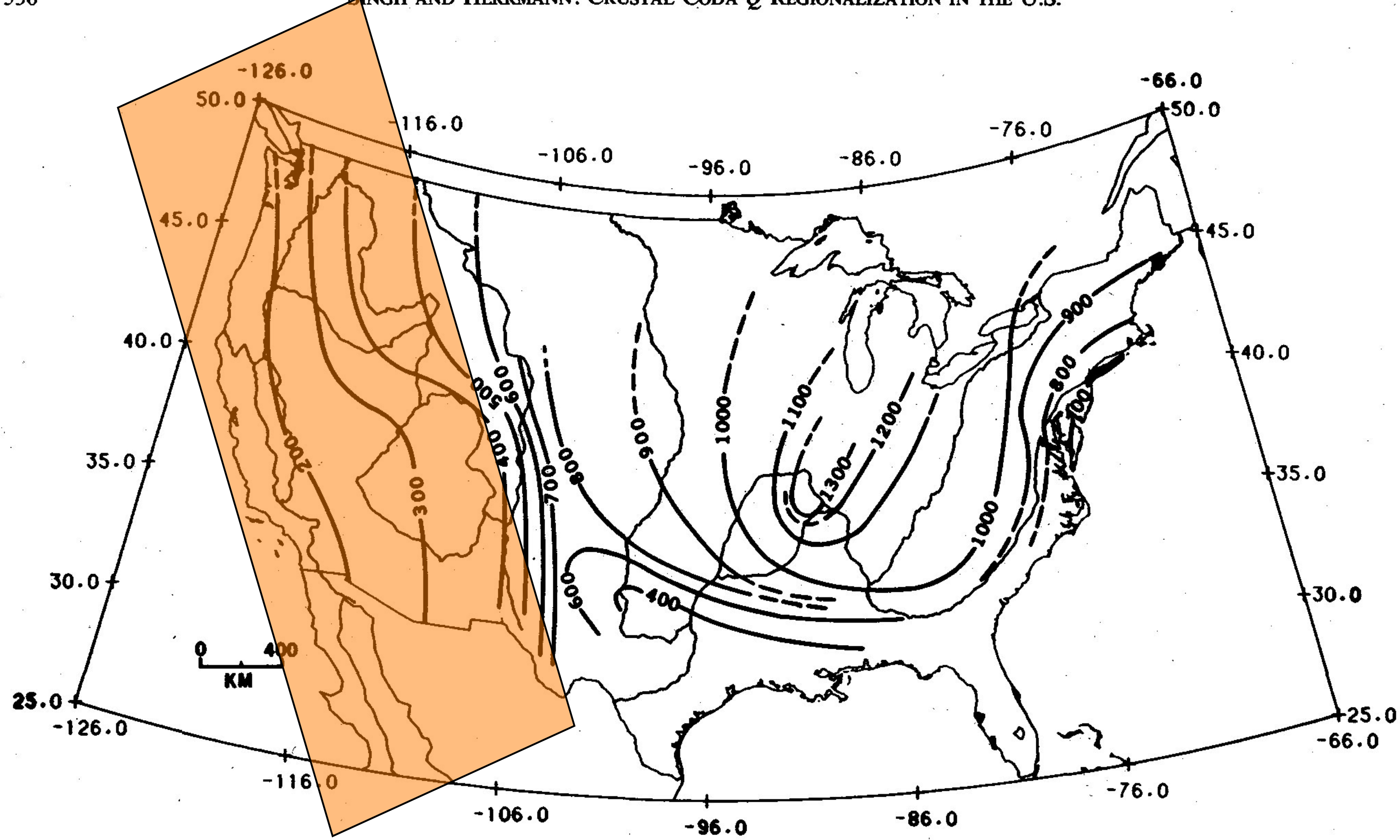
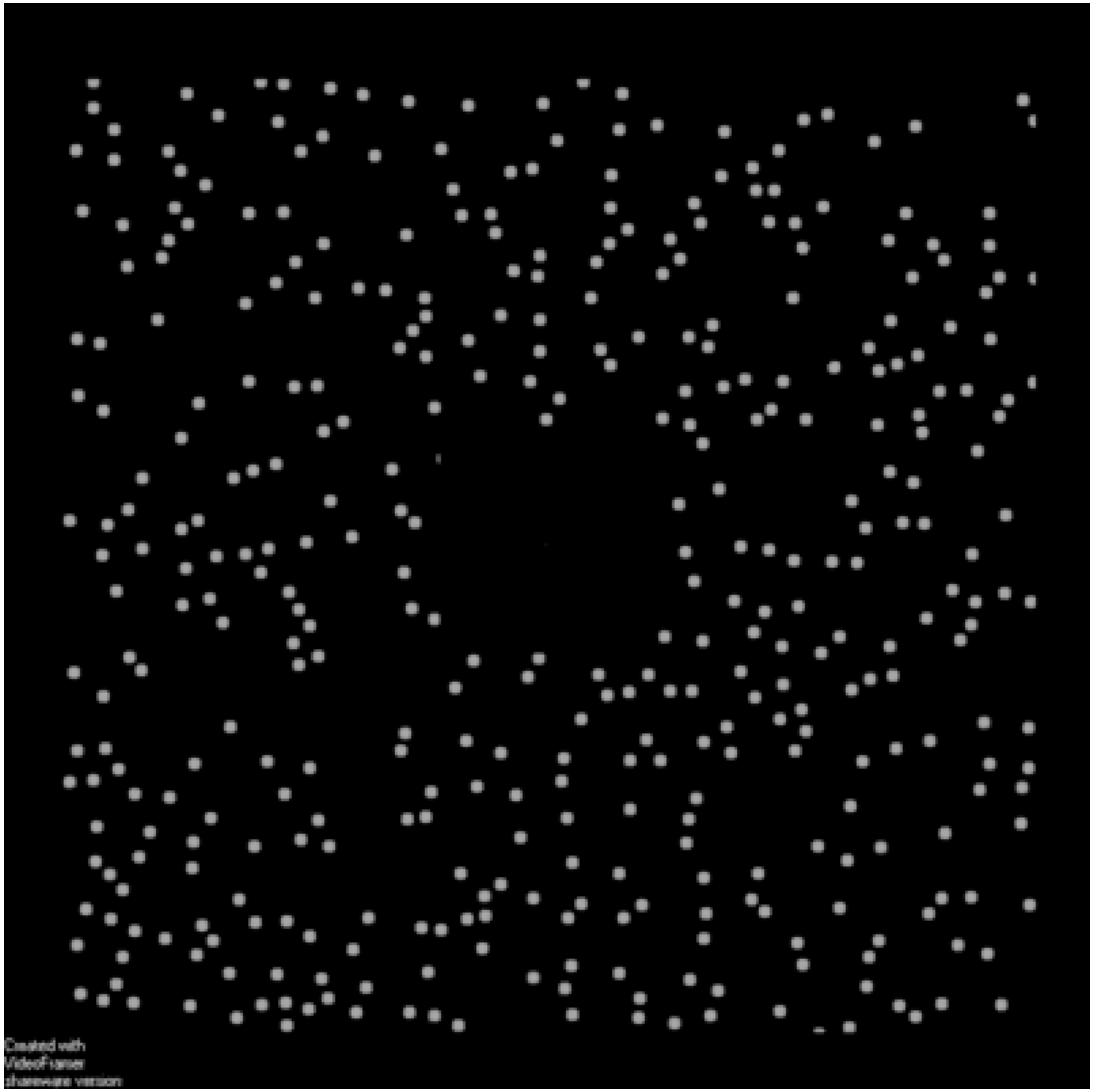


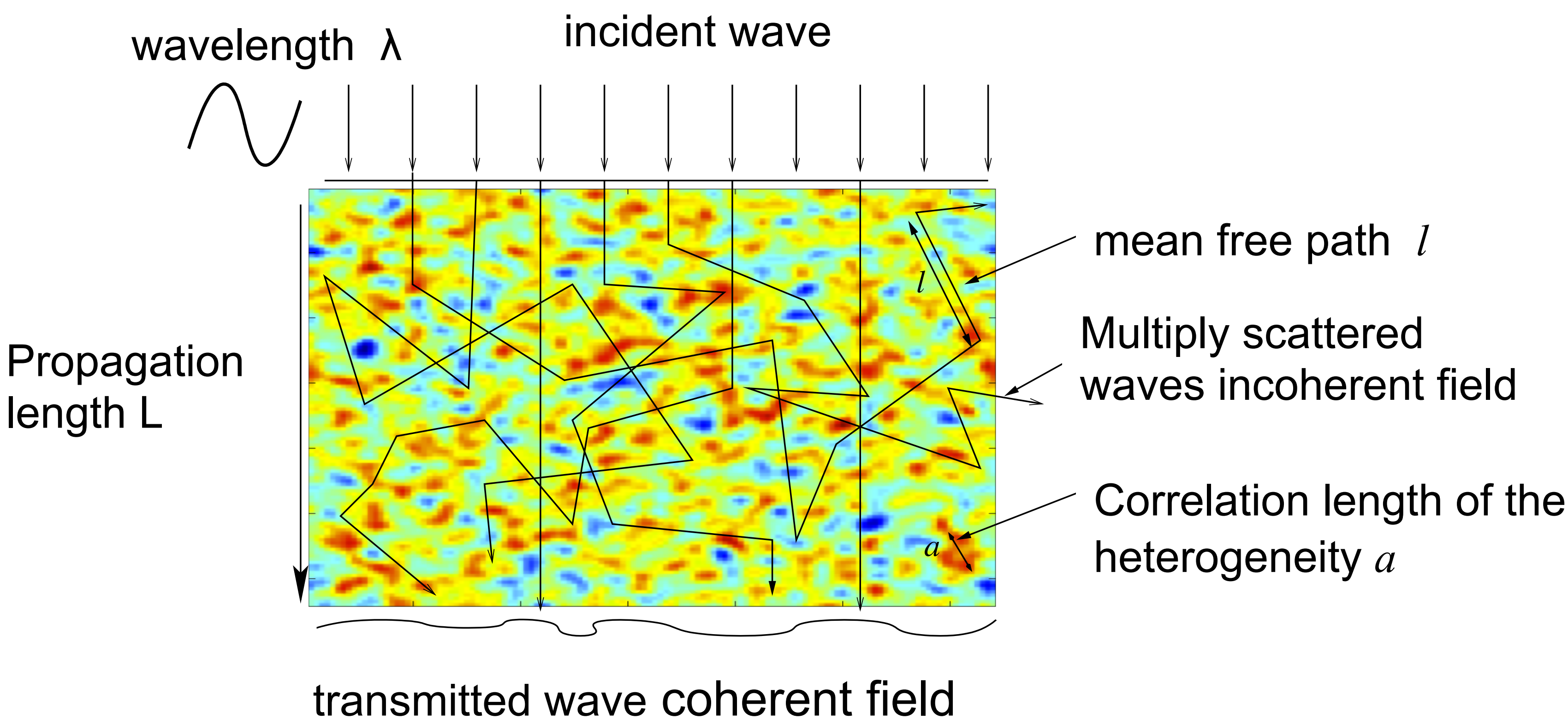
Fig. 15. Contour map of coda Q_0 for the entire continental United States.

Tectonically active western US: Qc=100-300

Transient signals in a complex medium…….



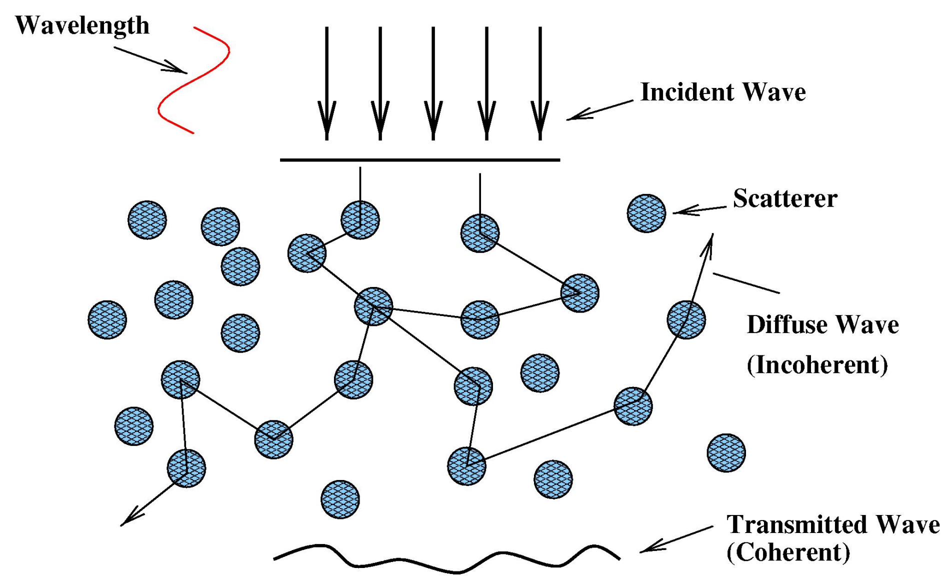
Propagation regimes



$L < l, l_a$ $\qquad\qquad$ $l_a > L \gg l$

coherent wave	single to multiple scattering	equipartition
mean field	radiative transfer	Diffusion

Wave Propagation through Random Media



Length Scales: λ, Correlation Length, Propagation Distance

Question: Ensemble Average Response?

Precise Definition of Coherent and Incoherent Waves

The Concept of Mean Free Path

First Moment of the Green Function:

- Dyson Equation

Ensemble average

$\langle G \rangle$: Coherent Field

$$\langle G \rangle = G_0 + G_0 M \langle G \rangle$$

M, Mass Operator Describes all Possible Scattering Situations

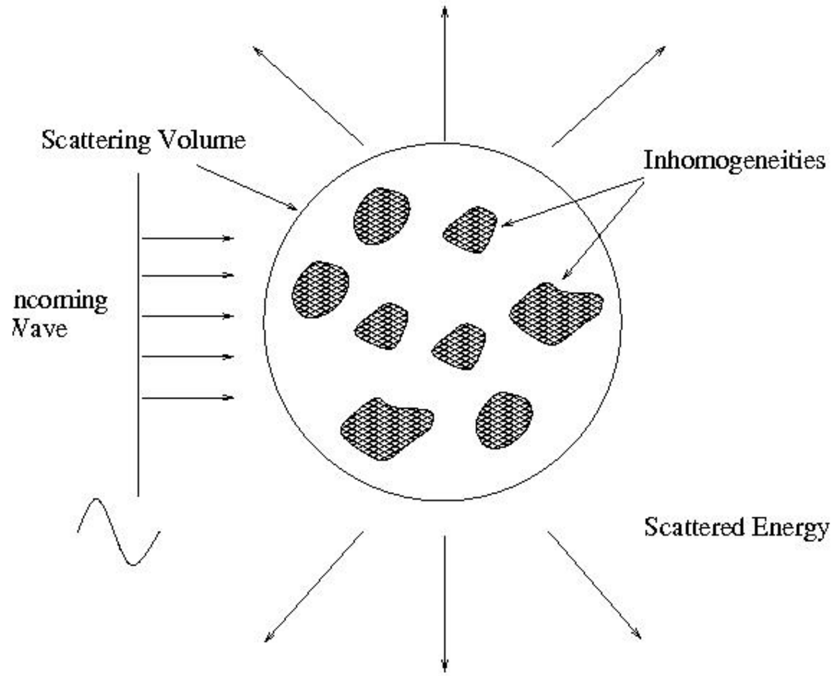
- Approximate Solution

$$\langle G(\vec{r};\vec{r_0}) \rangle = -\frac{1}{4\pi|\vec{r}-\vec{r_0}|} e^{ik|\vec{r}-\vec{r_0}|}$$

$$k = k_0 + \frac{i}{2l}$$

New Length Scale: Mean Free Path of Waves $l = f(\epsilon, a, \lambda)$

Exponential form of decay: exp-r/2l to be compared with attenuation



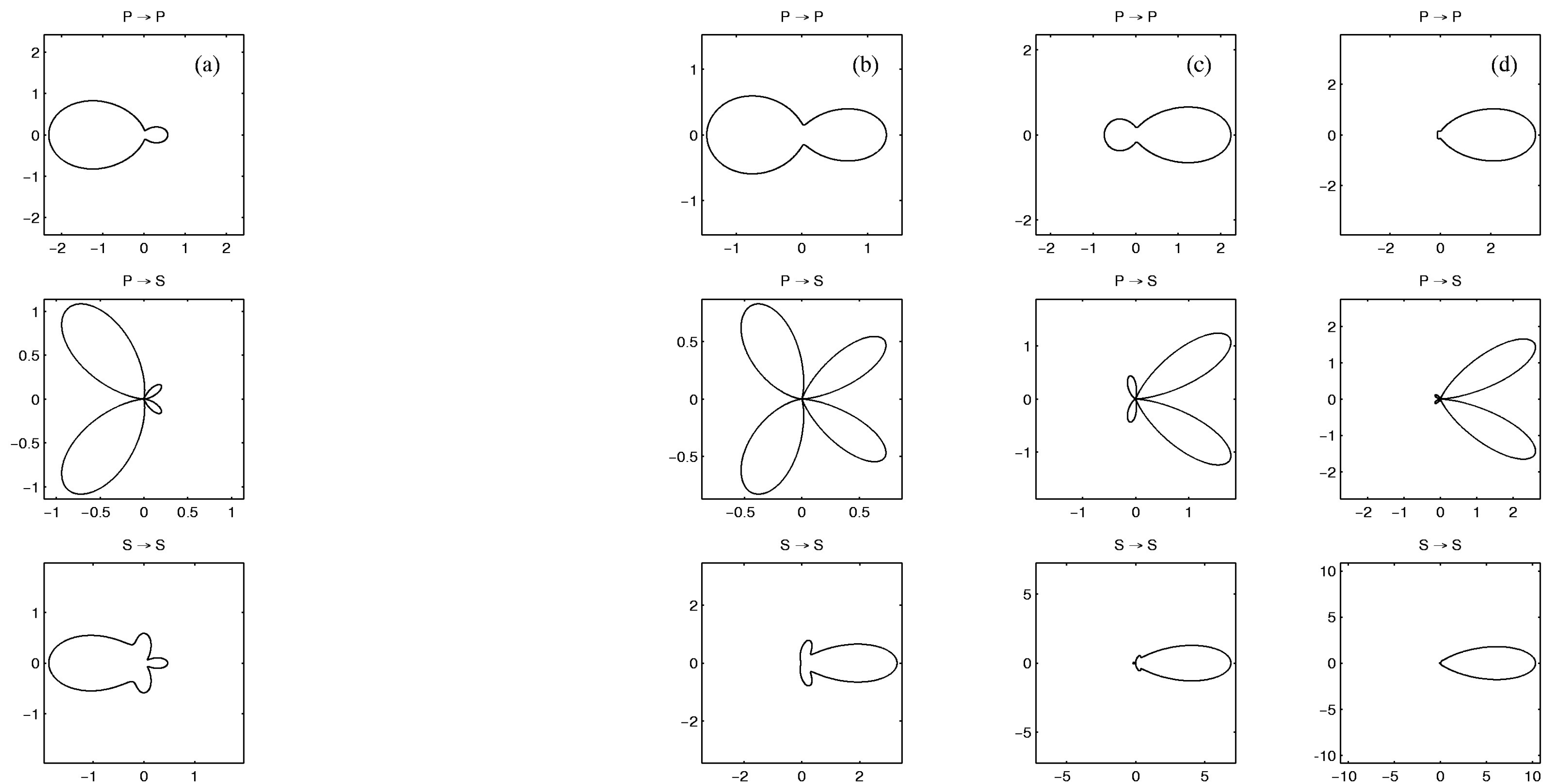
l=1/nσ

$$\frac{1}{l} = \frac{\text{Total Scattering Cross-Section}}{\text{Scattering Volume}}$$

Frequency Dependence:

- Rayleigh ($ka \ll 1$): $l \sim \omega^{-4}$
- High-Frequency ($ka > 1$): $l \sim \omega^{-2}$

Differential cross sections of scattering and conversion for a sphere of radius a

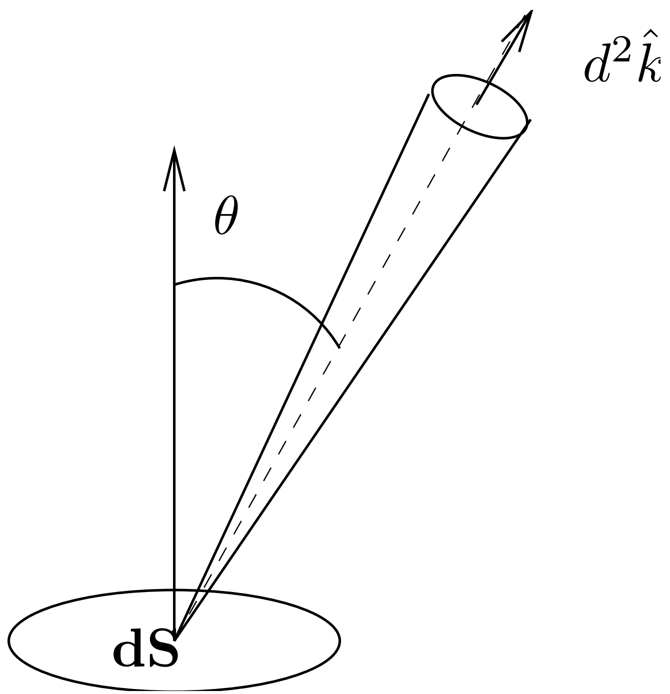


ka→0 : Rayleigh approximation

ka=1.2 ka=1.6 ka=2.0

(averaged in φ for S polarisation)

The specific intensity



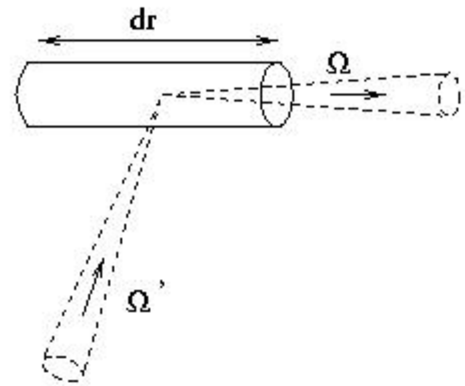
Angularly-resolved energy flux through a surface

Definition

$\mathcal{I}(\omega, t, \mathbf{r}, \hat{\mathbf{k}}) \times dS \times \cos(\theta) \times dt \times d\omega =$ Amount of energy within the frequency band $[\omega, \omega + d\omega]$ flowing through dS around direction $\hat{\mathbf{k}}$ during time dt

Radiative transfer equation

Energy balance of a beam of energy propagating a distance dr in the scattering medium



Variation of Intensity

=

Loss due to scattering into all space directions

+

Gain due to scattering from direction $\vec{\Omega}'$ to direction $\vec{\Omega}$

The Equation of Radiative Transfer

Scalar Case

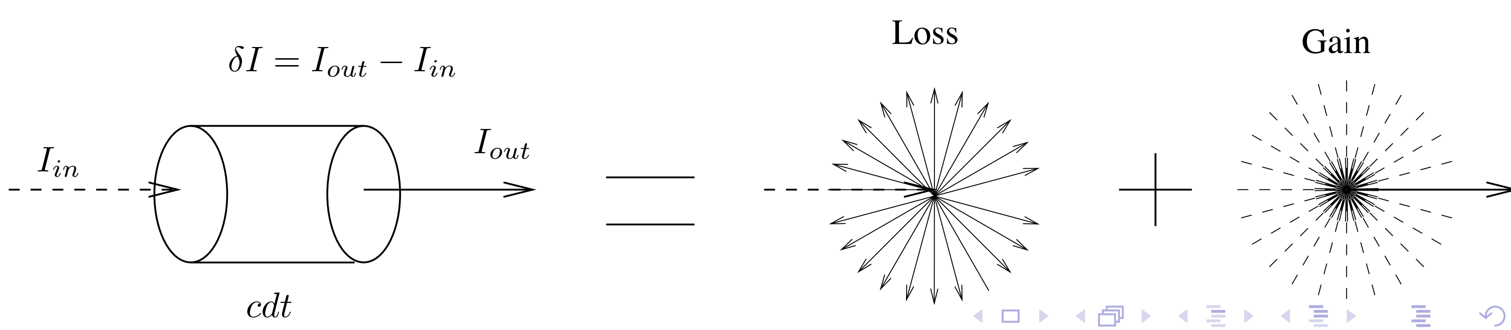
$$\left(\frac{1}{c}\frac{\partial}{\partial t} + \hat{\mathbf{k}} \cdot \nabla_{\mathbf{x}}\right) \mathcal{I}(t, \mathbf{x}, \hat{\mathbf{k}}) =$$
$$- \left(\frac{1}{l^s} + \frac{1}{l^a}\right) \mathcal{I}(t, \mathbf{x}, \hat{\mathbf{k}}) + \frac{1}{l^s} \oint p(\hat{\mathbf{k}}, \hat{\mathbf{k}}') \mathcal{I}(t, \mathbf{x}, \hat{\mathbf{k}}') d^2 \hat{k}'$$

l^s: scattering mean free path

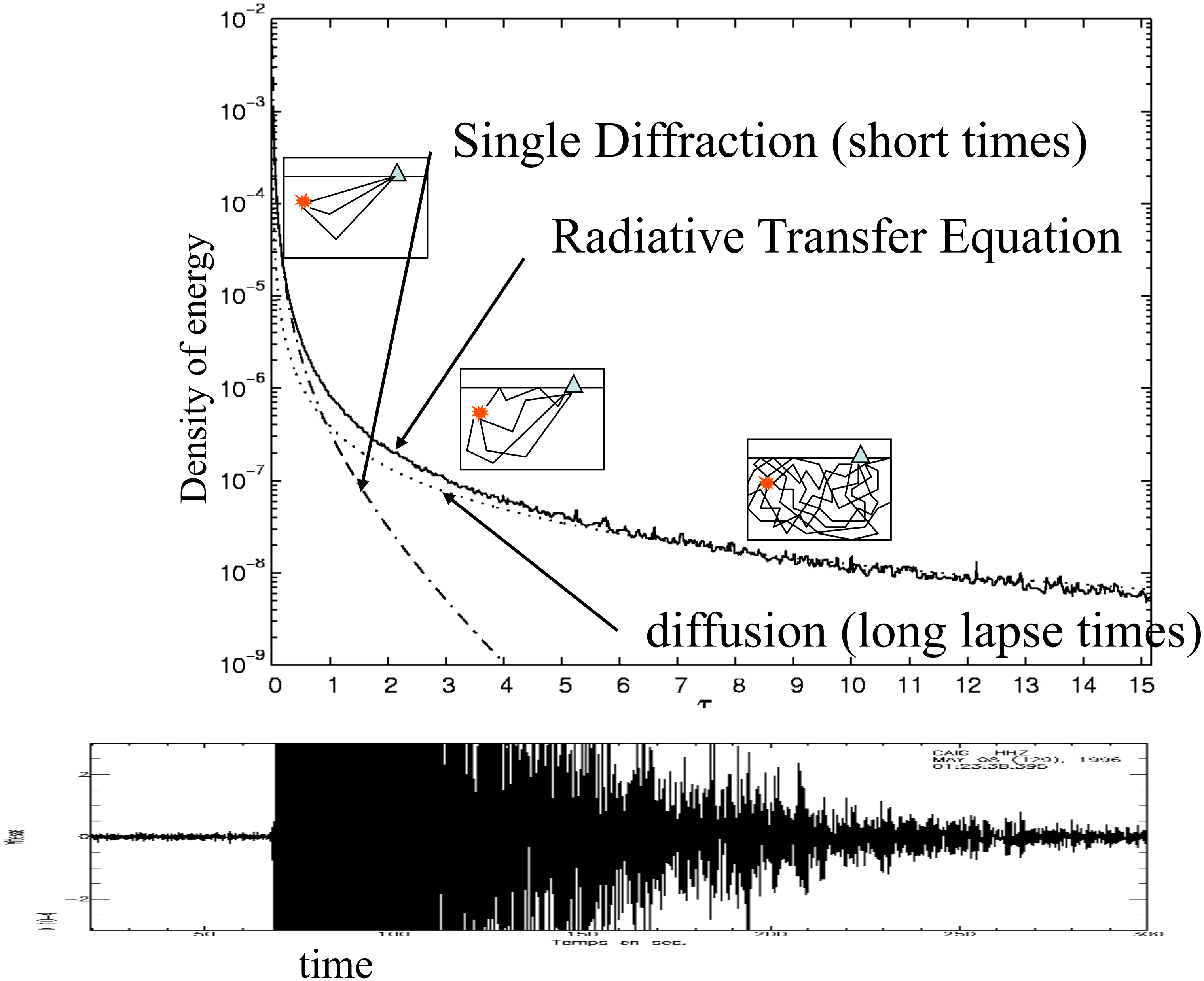
l^a: absorption length

c: wave speed

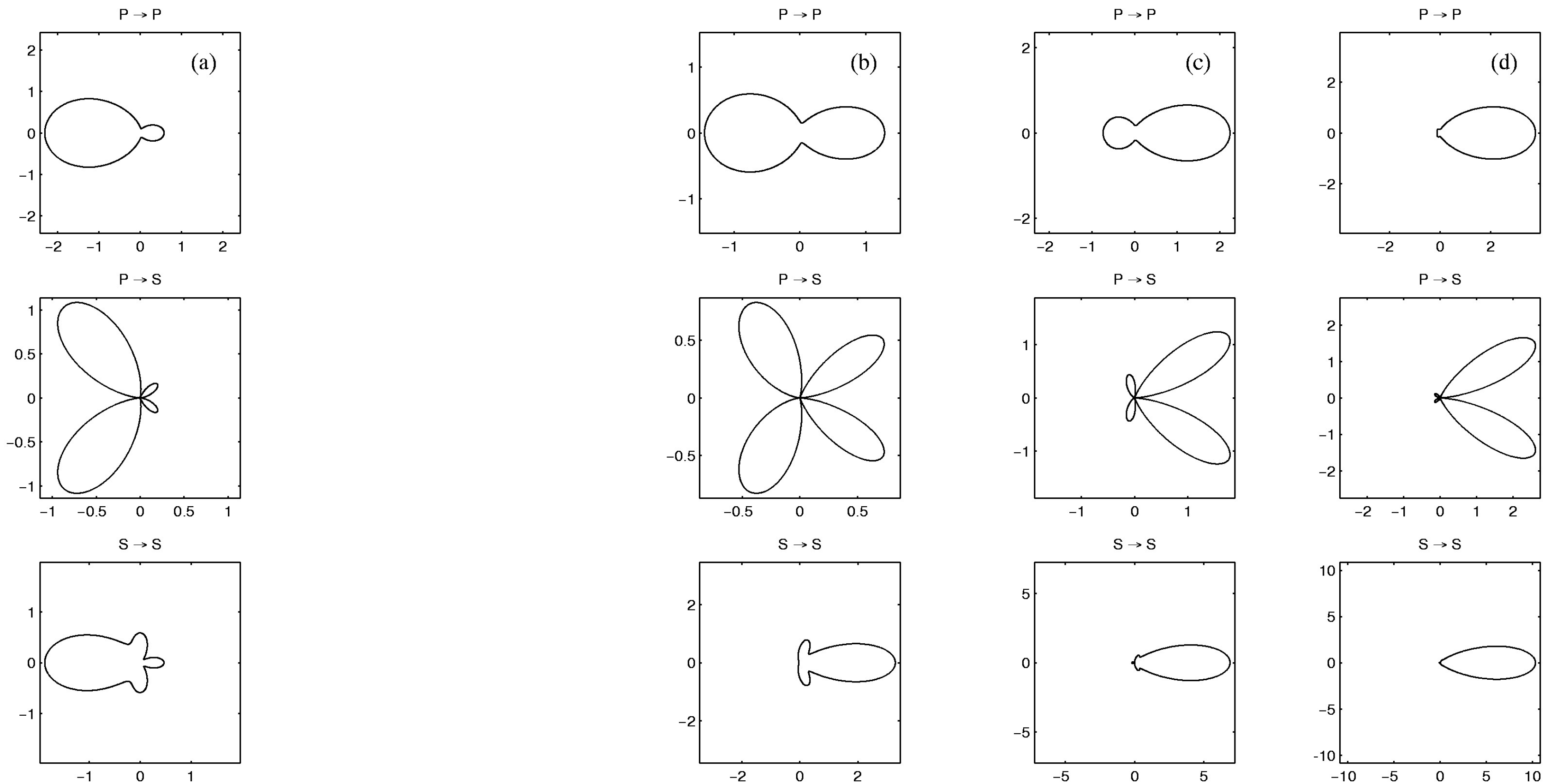
$p(\hat{\mathbf{k}}, \hat{\mathbf{k}}')$: scattering anisotropy



Propagation regimes and description of energy



Differential cross sections of scattering and conversion for a sphere of radius a



ka→0 : Rayleigh approximation

ka=1.2

ka=1.6

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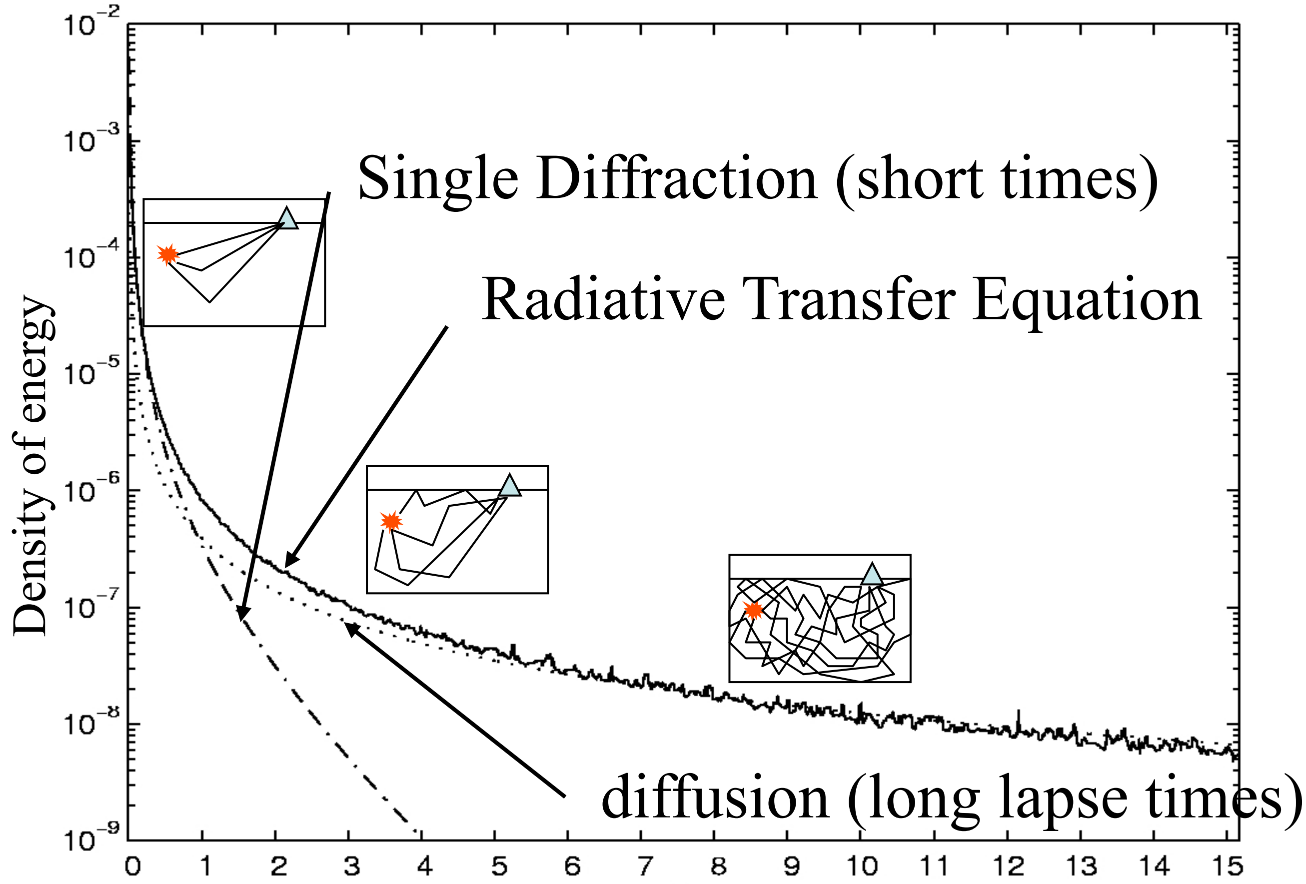
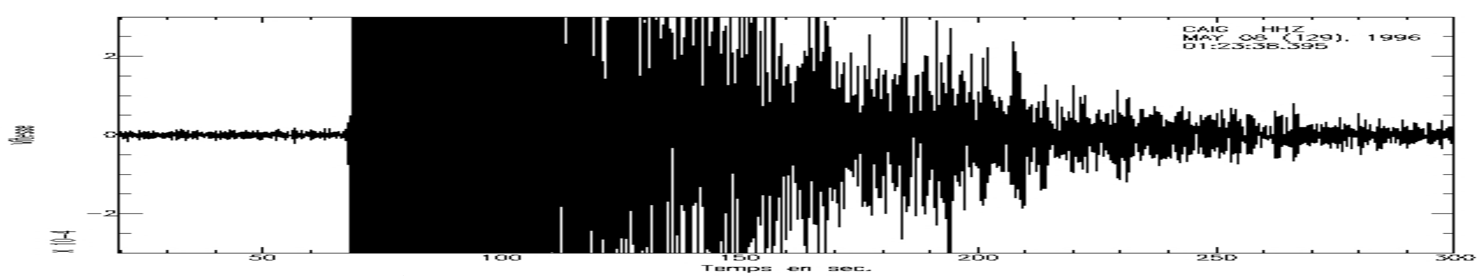
(averaged in φ for S polarisation)

Propagation regimes and description of energy





time

The Equation of Radiative Transfer

Second moment of the Green's function is governed by the Bethe-Salpeter equation:

$$\langle GG^\star \rangle = \langle G \rangle \langle G^\star \rangle + \langle G \rangle \langle G^\star \rangle K \langle GG^\star \rangle$$

K, Intensity Operator describes all scattering situations.

Neglecting recurrent scattering leads to:

$$\partial_t I(t, \vec{\Omega}, \vec{r}) + \vec{\Omega} \cdot \vec{\nabla}_r I(t, \vec{r}, \vec{\Omega}) \quad =$$

$$-\frac{1}{l} + \frac{1}{4\pi l} \int d\vec{\Omega}' I(t, \vec{\Omega}', \vec{r}) P(\vec{\Omega}, \vec{\Omega}')$$

Describes the transport of the incoherent part of the intensity.

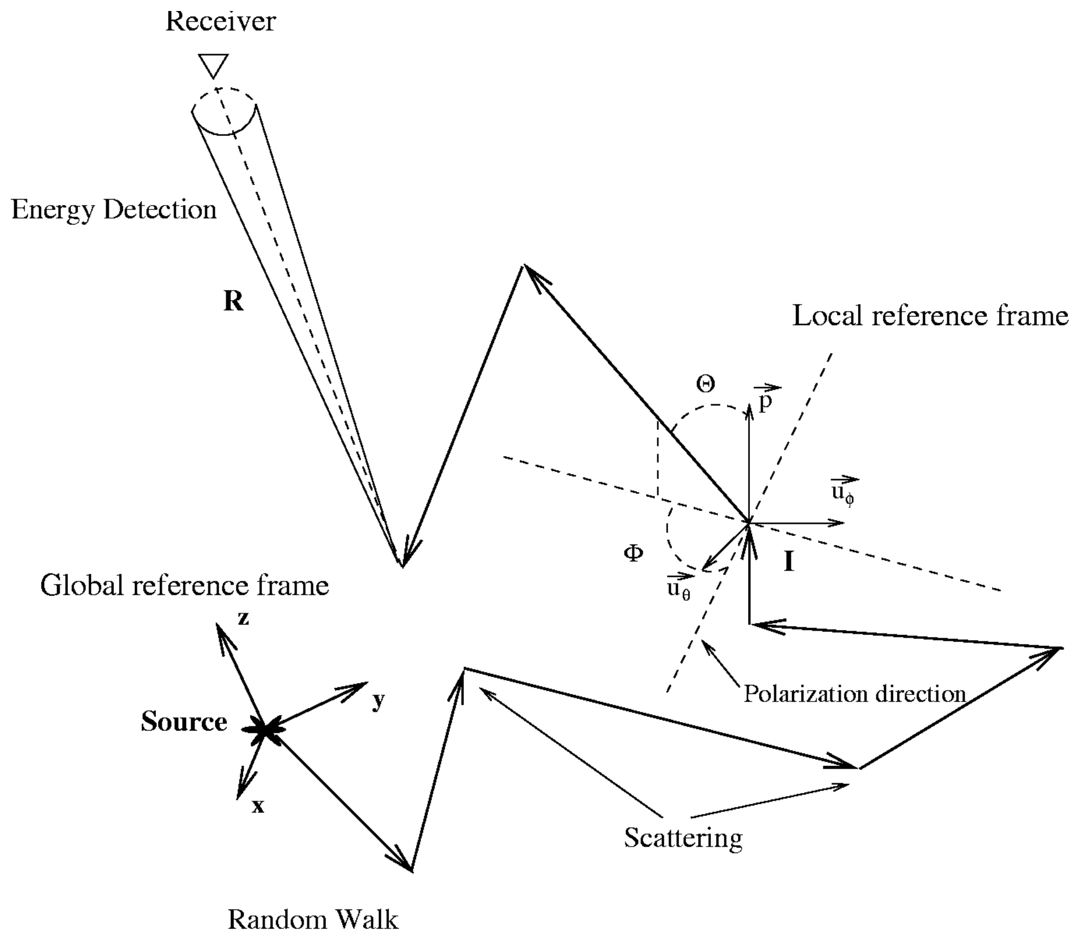
I, Specific Intensity function of space direction, time and position

$P(\vec{\Omega}, \vec{\Omega}')$, phase function (matrix) related to the power spectrum of the inhomogeneities

The radiative transfer equation

« particle analogy »

propagation under the ray theory assumptions



parameters: l_P, l_S ..., differential cross-sections

Single Scattering Approximation

The waves interact only once with the medium inhomogeneities

First term of an expansion of the intensity in a multiple scattering series:

$$I = I^0 + I^1 + \cdots + I^n + ...$$

I^0 : Coherent Intensity
I^n : Mean intensity of waves that have been scattered n times

$$I^1 \sim \frac{l}{t^2} e^{-vt/l}$$

When $vt \ll l$ reduces to the Born Approximation

The Diffusion Approximation

General Idea:

- Each scattering distributes energy over all space directions
- After several scatterings the intensity becomes almost isotropic

$$I(t,\vec{r},\vec{\Omega}) = \text{Angularly Averaged Intensity} + \text{constant} \times \vec{J}(t,\vec{r})\cdot\vec{\Omega}$$

The current density $\vec{J}(\vec{r},t)$, points in the direction of maximum energy flow. Integrating the RT Eq over all space directions leads to:

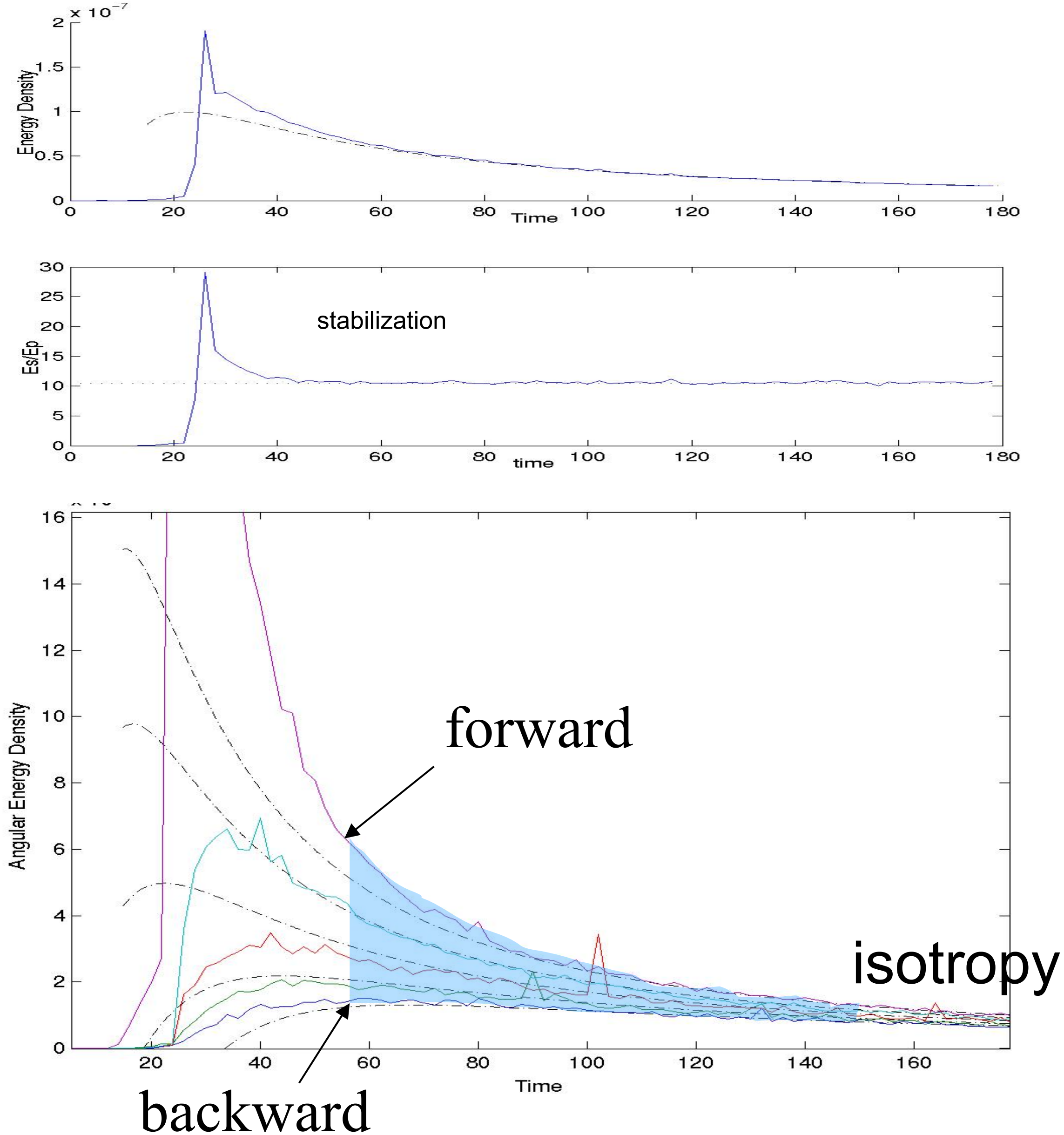
$$\partial_t \rho(t,\vec{r}) - D\nabla^2\rho(t,\vec{r}) = \delta(t,\vec{r})$$

where rho is the local energy density.

$$\rho(\boldsymbol{r}|\boldsymbol{r}',t) = \frac{1}{(4\pi Dt)^{d/2}} e^{-|\boldsymbol{r}-\boldsymbol{r}'|^2/4Dt}$$

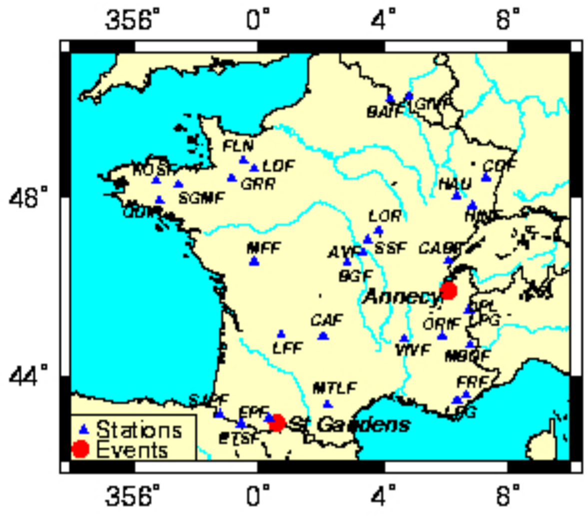
$\rho(t,\vec{r}) \sim \frac{1}{(Dt)^{3/2}}$ for large t.

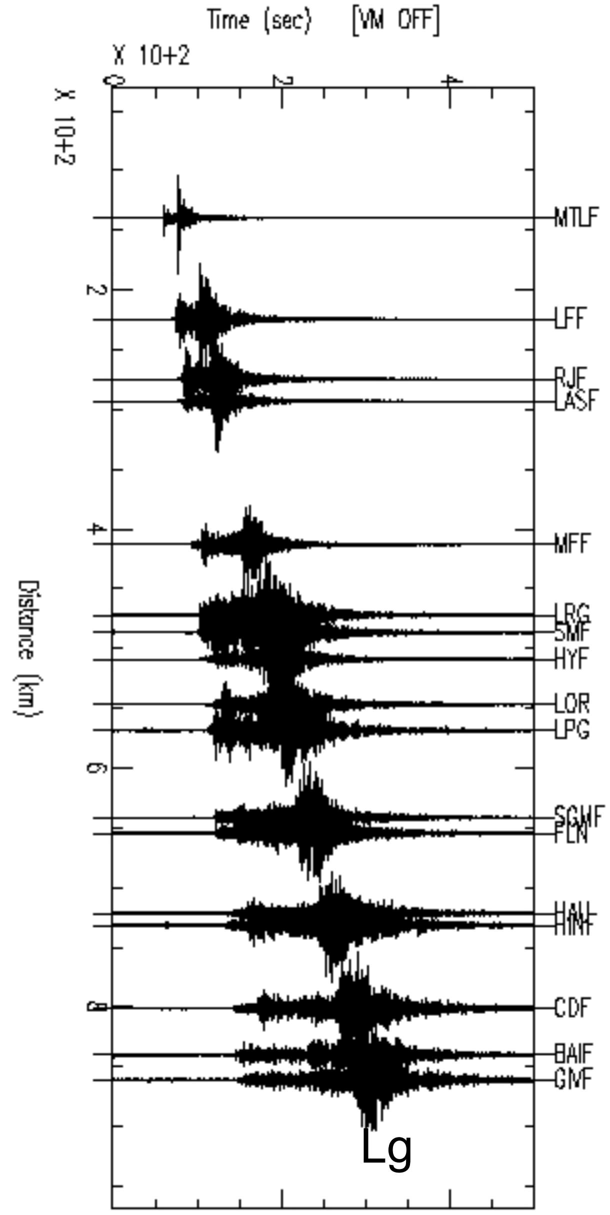
$D = vl/3$ is the diffusion constant of the waves.



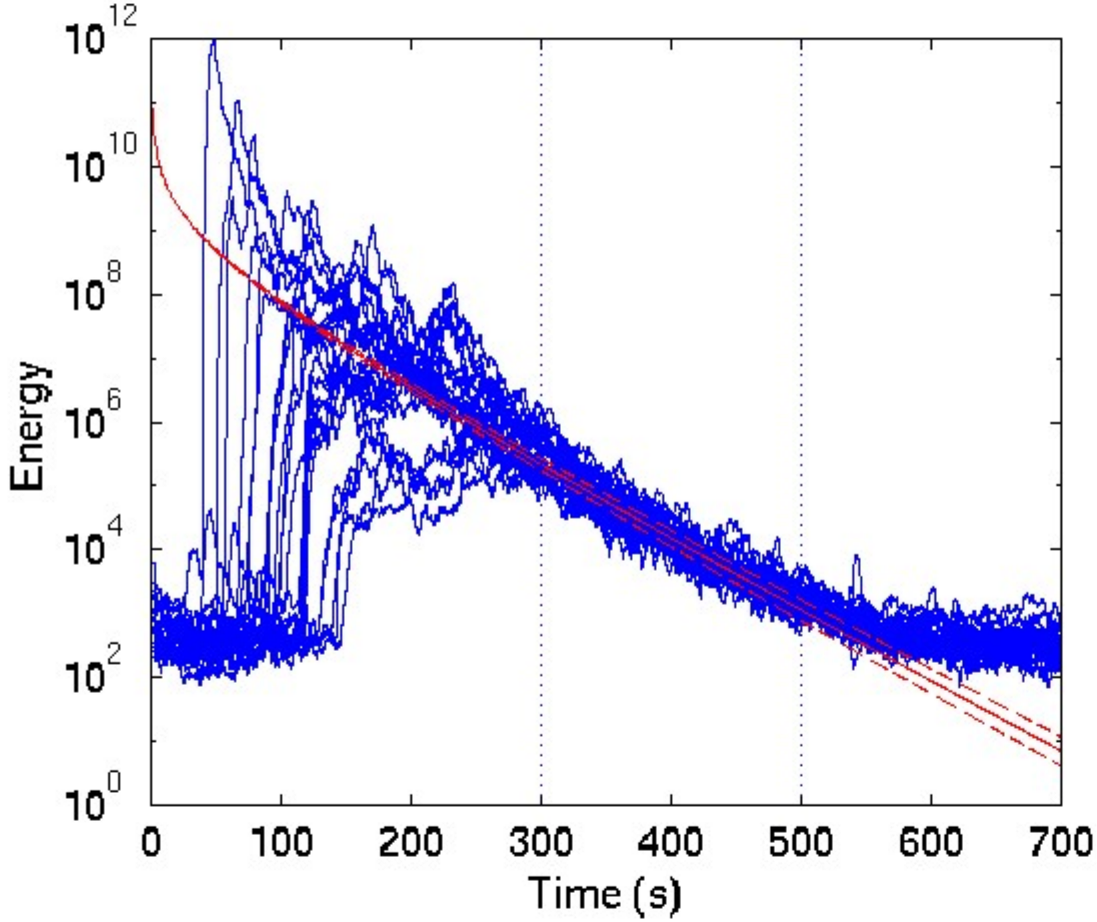
x 10^-7
Energy Density
Time
stabilization
Es/Ep
time
Angular Energy Density
forward
isotropy
Time
backward

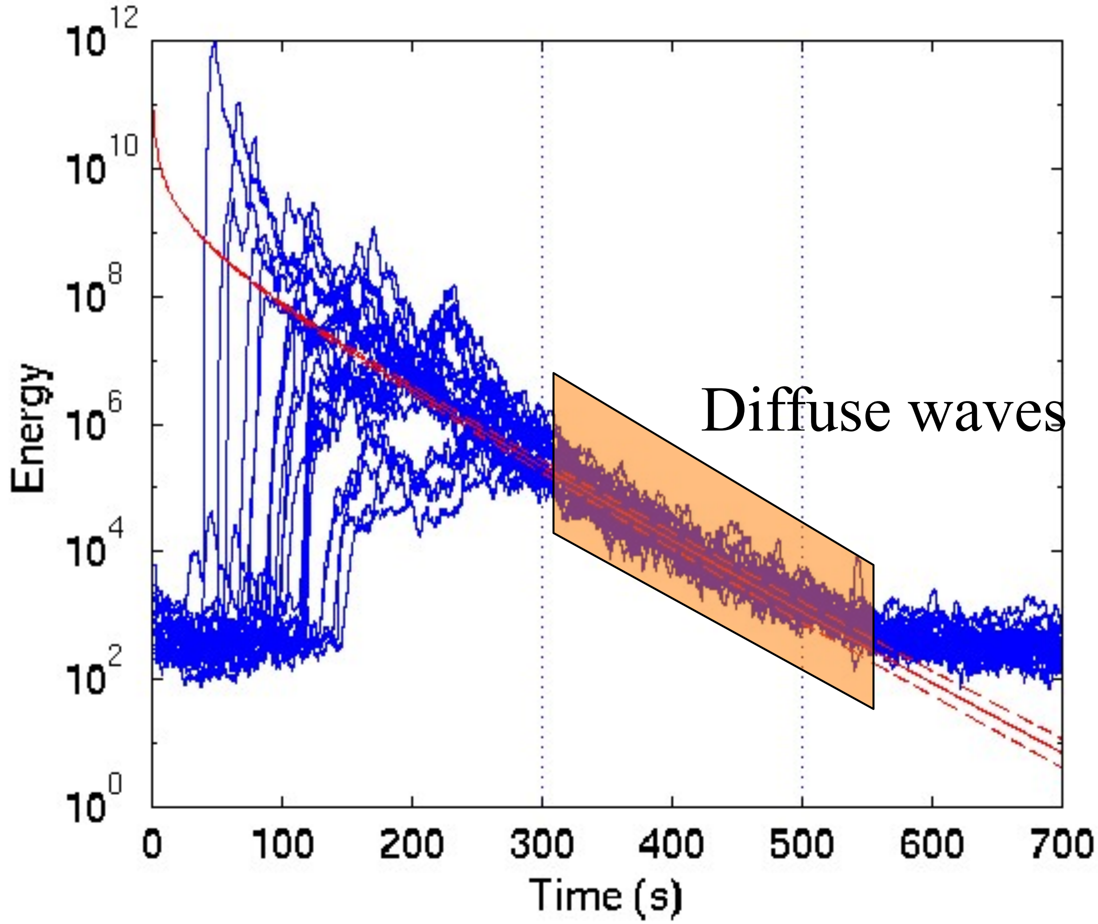
Regional seismograms





Observations at distances between 150 and 800 km!!





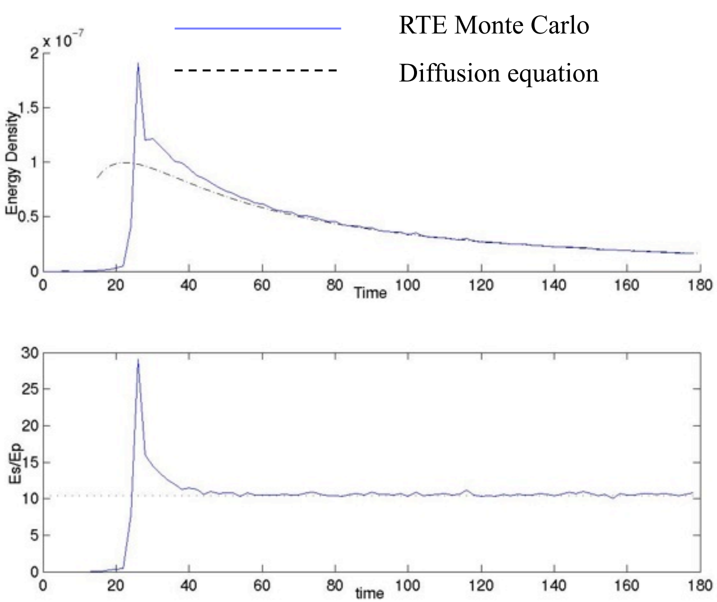
Diffuse waves
Energy
Time (s)
10^{12}
10^{10}
10^{8}
10^{6}
10^{4}
10^{2}
10^{0}
0
100
200
300
400
500
600
700

Searching for a marker of the regime of scattering...

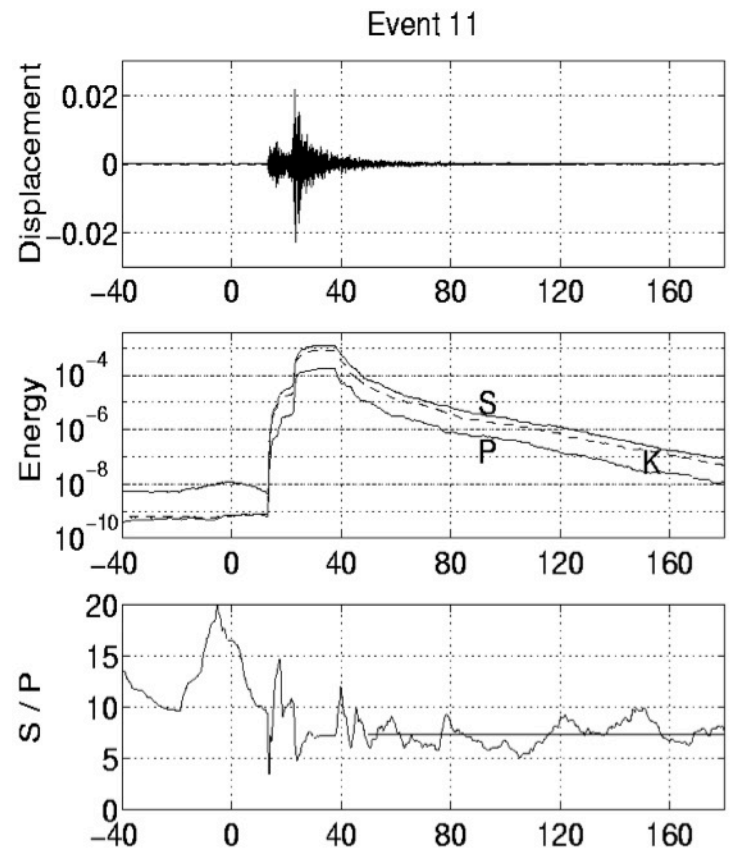
Equipartion principle for a completely randomized (diffuse) wave-field: in average, all the modes of propagation are excited to equal energy.

Implication for elastic waves (Weaver, 1982, Ryzhik et al., 1996): P to S energy ratio stabilizes at a value independant of the details of scattering!

Numerical simulation

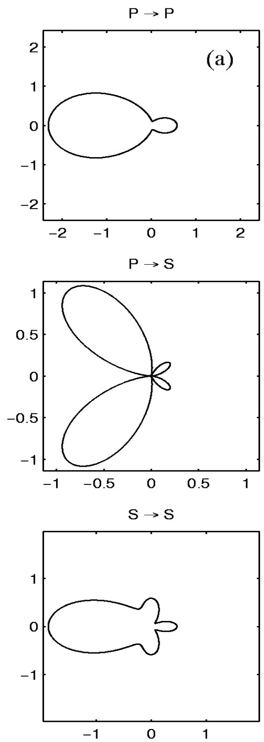


Observations

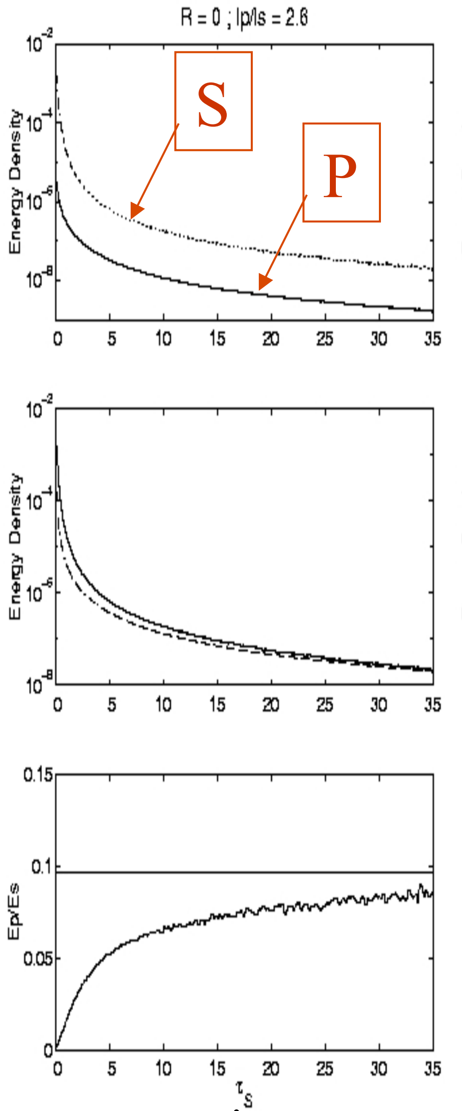


Cross sections

Numerical solutions of the RTE



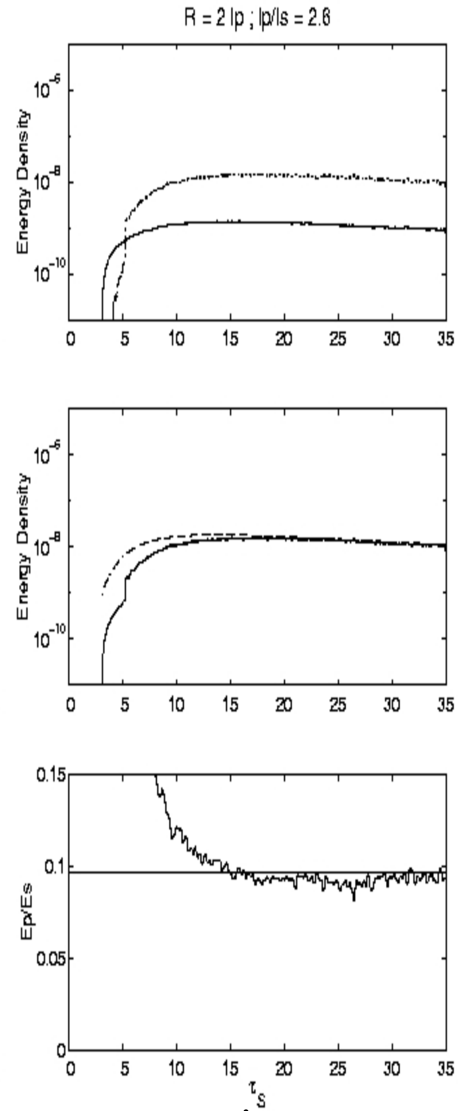
ka→0 : Rayleigh approximation



time

total energy
vs
diffusion app.

Energy ratio

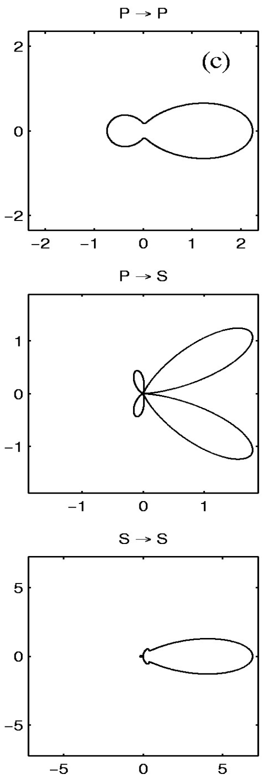


time

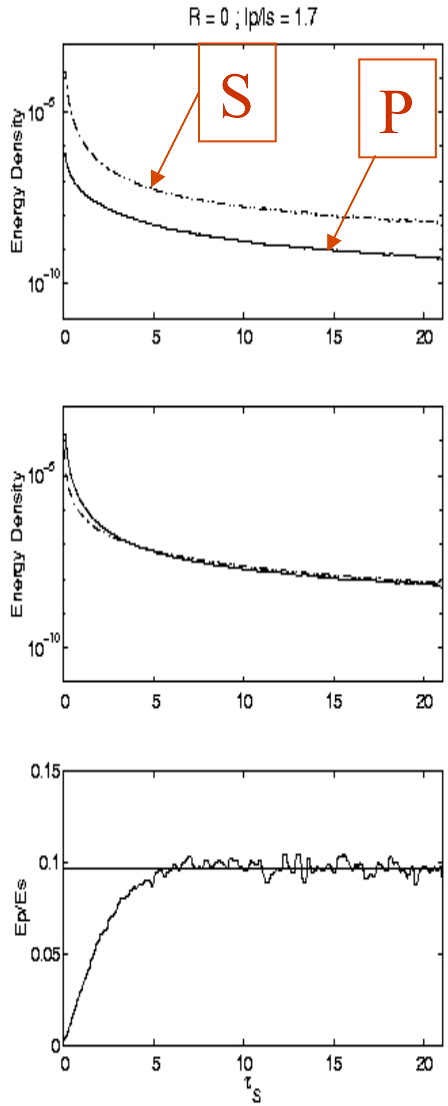
Margerin et al., 2000

Cross sections

RTE solutions (Monte Carlo)

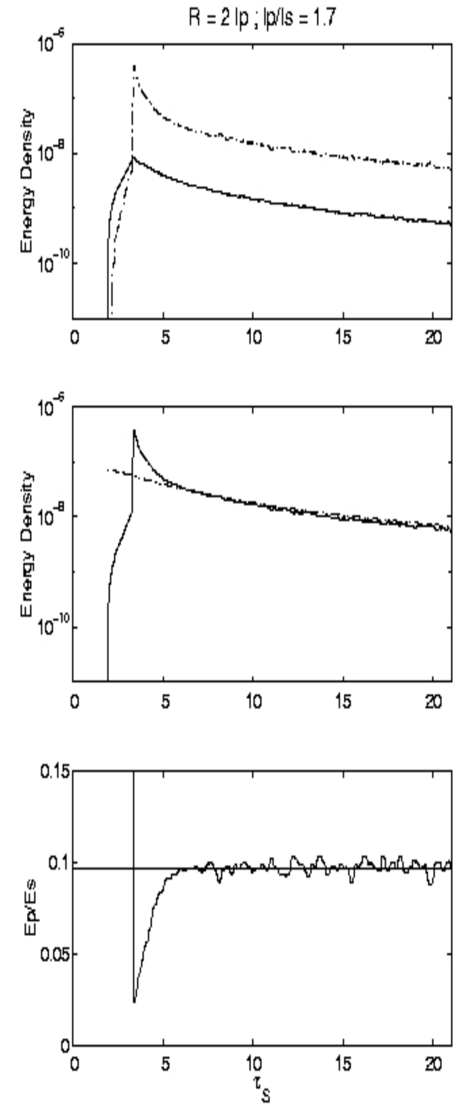


ka=1.6



total energy
vs
diffusion app.

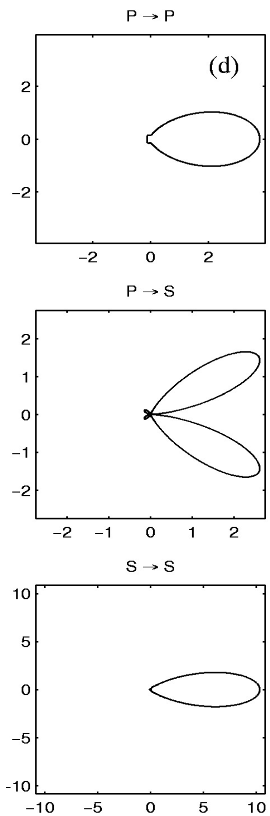
Energy ratio



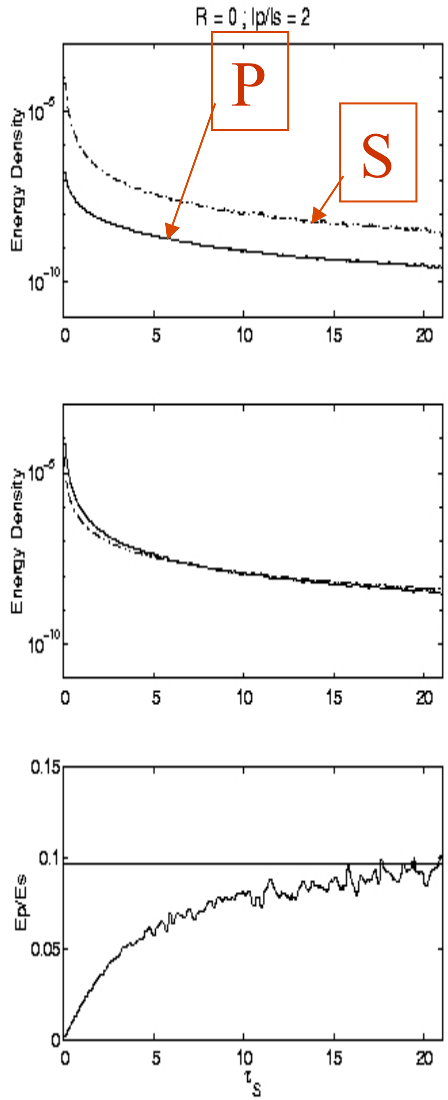
time

time

Cross sections



ka=2.0

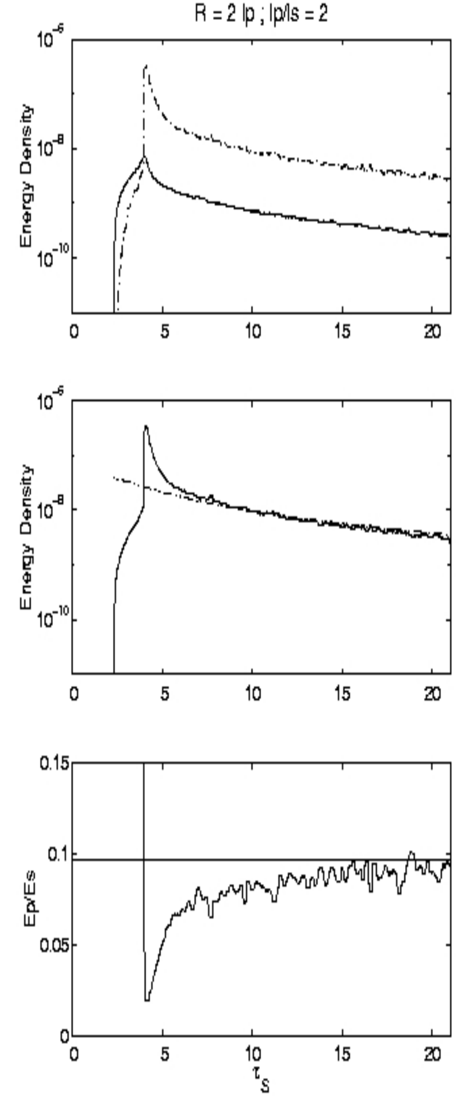


time

total energy
vs
diffusion app.

Energy ratio

is the same for different values of ka



time

Energy in an Elastic Solid

$$E = K + P + S + I$$

$$E = \frac{1}{2}\rho(\partial_t \mathbf{u})^2 + (\frac{\lambda}{2} + \mu)(\mathbf{div u})^2 + \frac{\mu}{2}(\mathbf{curl\ u})^2 + I$$

I contains mixed partial derivatives
$K = H^2 + V^2$

Focus on the ratios:
P/S, $K/(P+S)$,$I/(S+P)$,H^2/V^2

Equipartition predicts: Any Ratio of Energies Becomes Independent of Time

Measurement of the deformation energy requires evaluation of partial derivatives of the wavefield

S to P Energy ratio as a marker of the regime of scattering...

$$G_{i,j}(\vec{R},\vec{S},t) = \sum_n \varepsilon_n \Phi^n(\vec{R}) \exp(-i\Omega_n t)$$

where ε_n are random independent variables (finite body)

(ldos)

Consequence for an infinite inhomogeneous solid:

$$\frac{E_s}{E_p} = 2\left(\frac{v_p}{v_s}\right)^3$$

Independent of the Details of the Scattering !

Independent of the position in a full space with homogeneous reference

Partition of energy (Full elastic space)

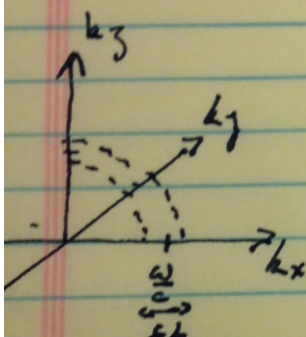
Multiple scattering, large t

→ "equipartition"

[reference medium + disorder]

Phase space of the full space elastic problem

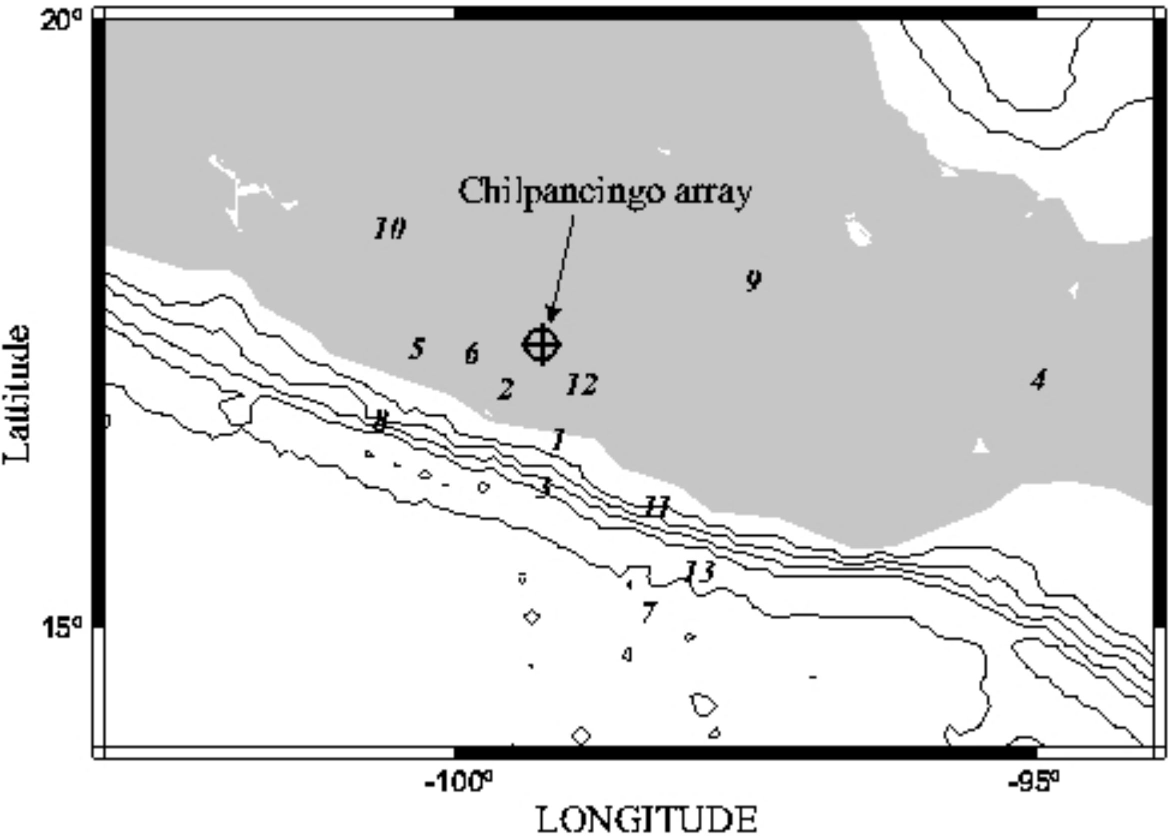
→ all propagating plane waves excited at same level of energy



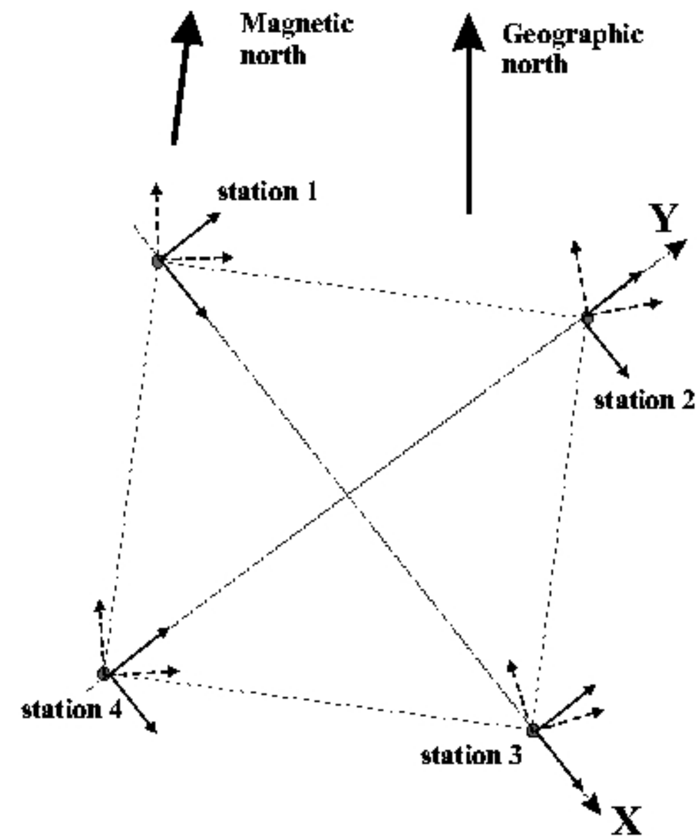
Energy in a band $\omega \pm \frac{\delta\omega}{2}$

→ Volume for P waves

$\delta k = \frac{\delta\omega}{\alpha}$

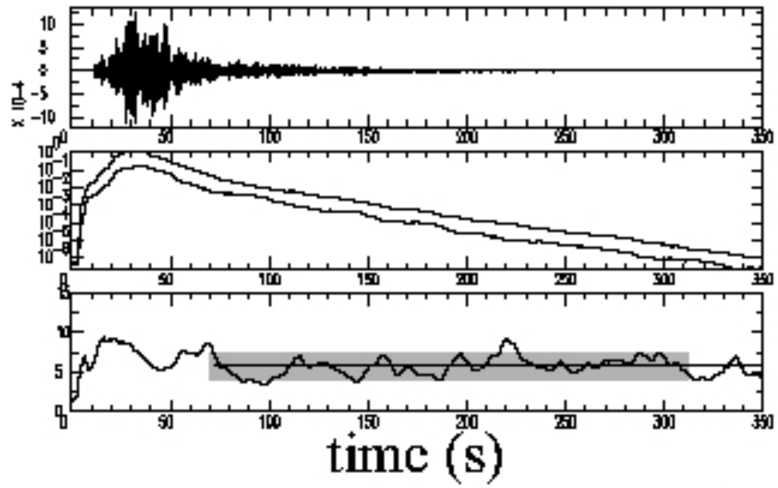


20°
15°
Latitude
Chilpancingo array
10
9
5
6
2
12
4
8
1
3
11
13
7
-100°
-95°
LONGITUDE

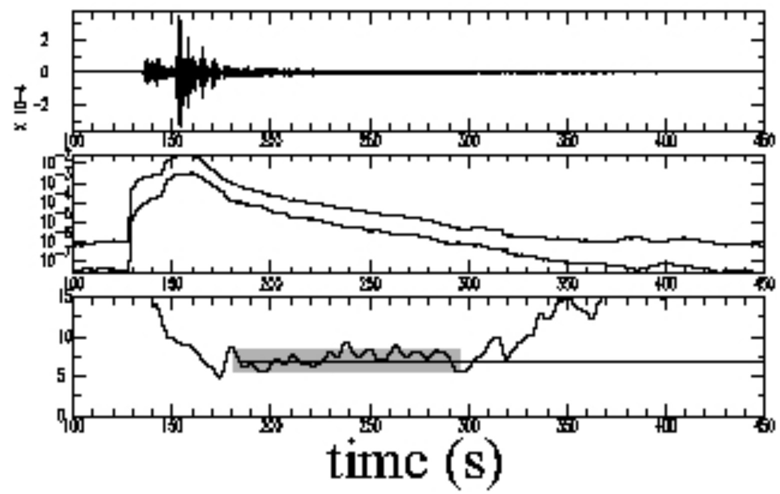


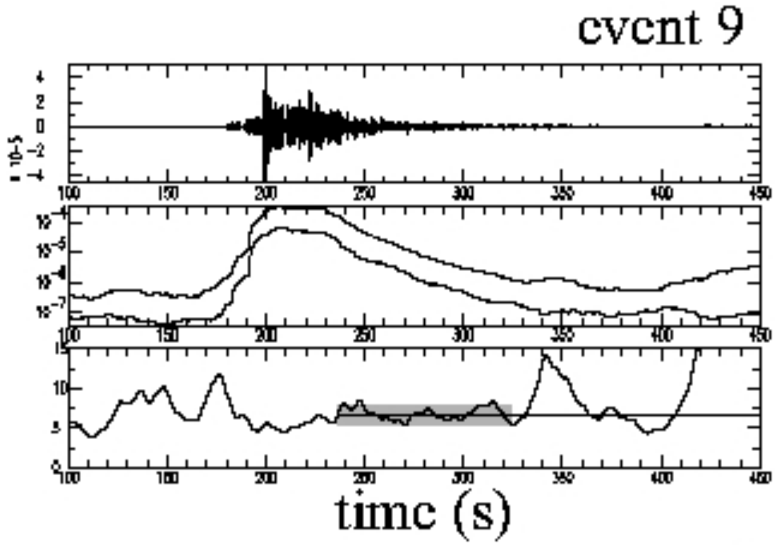
Magnetic north
Geographic north
station 1
station 2
station 3
station 4
Y
X

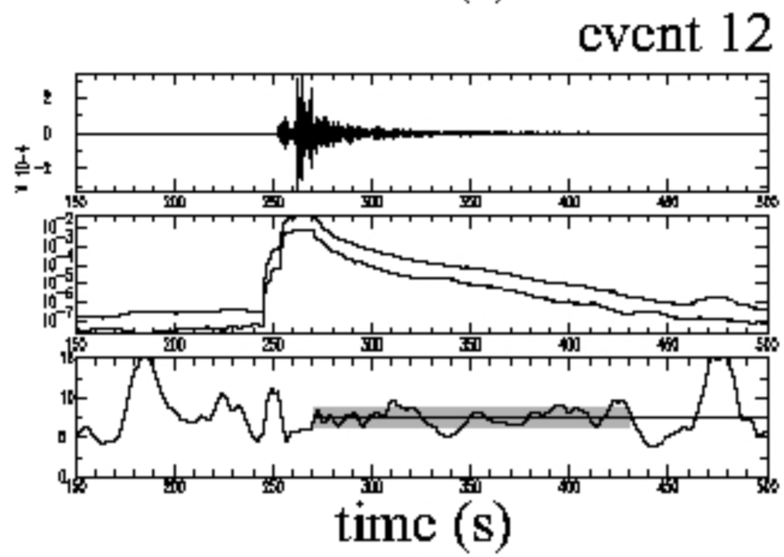
event 5



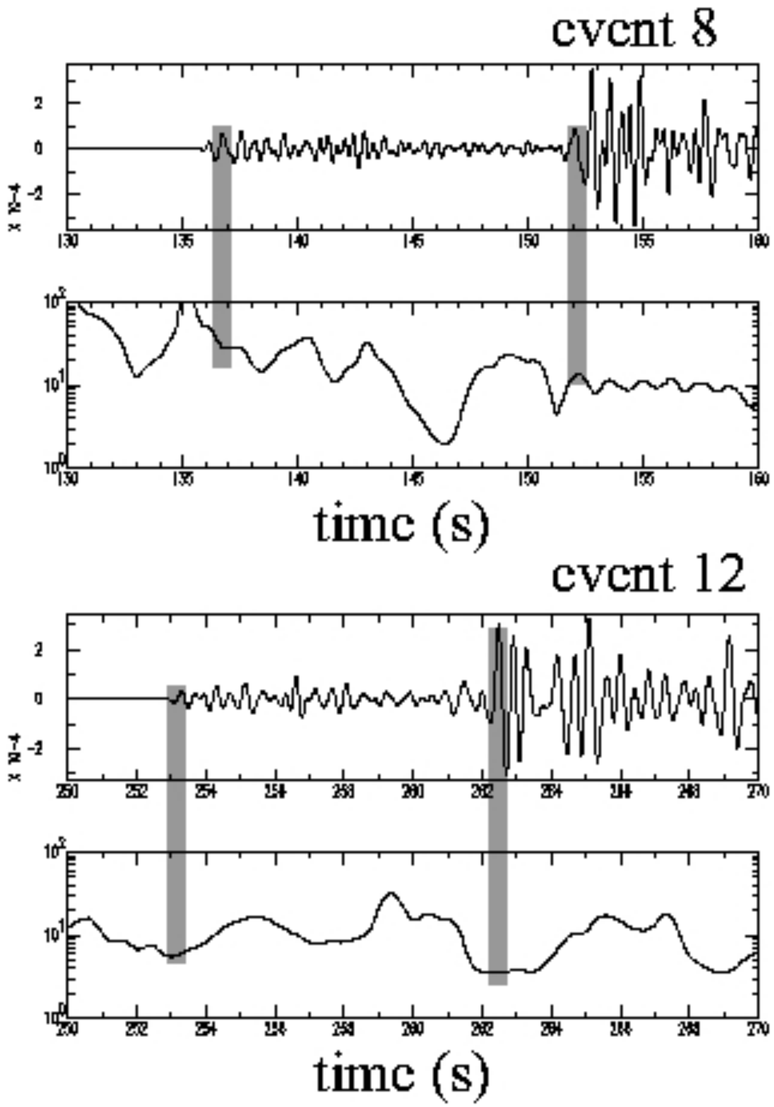
event 8

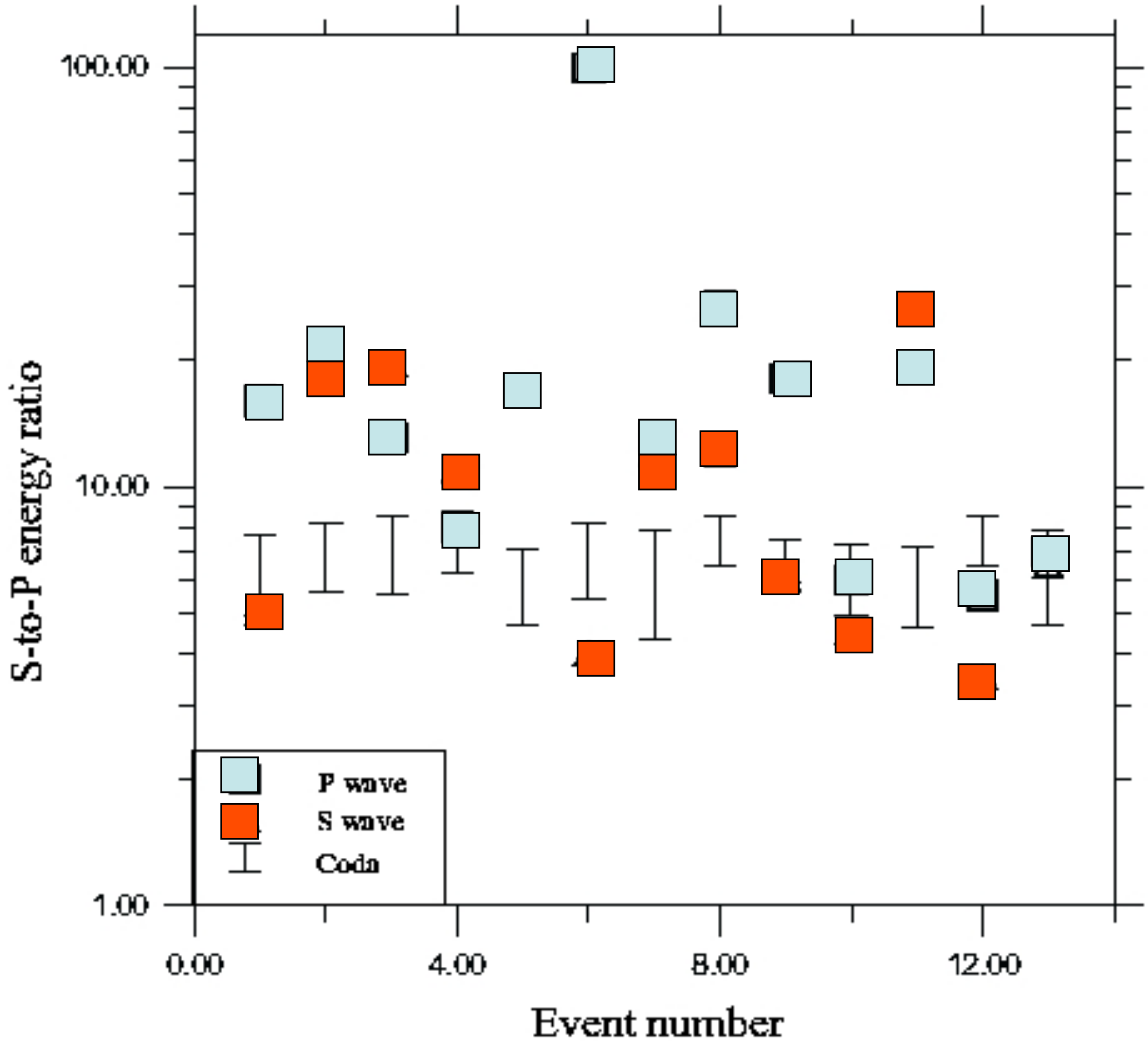



cvcnt 9
timc (s)


cvcnt 12
timc (s)

Direct waves

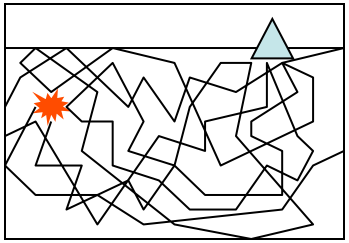




100.00
10.00
1.00
S-to-P energy ratio
0.00
4.00
8.00
12.00
Event number
P wave
S wave
Coda

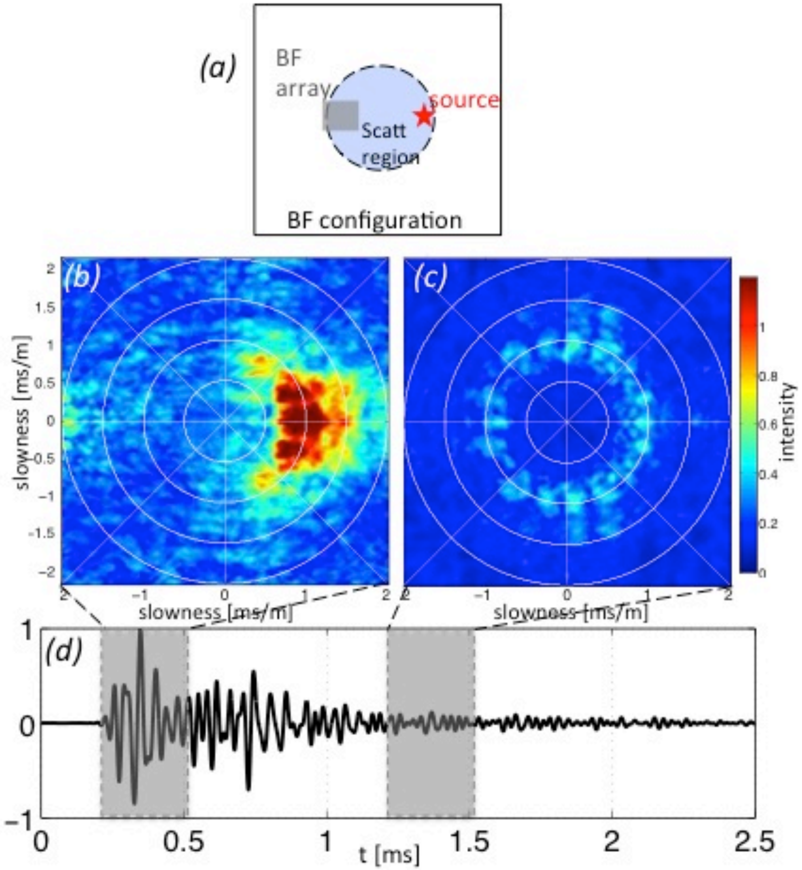
ENERGY RATIO	DATA	THEORY FULL SPACE	THEORY HALF SPACE BULK WAVES	THEORY HALF SPACE with RAYLEIGH WAVES
S/P	7.3	10.39	9.76	7.19
K/(S+P)	0.65	1	1.19	0.534
I/(S+P)	-0.62	0	-0.336	-0.617

For asymptotically long lapse time (diffusion), the disorder produces a completely randomized wave-field. such that , all the modes of propagation are excited in average to equal energy (the equipartition principle).

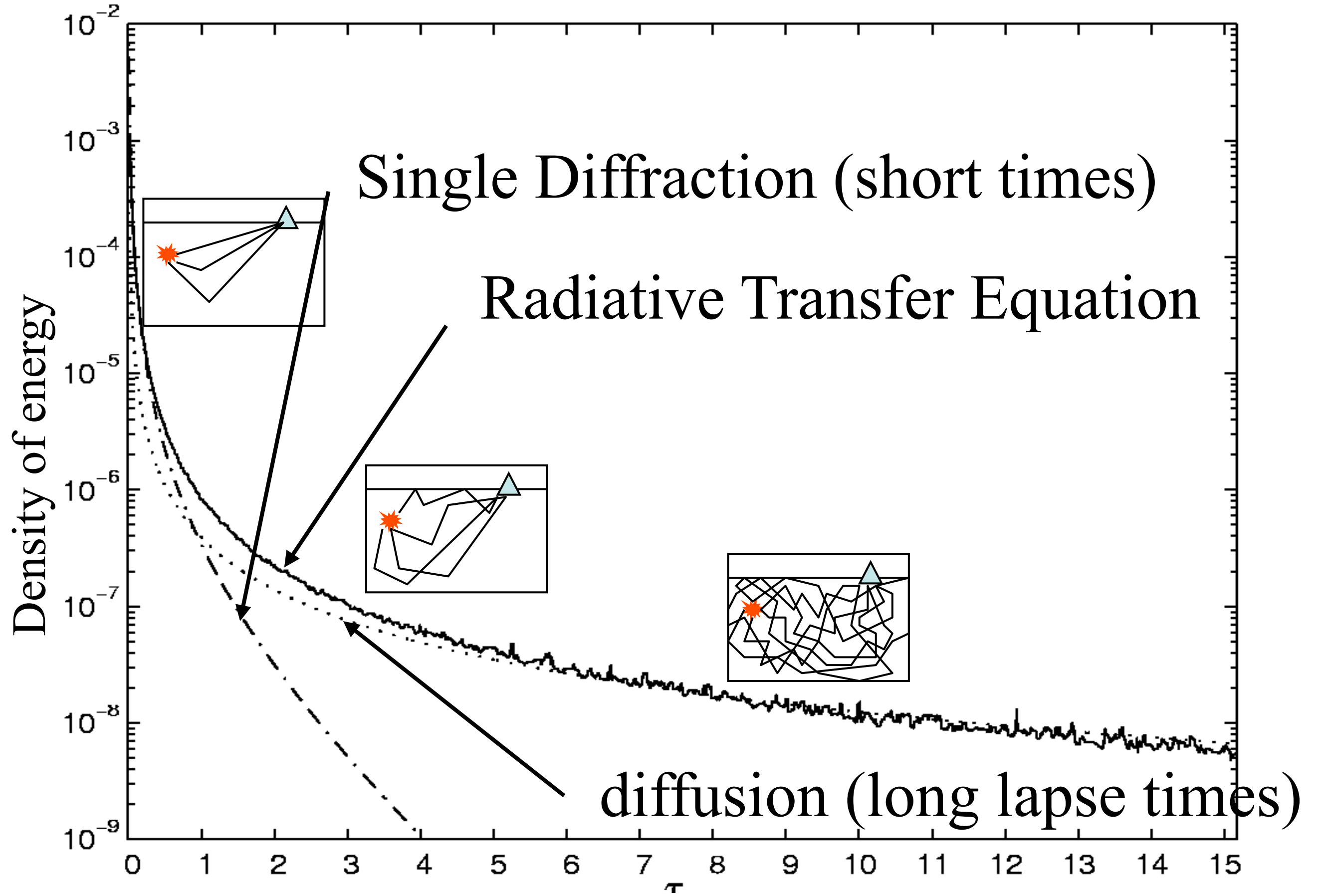


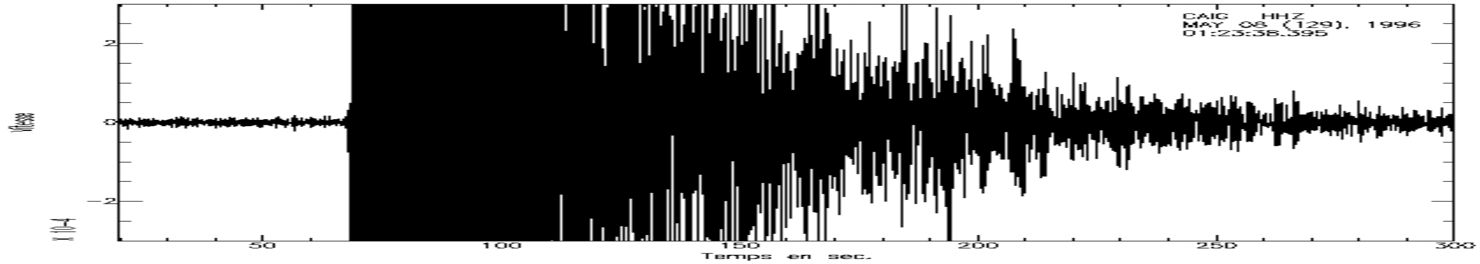
Numerical example:
2D scalar waves

➔intensity isotropy



Propagation regimes and description of energy





time

The Diffusion Approximation

General Idea:

- Each scattering distributes energy over all space directions
- After several scatterings the intensity becomes almost isotropic

$$I(t, \vec{r}, \vec{\Omega}) = \text{Angularly Averaged Intensity} + \text{constant} \times \vec{J}(t, \vec{r}) \cdot \vec{\Omega}$$

The current density $\vec{J}(\vec{r}, t)$, points in the direction of maximum energy flow.
Integrating the RT Eq over all space directions leads to:

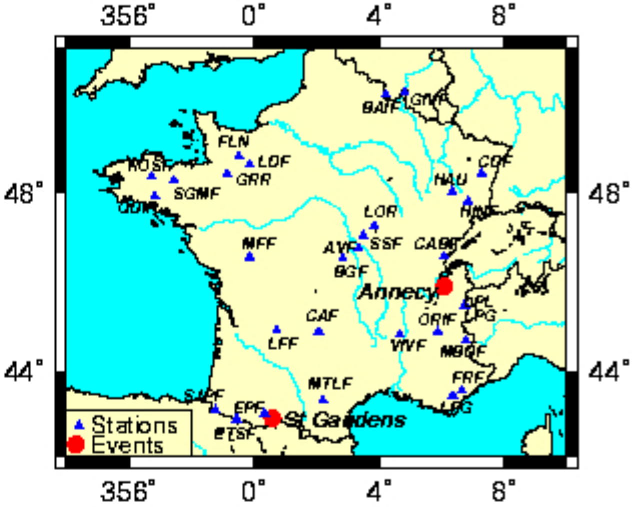
$$\partial_t \rho(t, \vec{r}) - D\nabla^2 \rho(t, \vec{r}) = S(t, \vec{r})$$

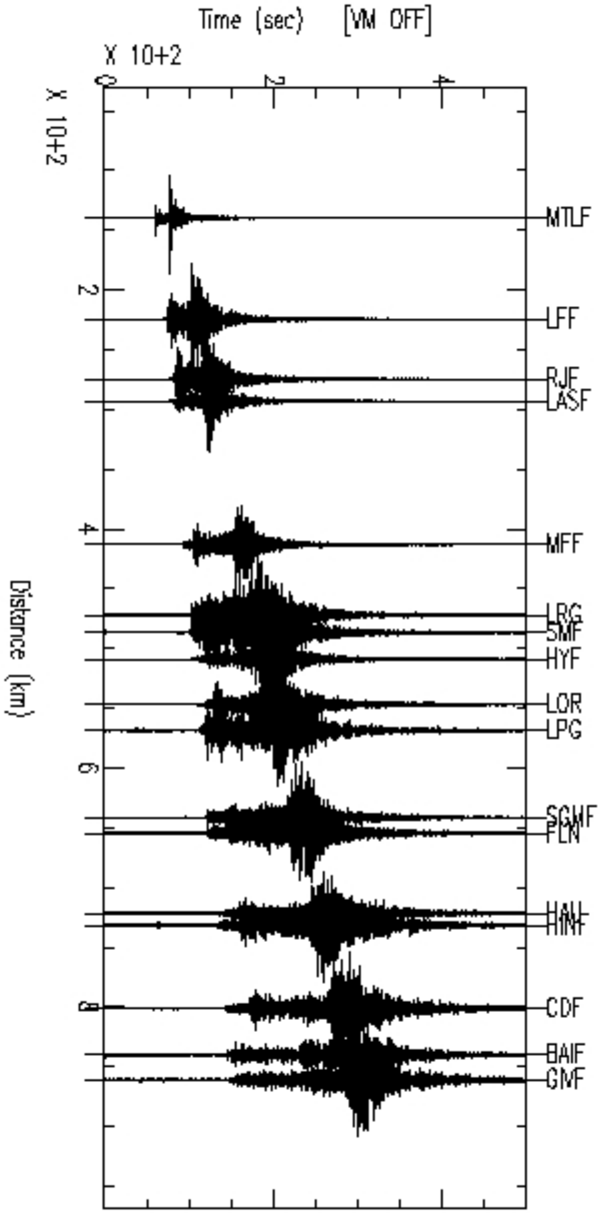
where rho is the local energy density.

$\rho(t, \vec{r}) \sim \frac{1}{(Dt)^{3/2}}$ for large t.

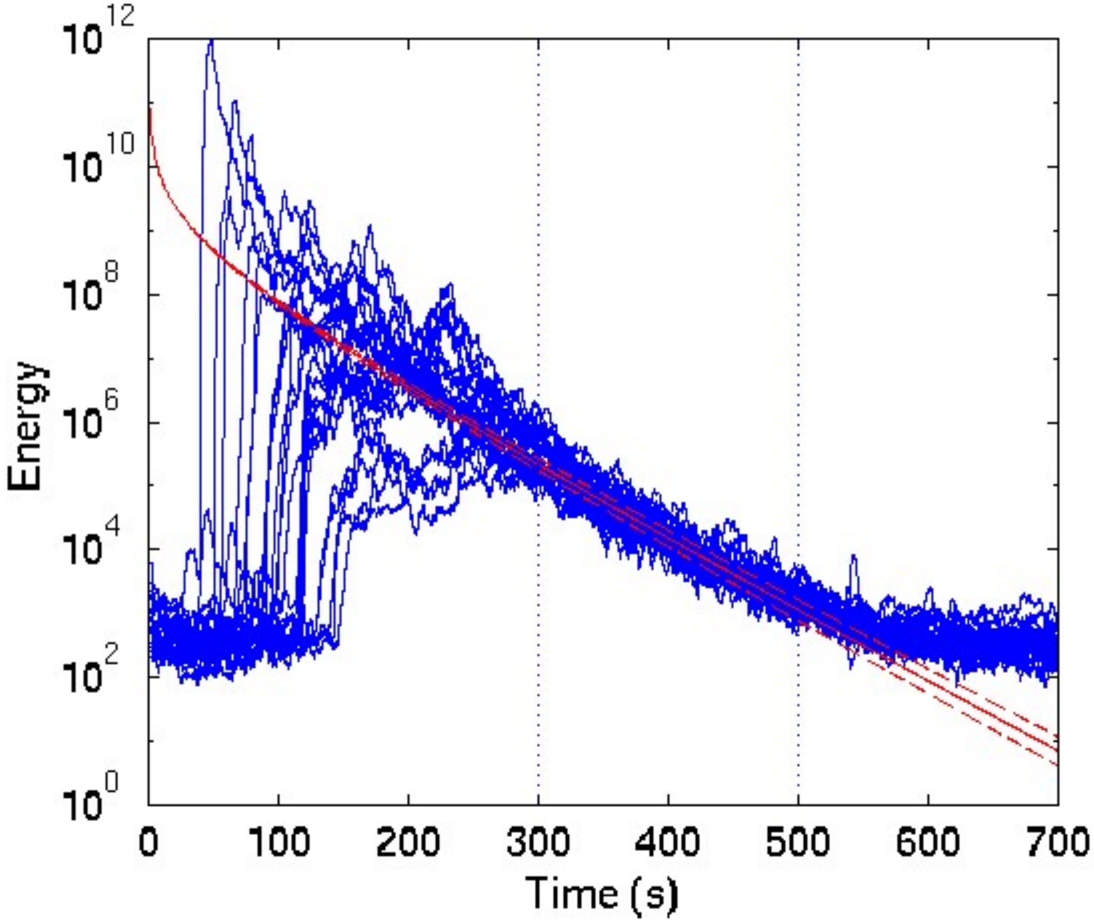
$D = vl/3$ is the diffusion constant of the waves.

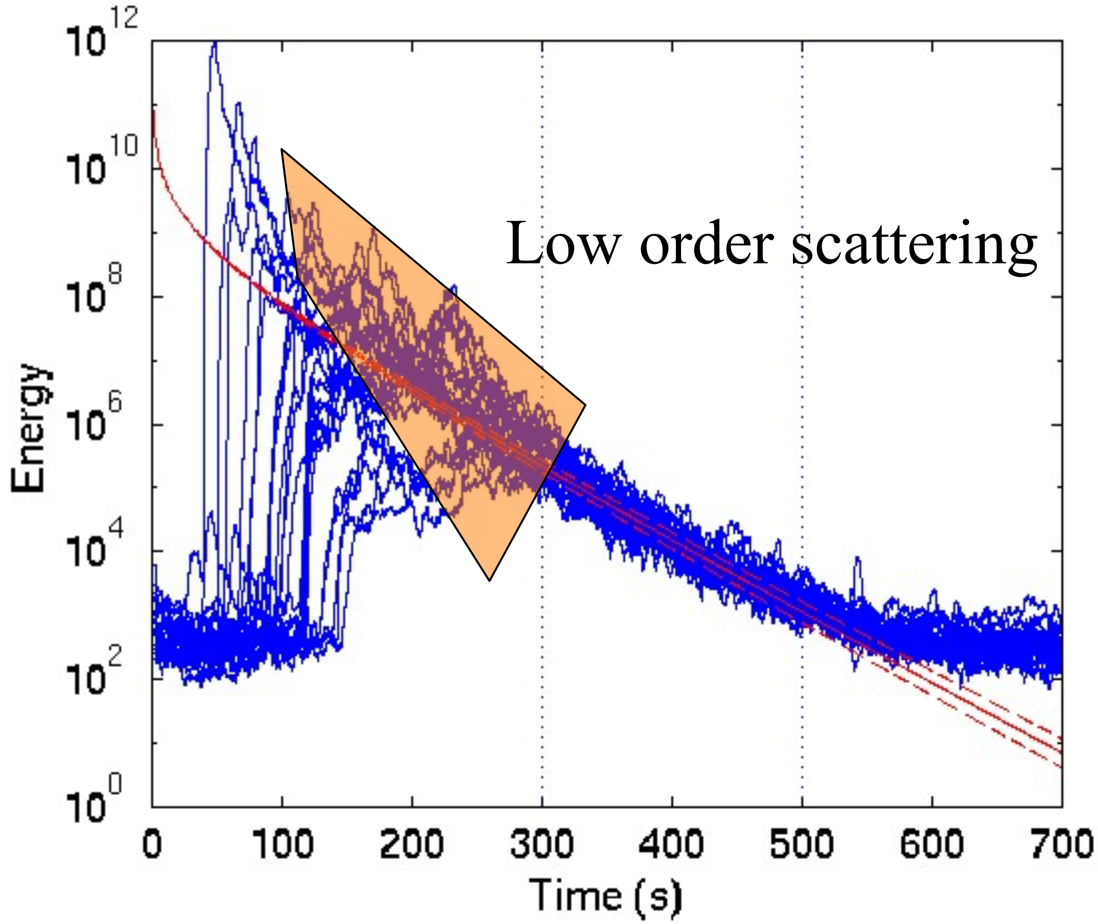
Coda of regional seismograms





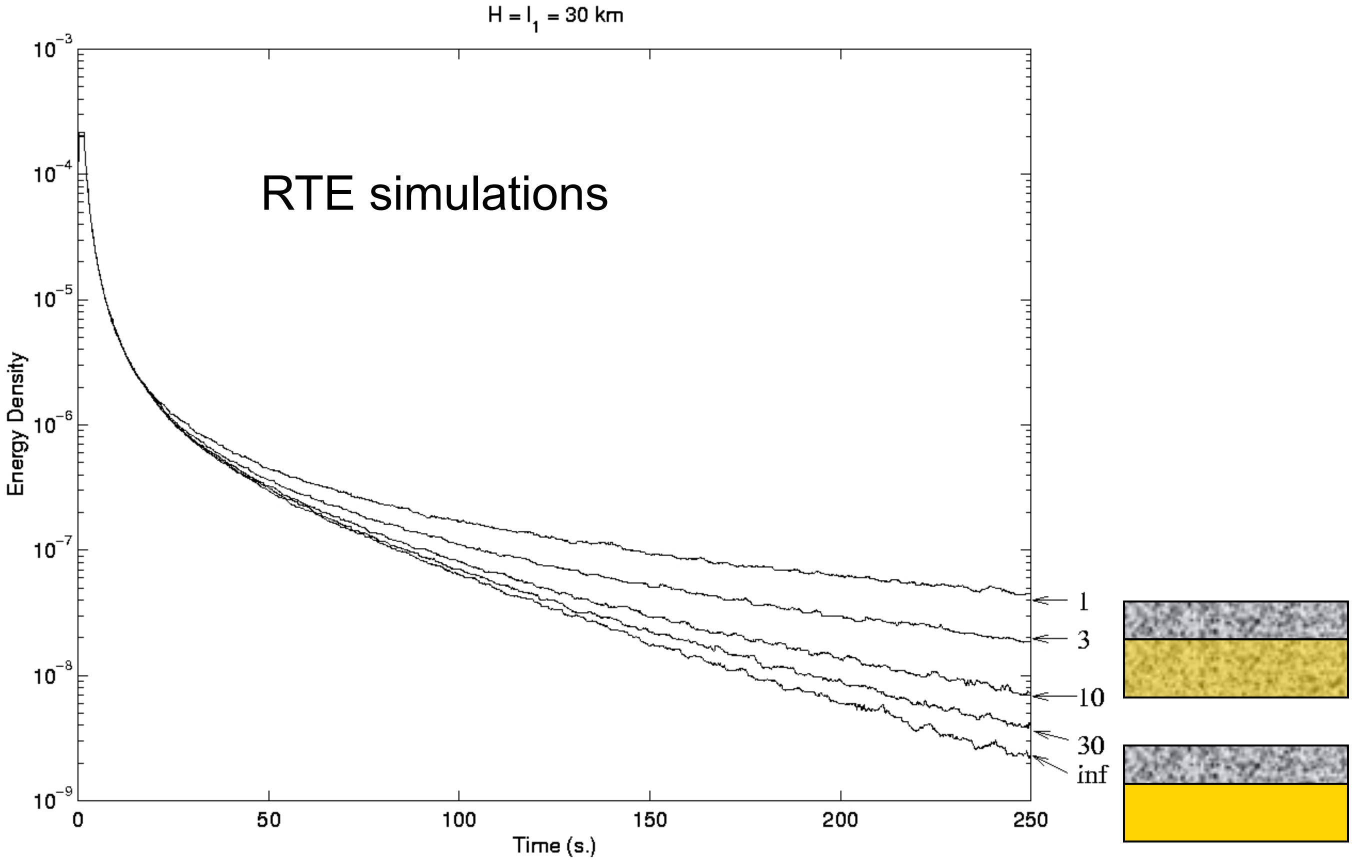
Observations at distances between 150 and 800 km!!



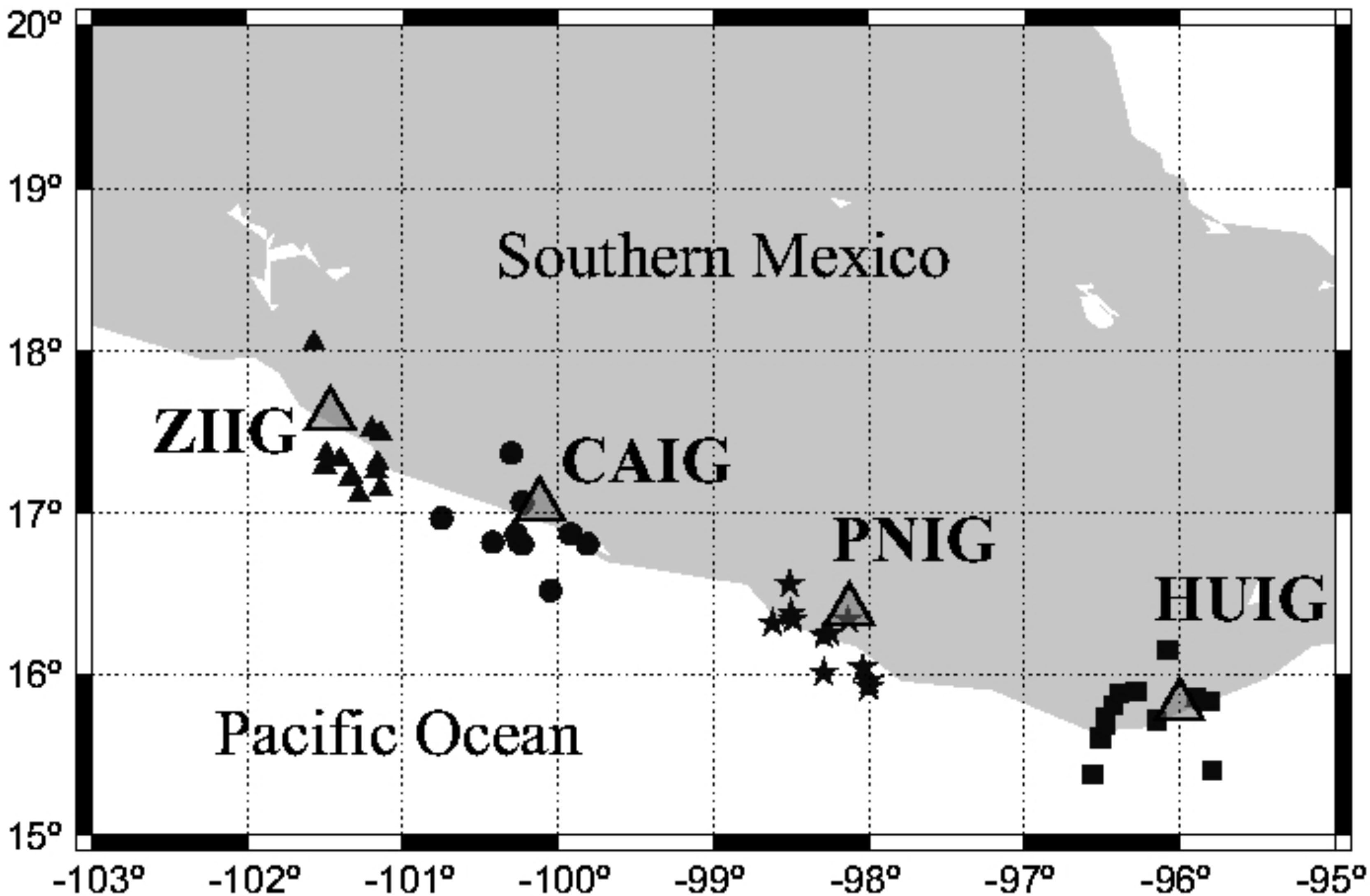


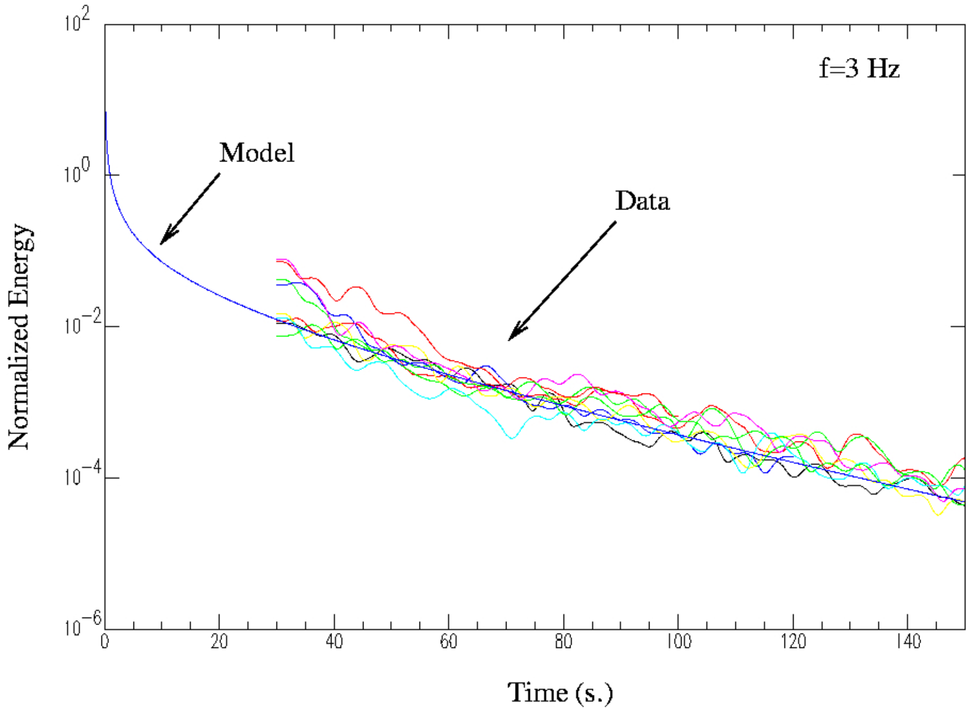
requires radiative transfer equation

Influence of the value of mantle mean free path



Leakage of energy in the mantle

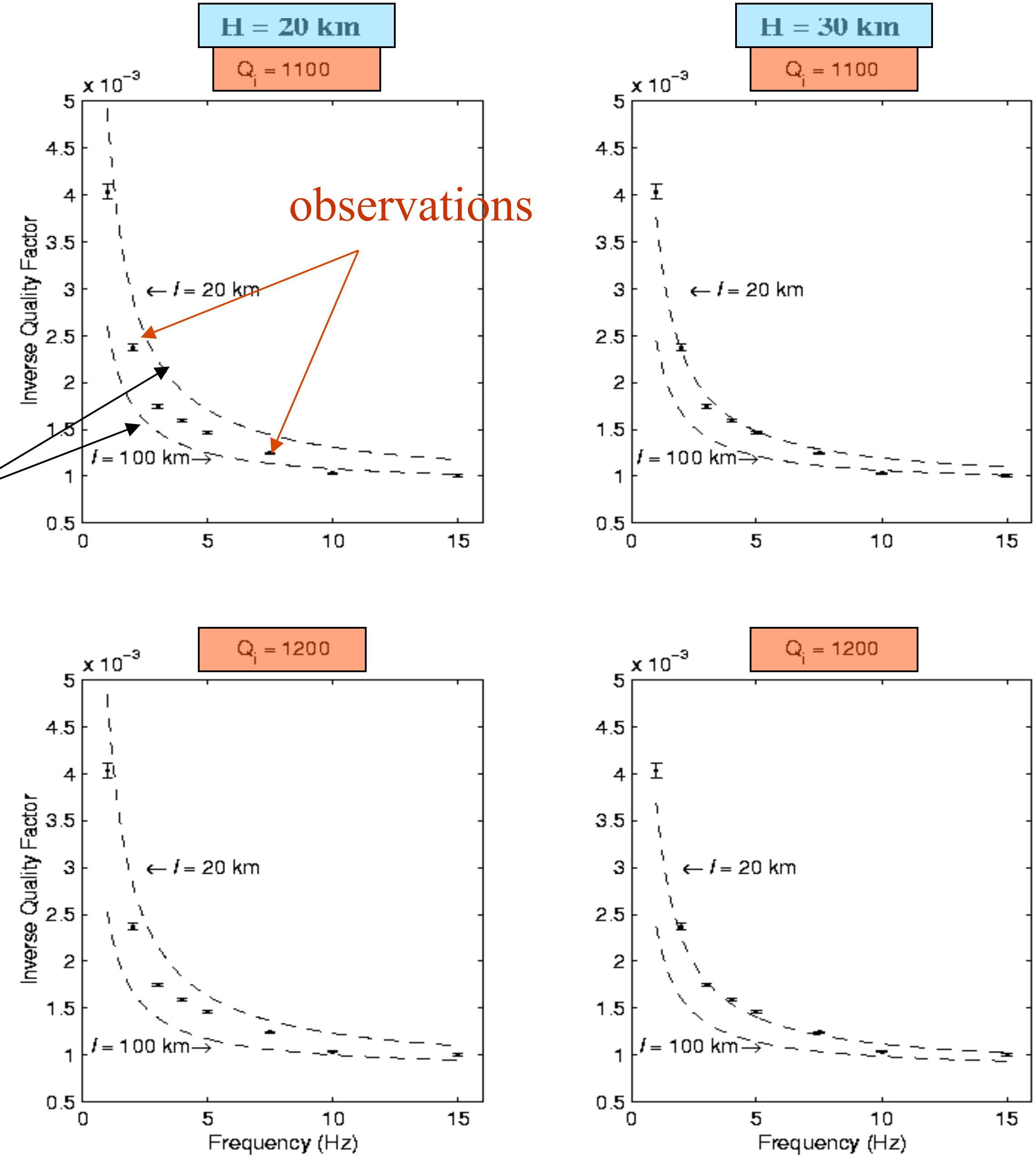

Southern Mexico
Pacific Ocean
ZIIG
CAIG
PNIG
HUIG
20º
19º
18º
17º
16º
15º
-103º
-102º
-101º
-100º
-99º
-98º
-97º
-96º
-95º


f=3 Hz
Model
Data
Normalized Energy
Time (s.)
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
0
20
40
60
80
100
120
140

With dissipation

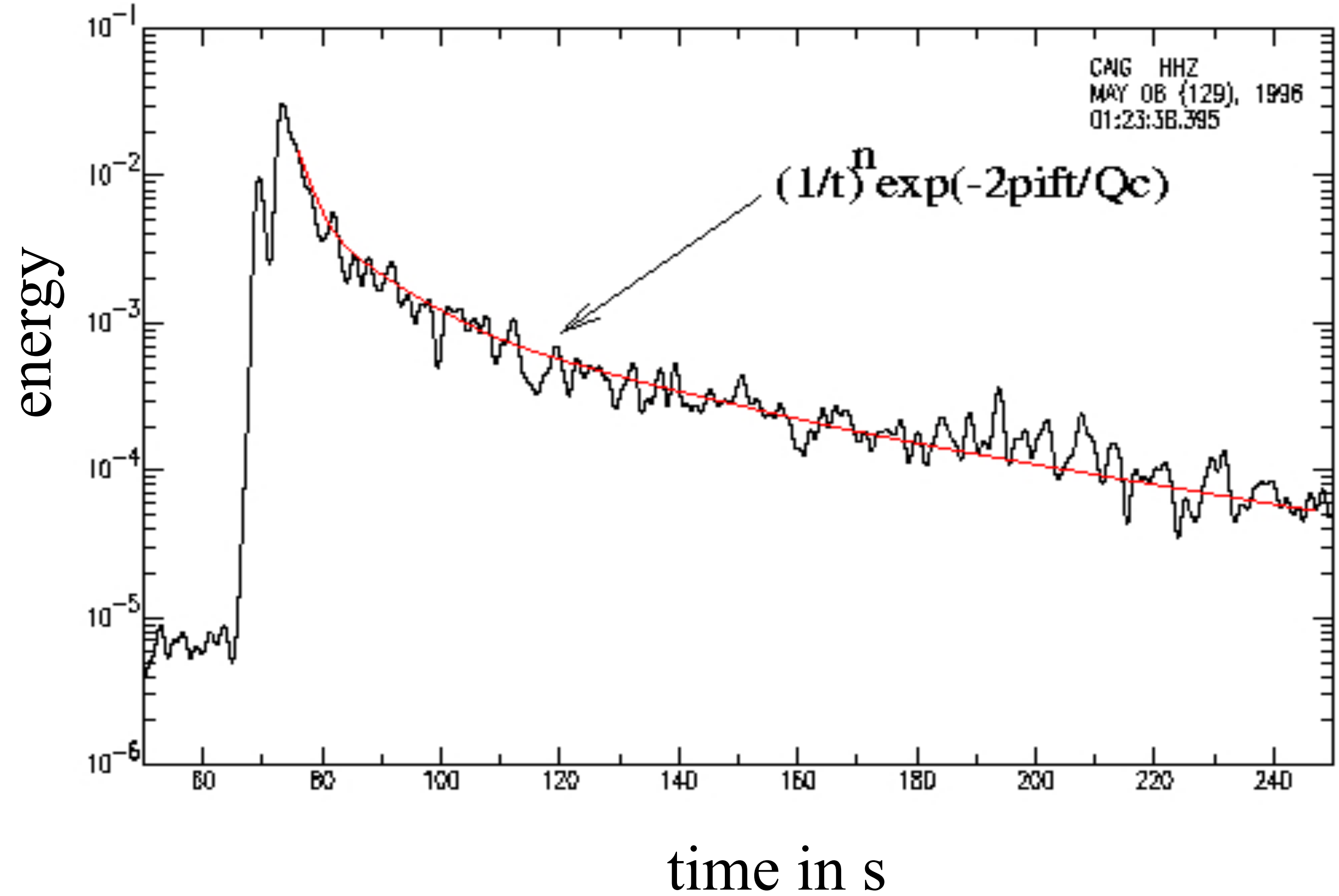
Q > 1000

RTE solutions



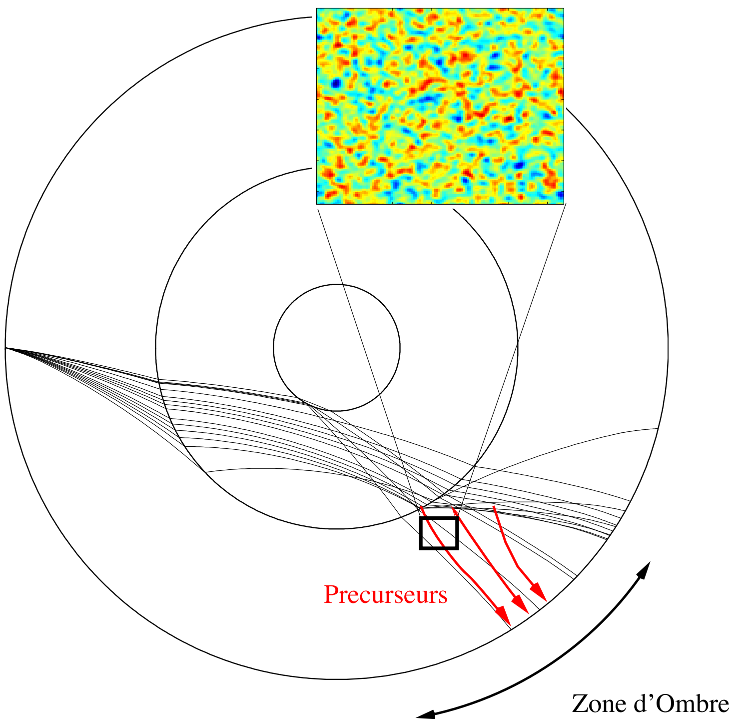
l(f) in the range 20-80 km

Energy decay in the coda (Aki and Chouet, 1975)

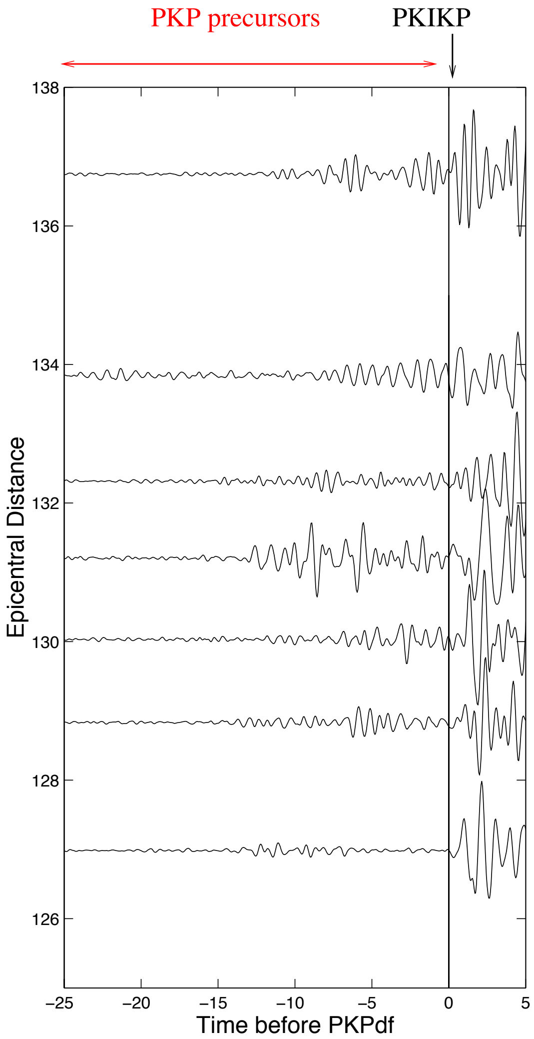


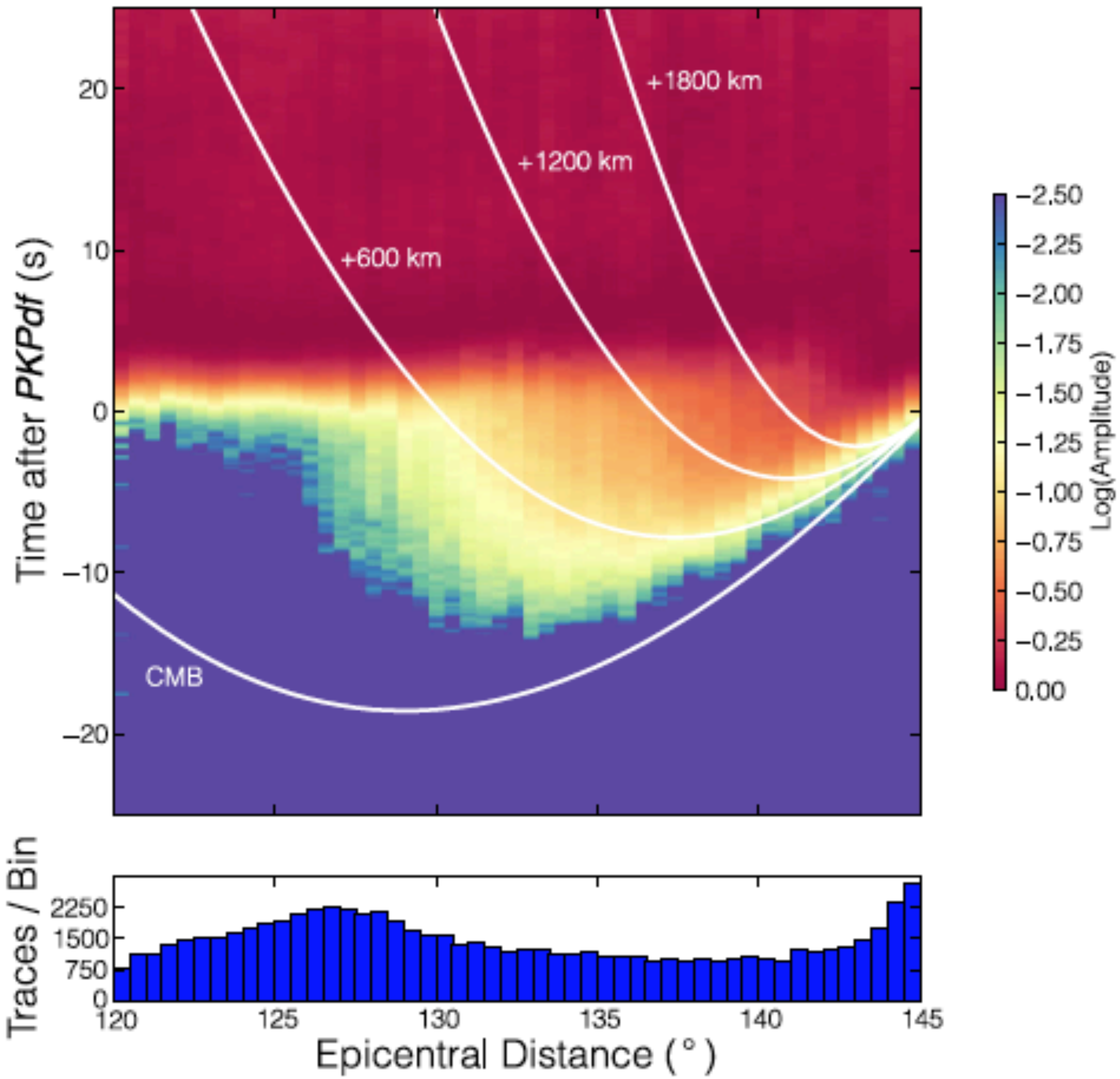
Hétérogénéité à grande profondeur

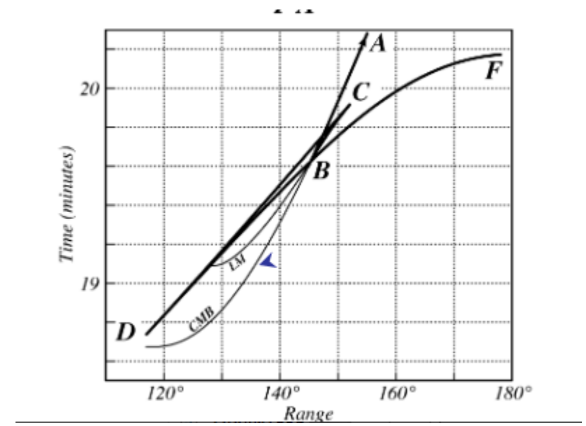
Précurseurs des PKIKP



Margerin & Nolet, 2003





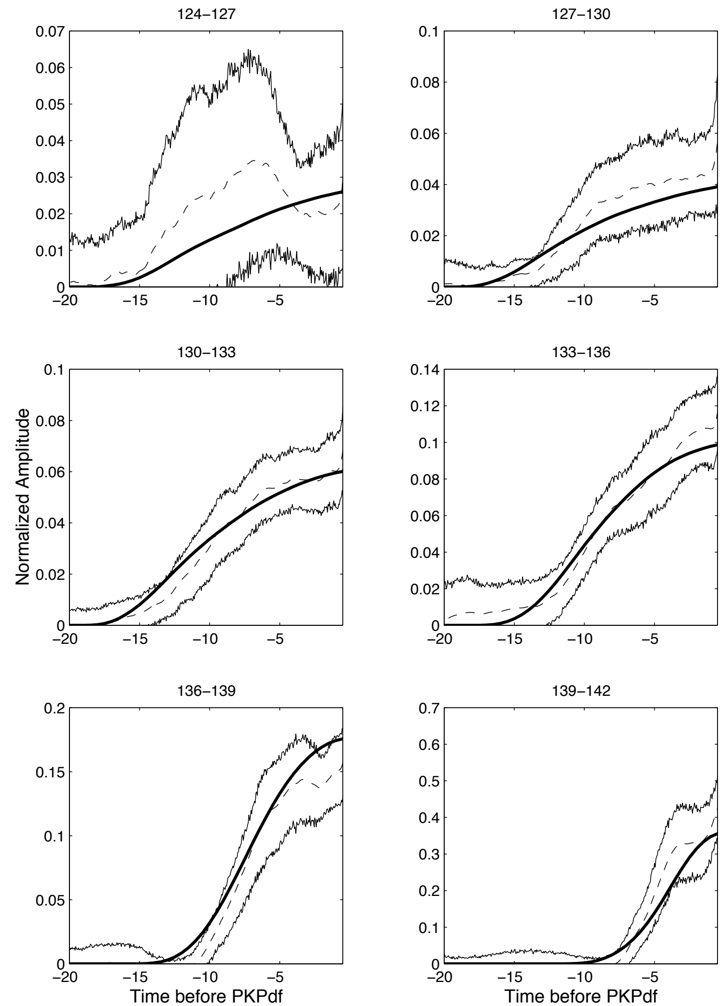


N. J. Mancinelli and P. M. Shearer

Fit of the observations with RTE and heterogeneity in the whole mantle described by a Henyey and Greenstein (H-G) correlation function.

Perturbations are of the order of 0.1%, correlation length is model dependent but the heterogeneity is rich in small scales (0.1-10km).

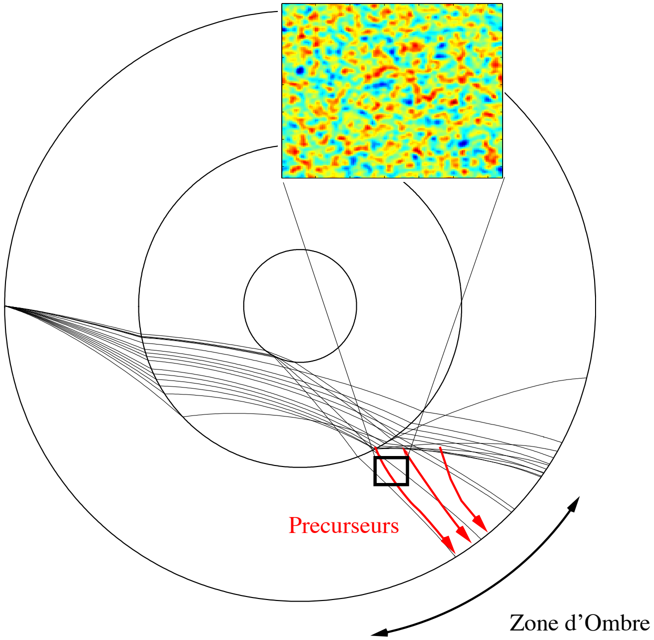
mft of the order of 10^3s



Margerin and Nolet, 2003

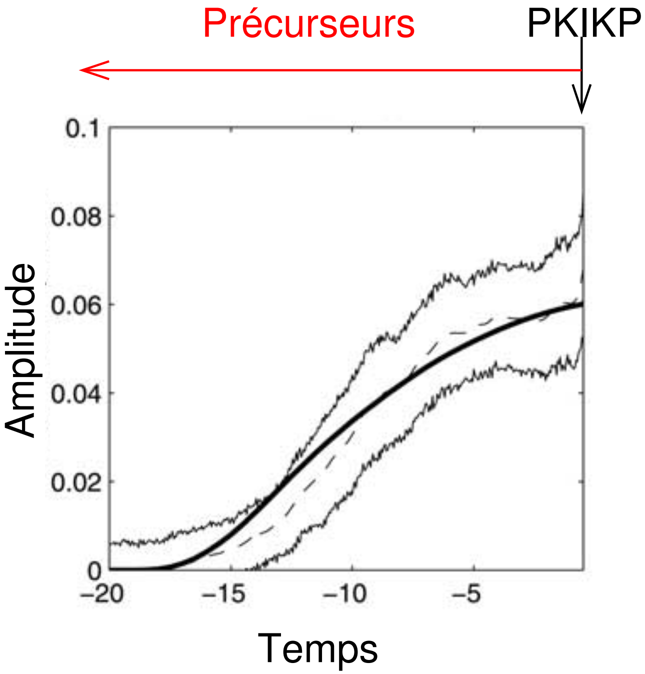
Transfert radiatif et hétérogénéité du manteau inférieur

Précurseurs des PKIKP



Margerin & Nolet, 2003

Observations vs Modèle



- - - - - - : Moy. des données ——- :Transfert radiatif

- Première application du Transfert Radiatif à l'échelle du globe
- Distribution d'hétérogénéité dans tout le manteau Inf
- Faibles fluctuations (0.1%) riches en petites échelles : 0.1km–10km, spectre k^{-3}