



Jean Virieux after Anne Obermann

# Part I: Seismic Refraction



Students M1

Students M2

# Overview

- Introduction – historical outline
- Chapter 1: Fundamental concepts
- Chapter 2: Data acquisition and material
- Chapter 3: Data interpretation
  - A: Geophysical Interpretation
  - B : Geological Interpretation

# Overview

- Introduction – historical outline
- Chapter 1: Fundamental concepts
- Chapter 2: Data acquisition and material
- Chapter 3: Data interpretation

# Towards refraction seismology

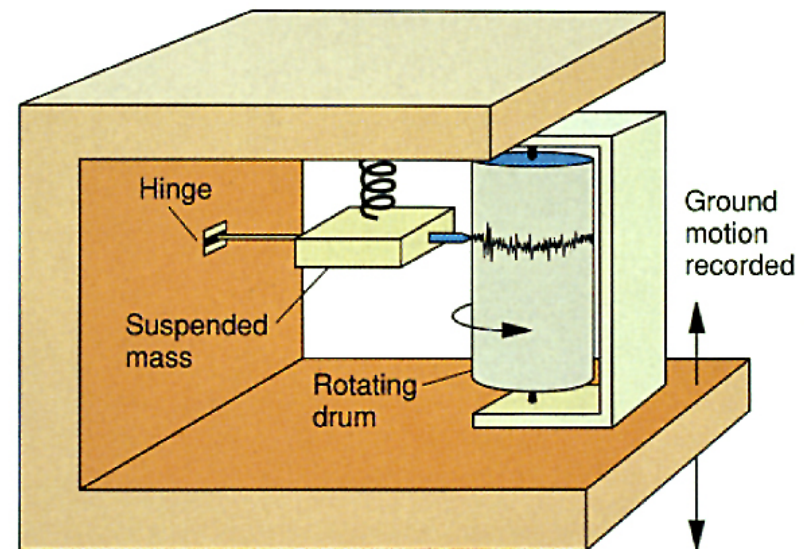
- 1885 all that was known about the Earth structure was a vague idea that the density inside had to be much greater than at the surface
- ➔ within 50 years an incredible amount more had been learned using seismology

Breakthrough: Seismometer (late 1800')

Instrumental challenge: how to measure ground motion given that the seismometer sits on the ground?

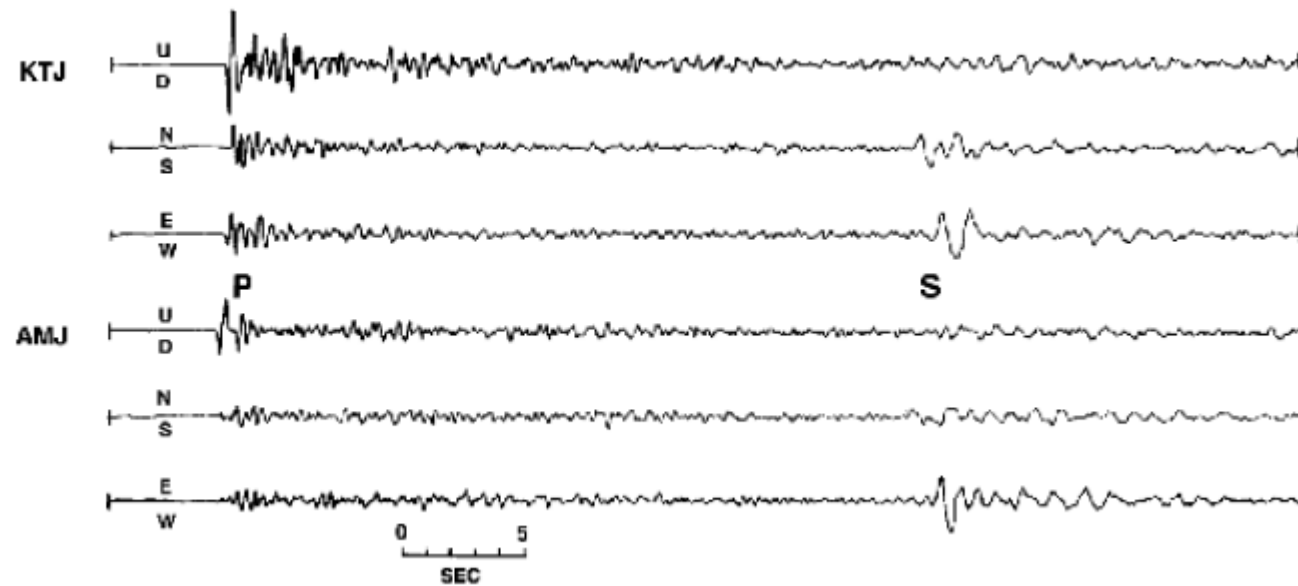
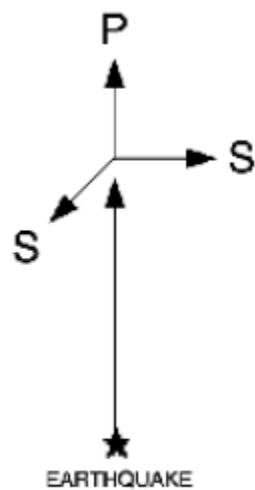
Record very small ground motions on the order of 10<sup>-3</sup> cm for distant earthquakes

Inertial principle



# Towards refraction seismology

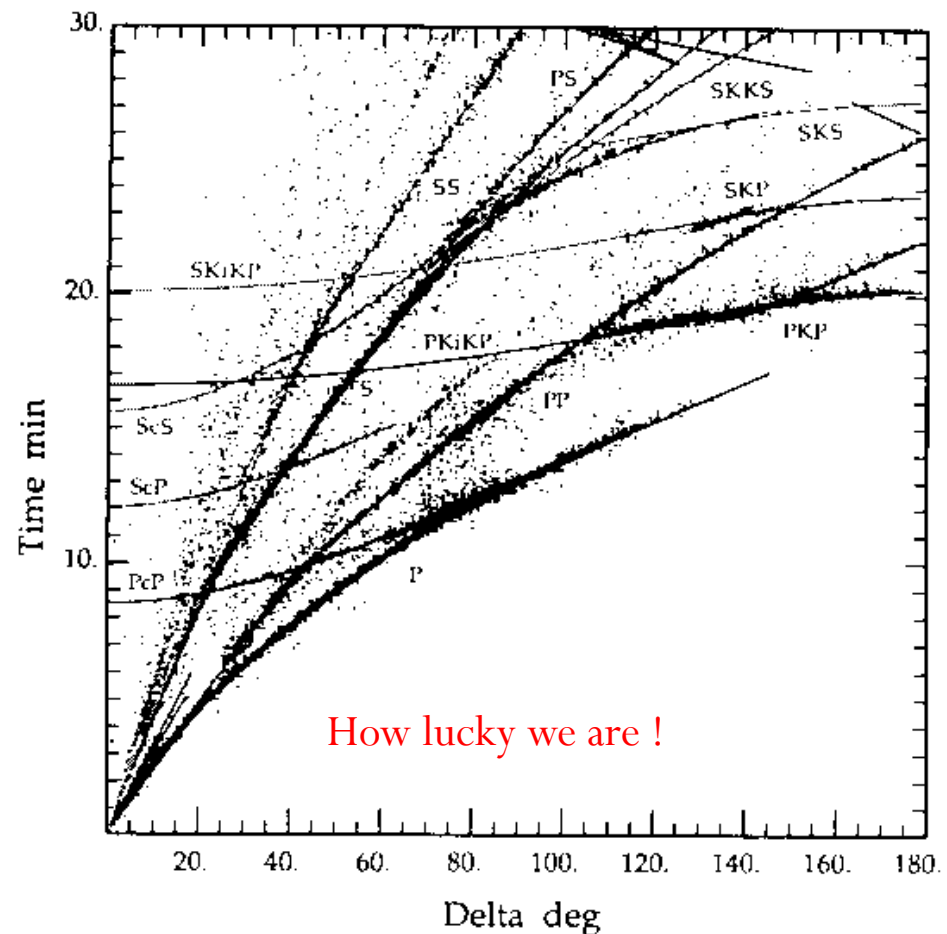
- Seismometers were developed to record vertical and horizontal motions.
- Precise timing, nowadays done using GPS (Global Positioning System) clocks - so that records can be compared between stations. Data are now recorded digitally and made available on the web.



# Wave packets : time separation

- More than 6000 arrival times of different phases coming from discontinuities inside the Earth.
- Building up velocity models which verify these propagation times

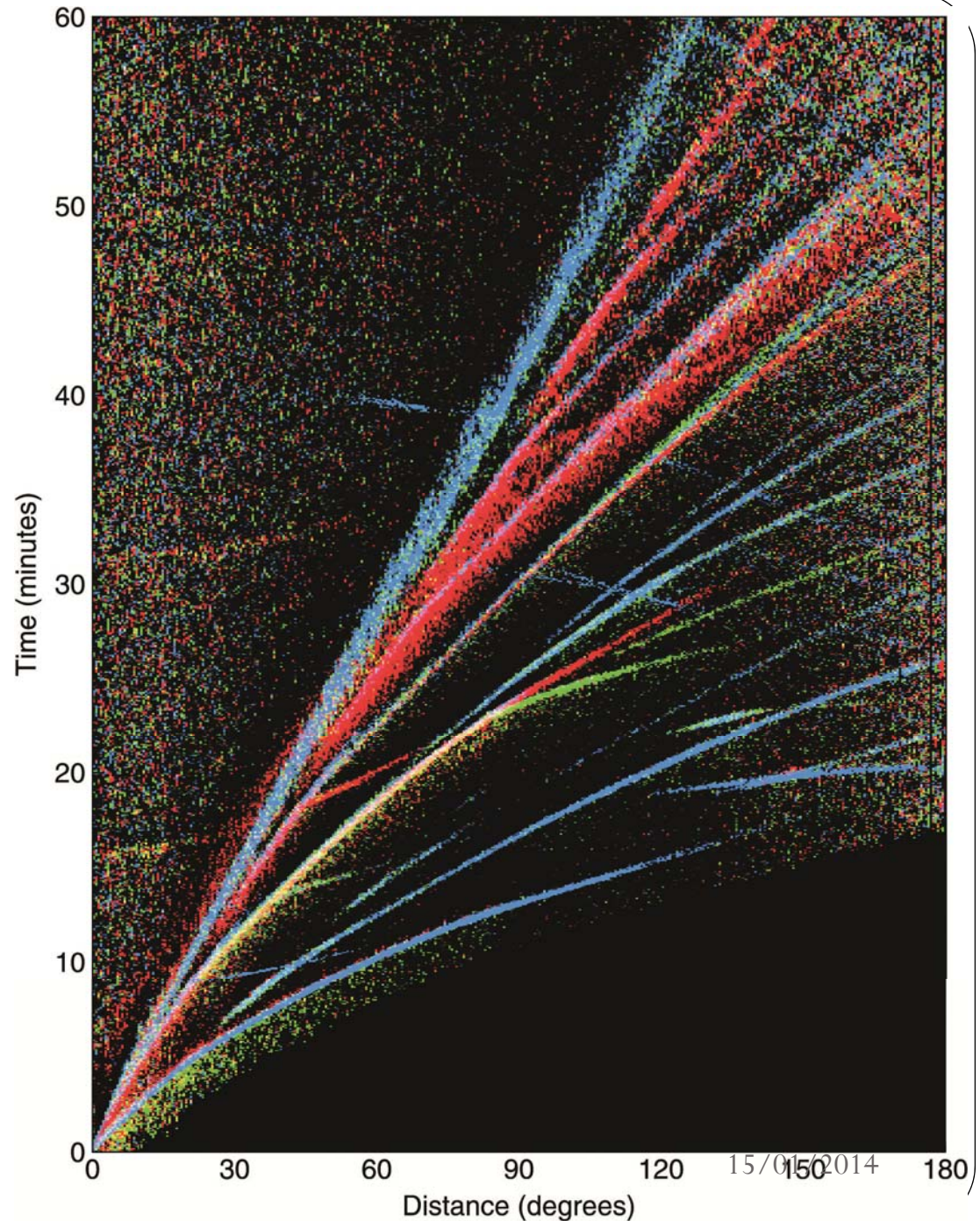
Modèle JB ( 2 sec shift)



# Wave packets : time separation

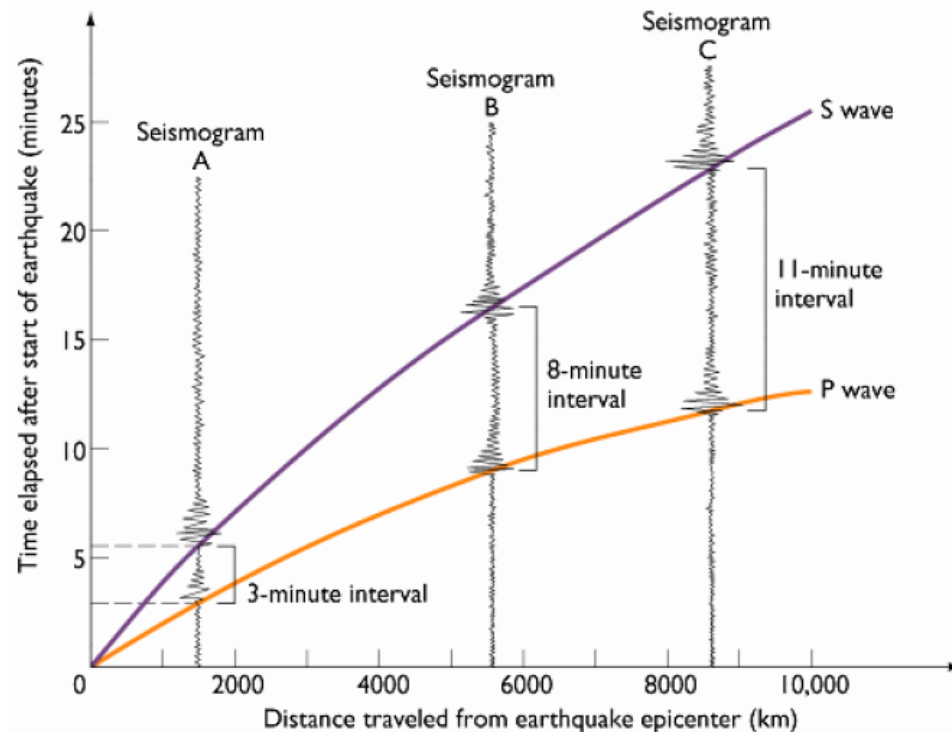
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Modèle JB ( 2 sec shift)

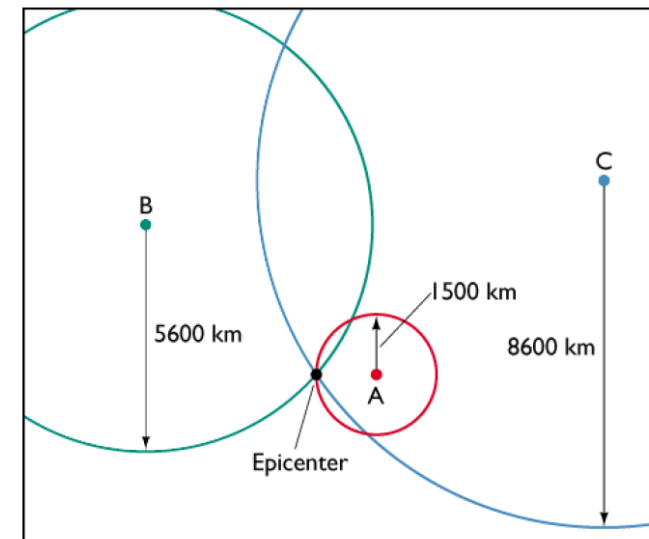


# Towards refraction seismology

- In 1889, an earthquake in Japan was recorded successfully on several seismometers in Germany.
- Milne discovered that observations showed that the time separations between P and S wave arrivals increased with distance from the earthquake.



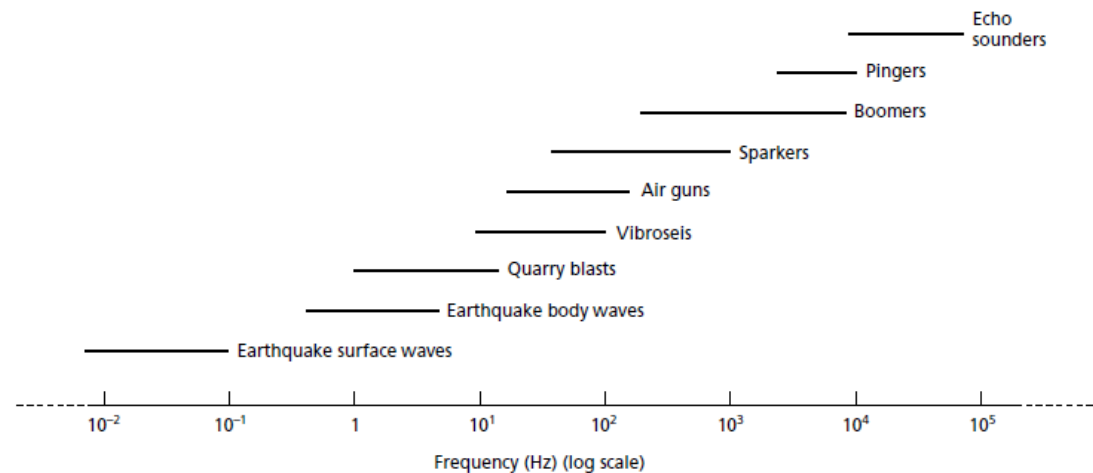
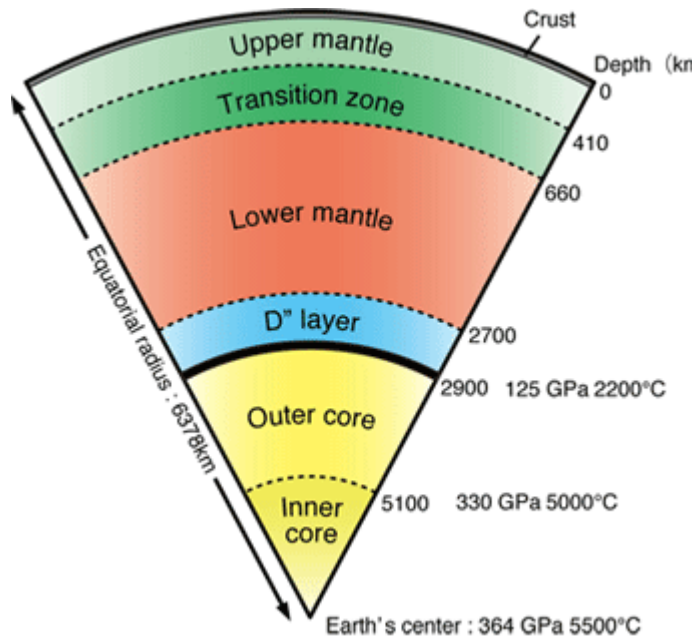
- Thus, the S-P time could be used to measure the distance to the earthquake.





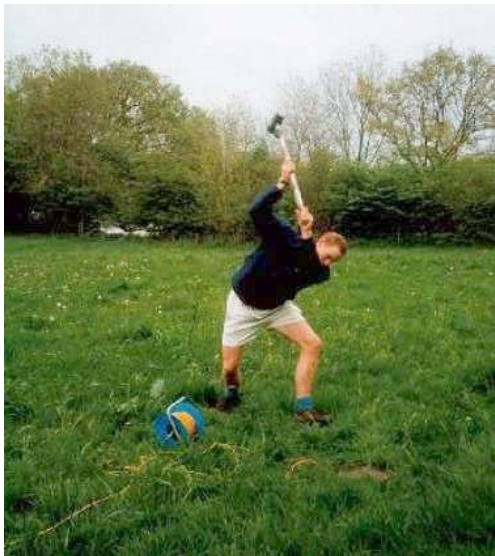
# Towards refraction seismology

- Next step: Infer the velocity structure of the Earth as a function of depth from the seismograms that were recorded from many different earthquakes (Inverse Problem).
- The simplest approach to the inverse problem treats the earth as flat layers of uniform velocity material. The basic geometry is a layer of thickness  $z$ , with velocity  $v_1$ , overlying a halfspace with a higher velocity  $v_2$ .



**PROBLEM: DATA QUANTITY DEPENDENT ON LARGE EARTHQUAKES – DIFFERENT SOURCES NEEDED!**

# Towards refraction seismology

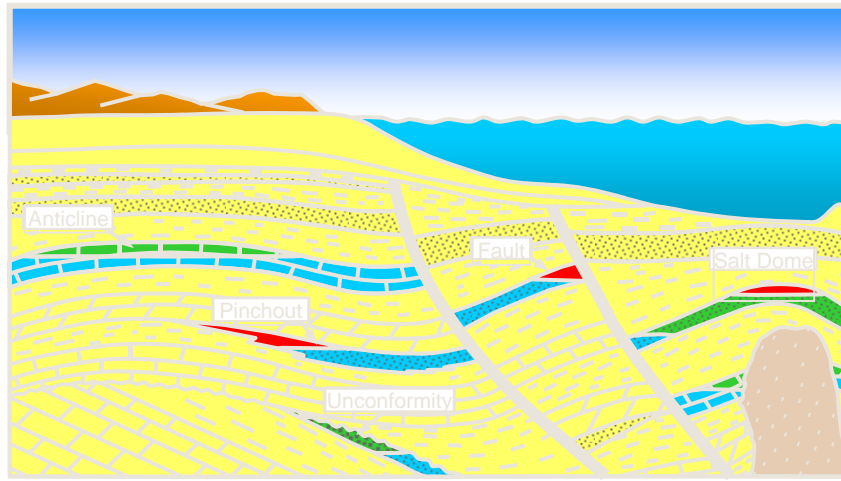


# Industrial context: exploration

- Industry focus: Increase Production vs. find new reserves



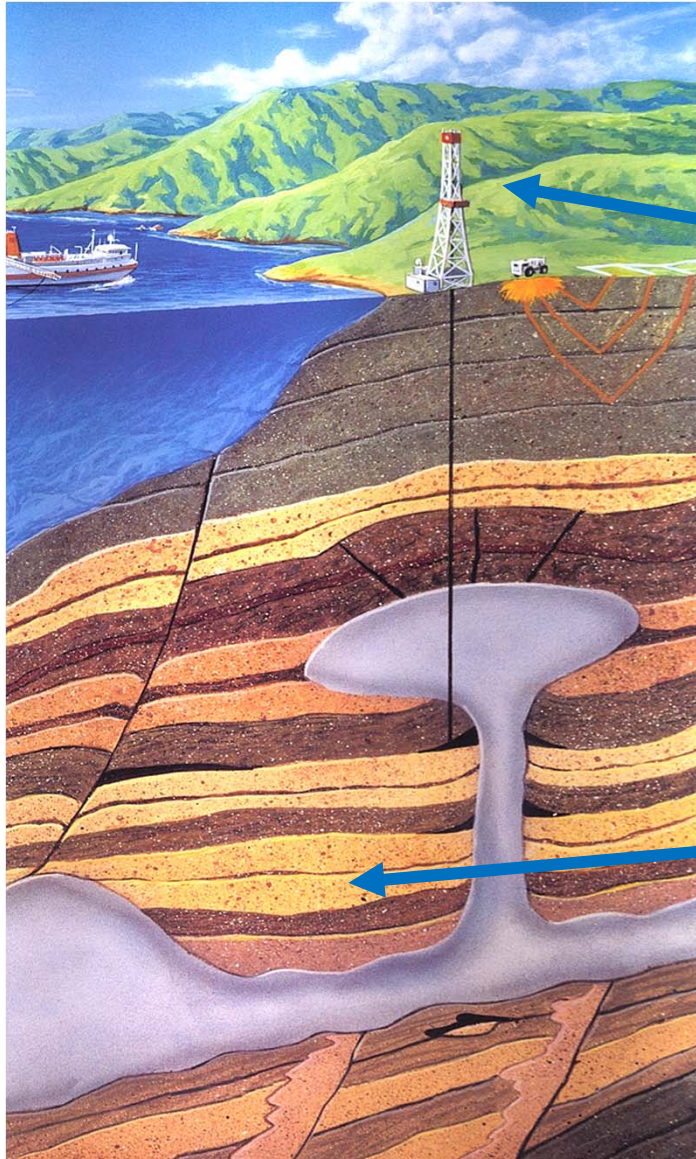
Improve the geological model



American Petroleum Institute, 1986



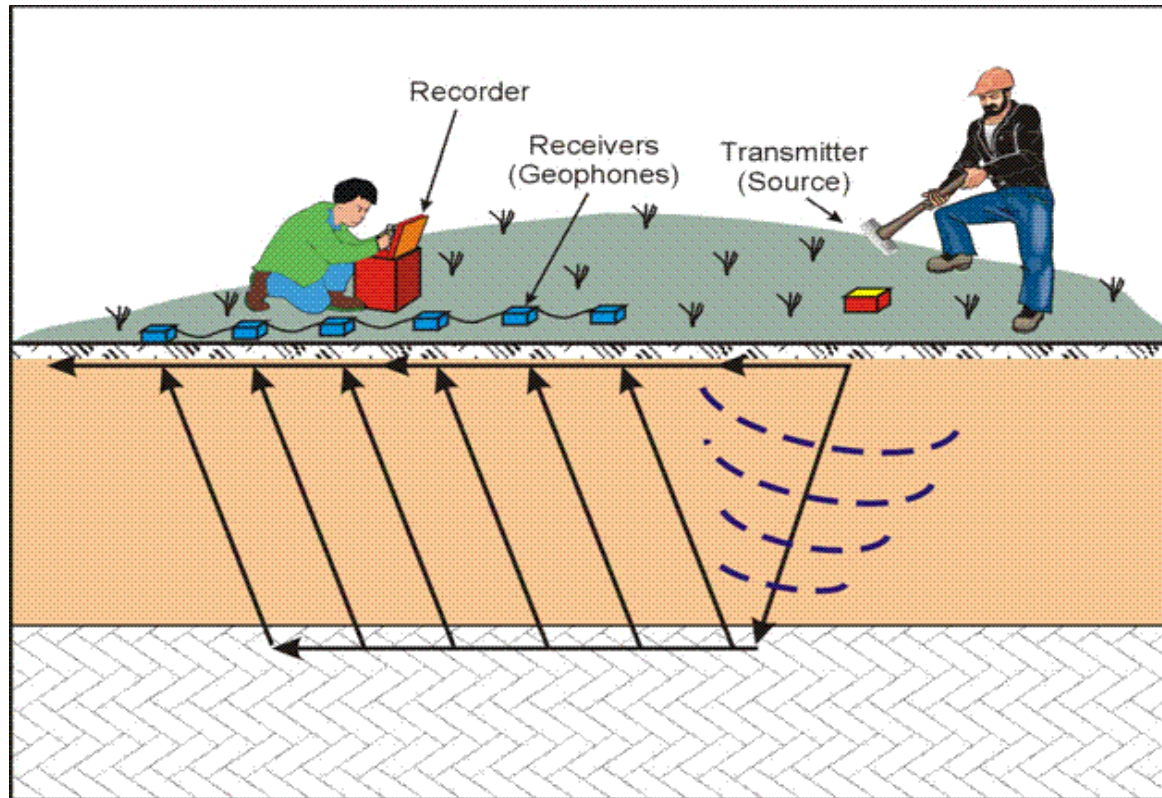
# Context: Interpreting the Unseen



- Surface Geology
  - Aerial photos
  - Geologic maps
  - Out-crops

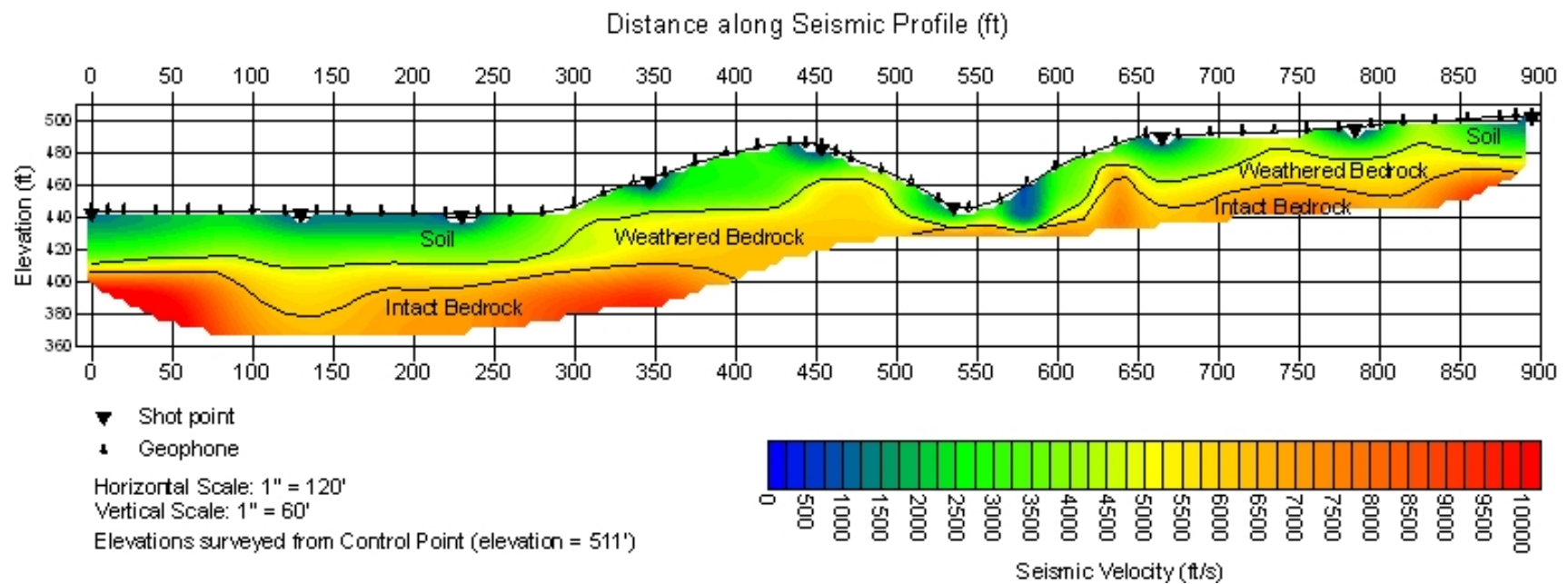
- Subsurface Studies
  - Gravity
  - Magnetics
  - Seismic reflection
  - Wells

# Towards refraction seismology



- Set out a line or array of geophones
- Input a pulse of energy into the ground
- Record the arrival times to interpret the velocity structure

# Towards refraction seismology



# Seismic methods and scales

- Controlled source seismology
  - allows higher resolution studies (m to 100s km)
  - can carry out experiments away from tectonic regions
  - source position and signal under control



- Global seismology (earthquakes)
  - provides information on global earth structure and large scale velocity anomalies (100s to 1000s km)
  - difficult to image smaller scale structure, particularly away from earthquake source regions
  - source and velocity structure imaging: difficult problem



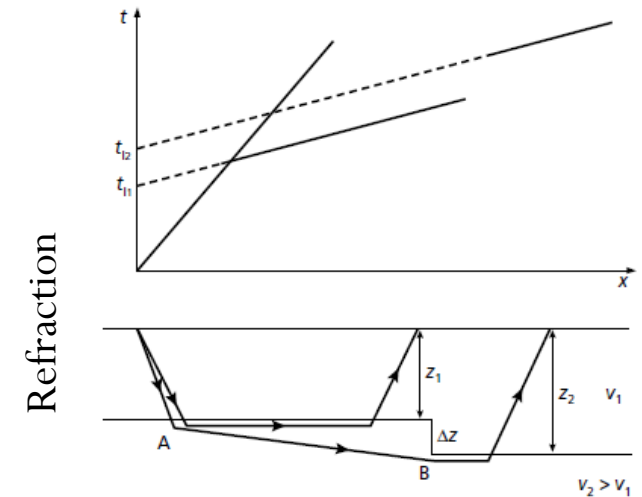
# Seismic methods and scales

- **Seismic refraction**

- Used to study large scale crustal layering: thickness and velocity

- **Seismic reflection**

- “Imaging” of subsurface reflectors
- Difficult to determine accurate velocities and depths
- (see courses given by R. Brossier (next week)!





# Applications

?

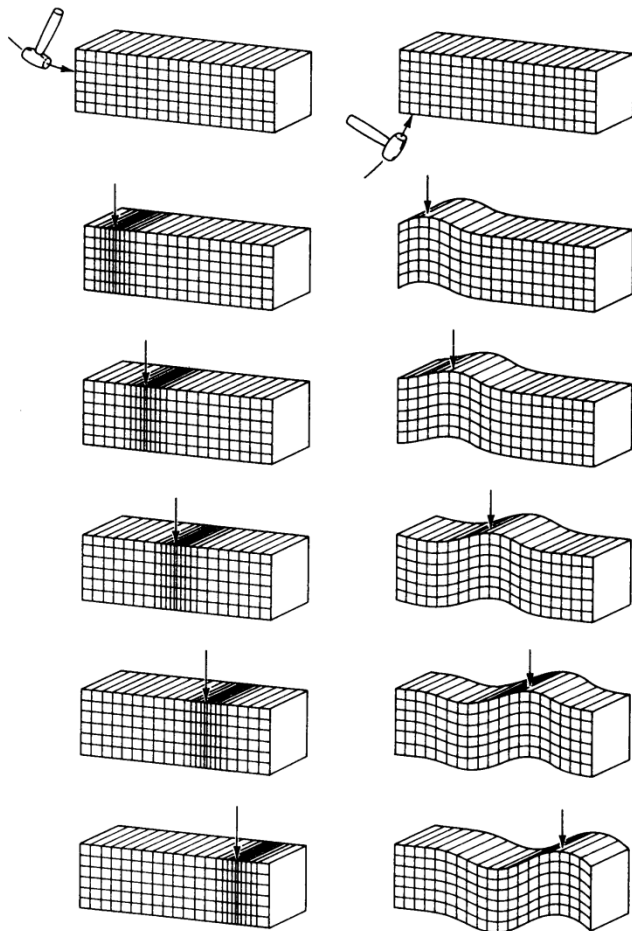


# Overview

- Introduction – historical outline
- Chapter 1: Fundamental concepts
  - Physical notions
  - Two-layered model
  - Special cases
- Chapter 2: Data acquisition and material
- Chapter 3: Data interpretation

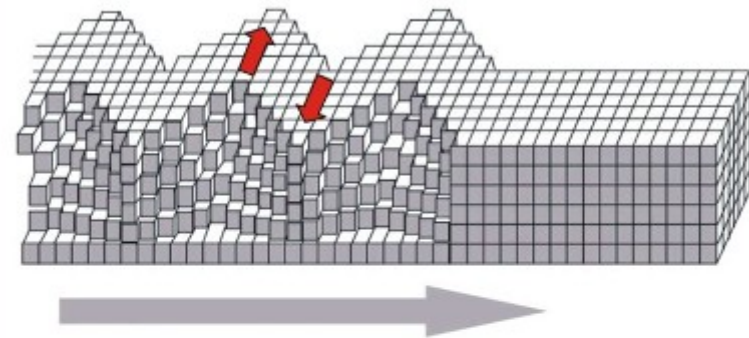
# Different waves

P (compression) + S (shear) waves

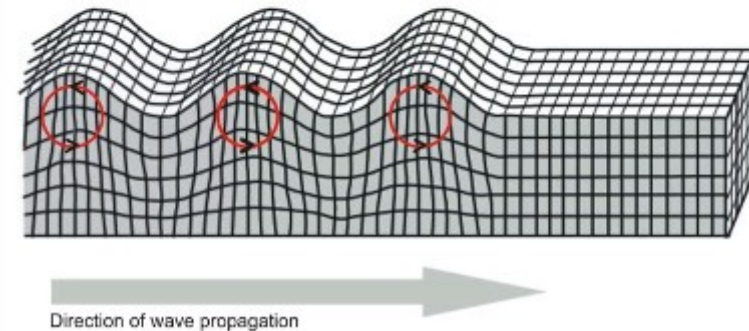


Surface waves

Love wave



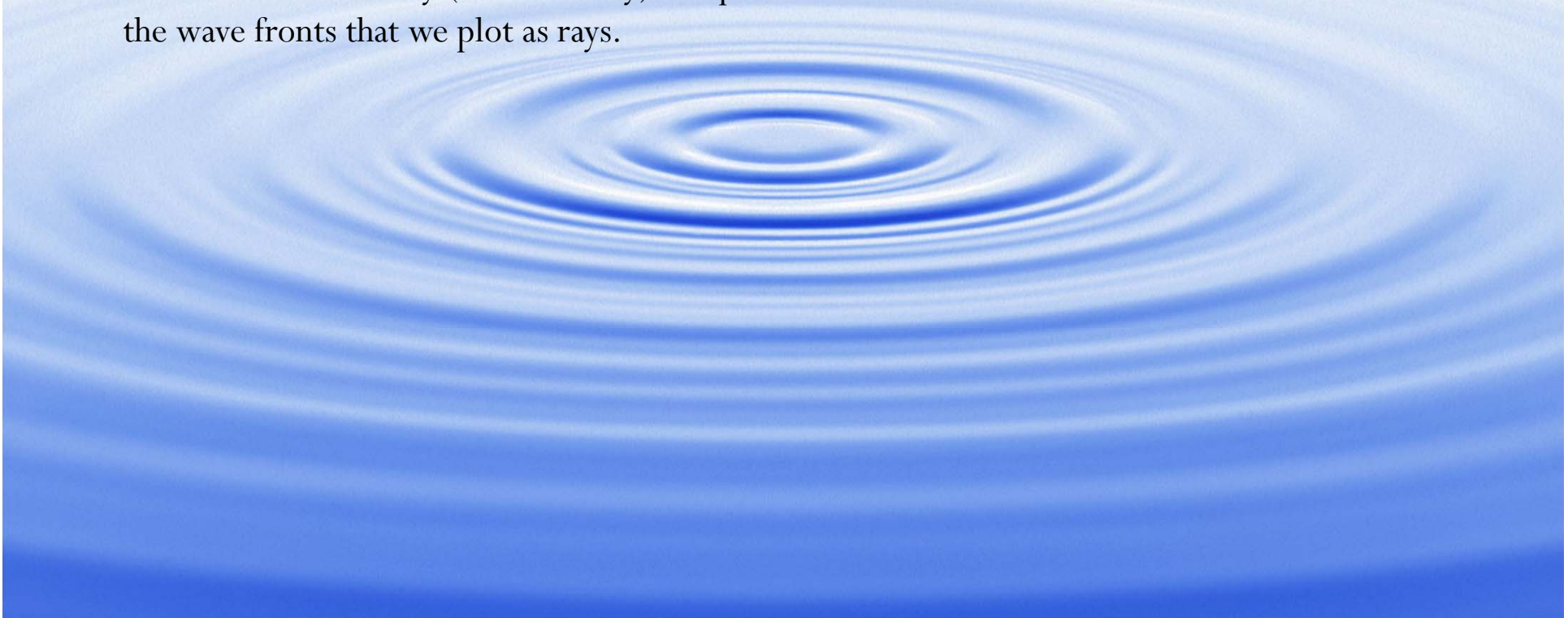
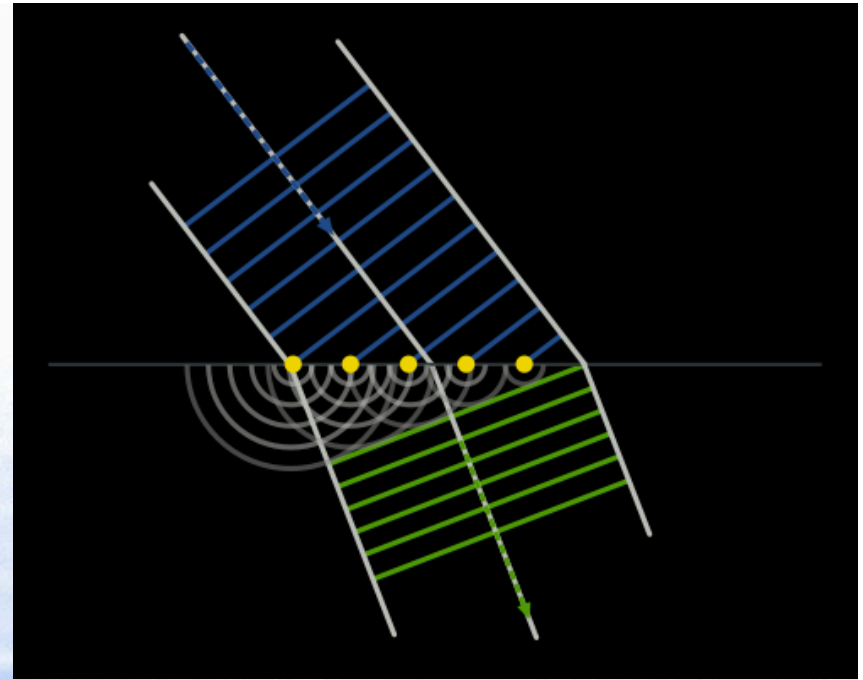
Rayleigh wave



# Huygens Principle

Each point along a material acts like a point source of waves.

Waves have circular (spherical) wave fronts, these interact constructively (destructively) and produce the wave fronts that we plot as rays.



# Snell's Law

**WRONG REPRESENTATION !! WHY? -**



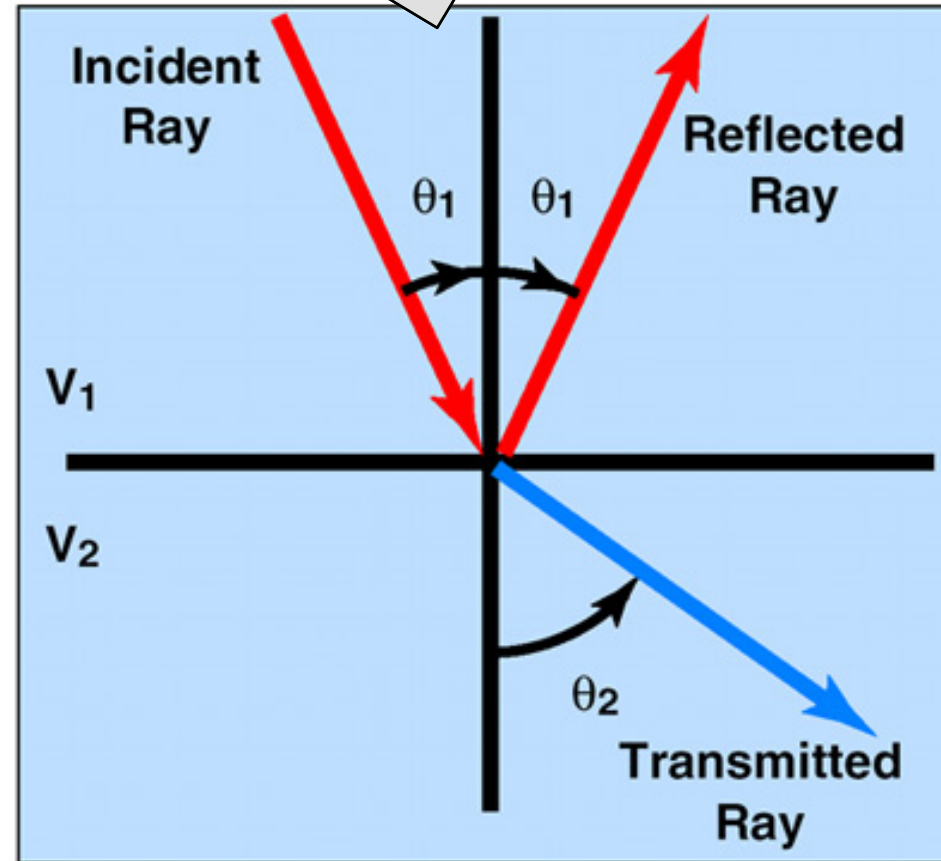
Seismic rays obey Snell's law

The angle of incidence equals the angle of reflection.

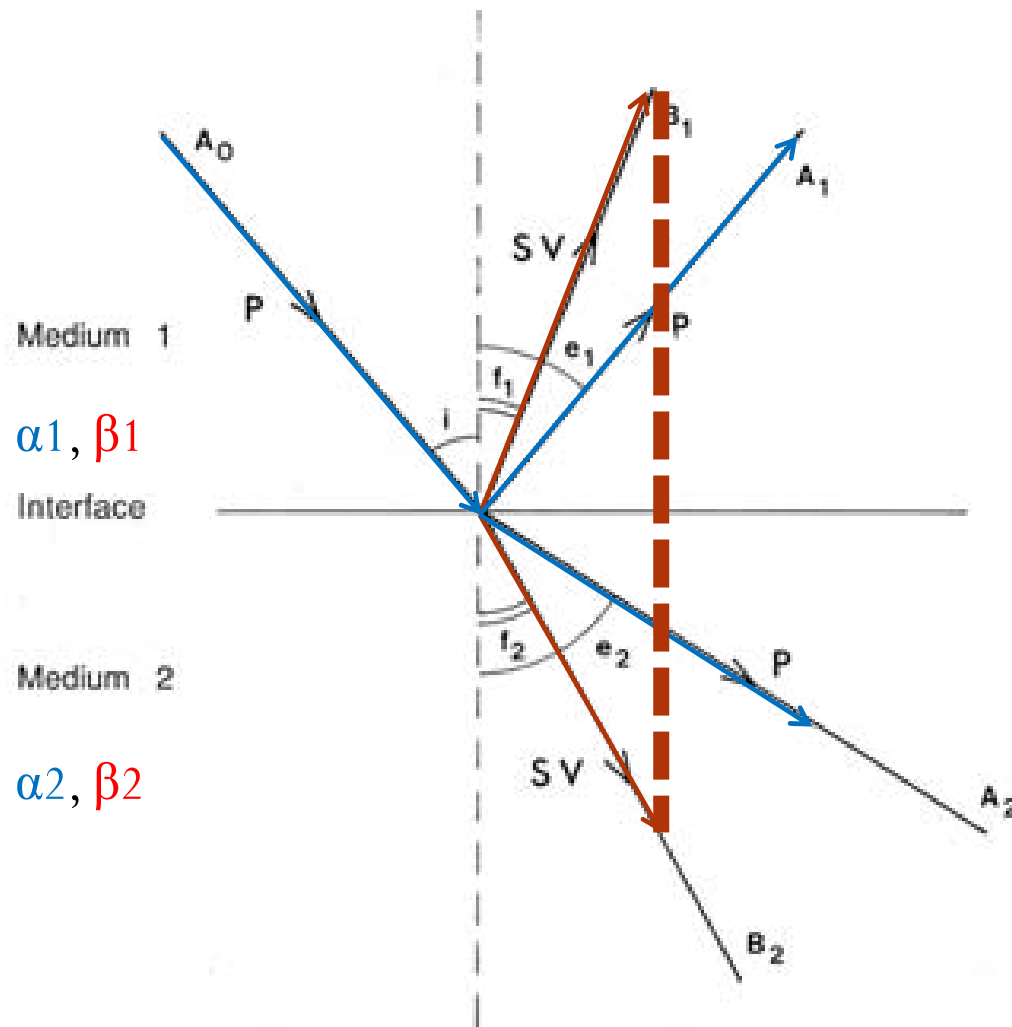
The angle of transmission is related to the angle of incidence through the velocity ratio.

$$\frac{\sin(\theta_1)}{v_1} = \frac{\sin(\theta_2)}{v_2}$$

Note: the transmitted energy is refracted



# Snell's law: S wave conversion



Medium 1

$\alpha_1, \beta_1$

Interface

Medium 2

$\alpha_2, \beta_2$

A conversion from P to S or vice versa can also occur. Still, the angles are determined by the velocity ratios.

$p$  is the ray parameter and is constant along each ray.

$$\frac{\sin(i)}{\alpha_1} = \frac{\sin(e_1)}{\alpha_1} = \frac{\sin(e_2)}{\alpha_2} = \frac{\sin(f_1)}{\beta_1} = \frac{\sin(f_2)}{\beta_2} = p$$



# Anatomy of seismic waves phases

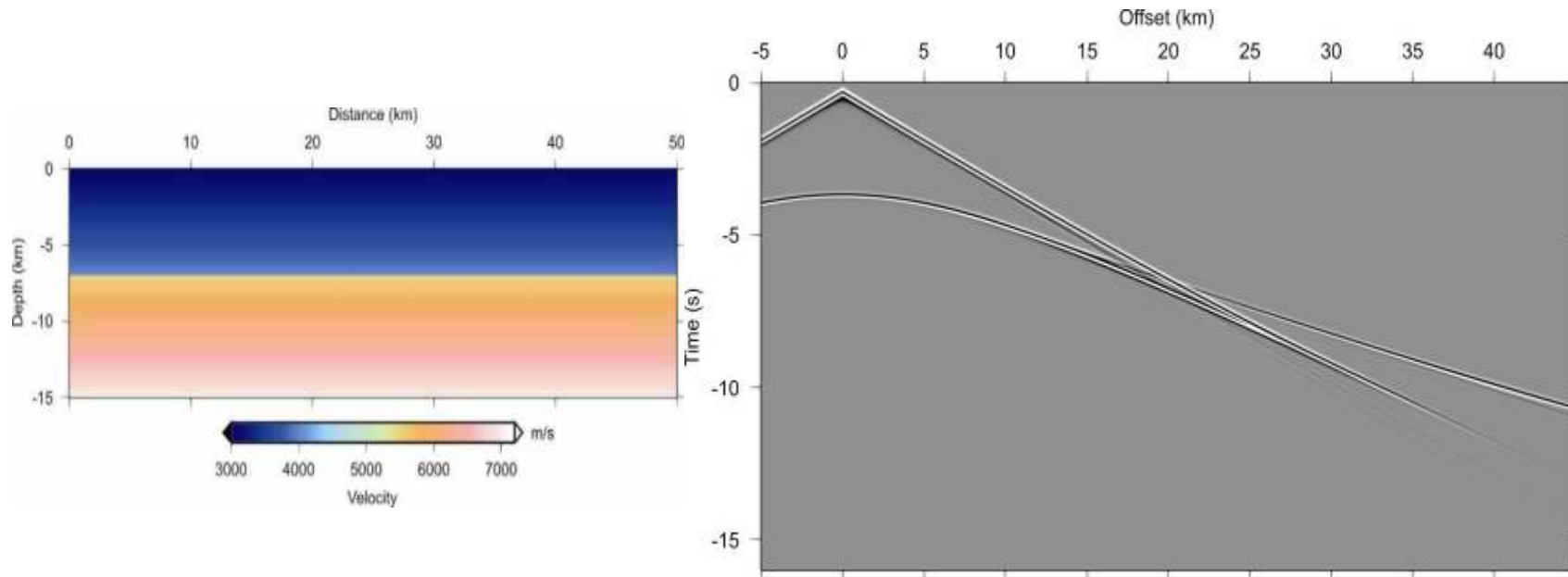
The diagram illustrates seismic wave propagation in a two-layer Earth model. The upper layer is blue and the lower layer is orange. A vertical line represents the source. From the source, several rays emerge, some reflecting off the interface and some refracting into the lower layer. The wavefronts are shown as curved lines that expand outwards from the source. The rays and wavefronts are color-coded to match the layers they are in.

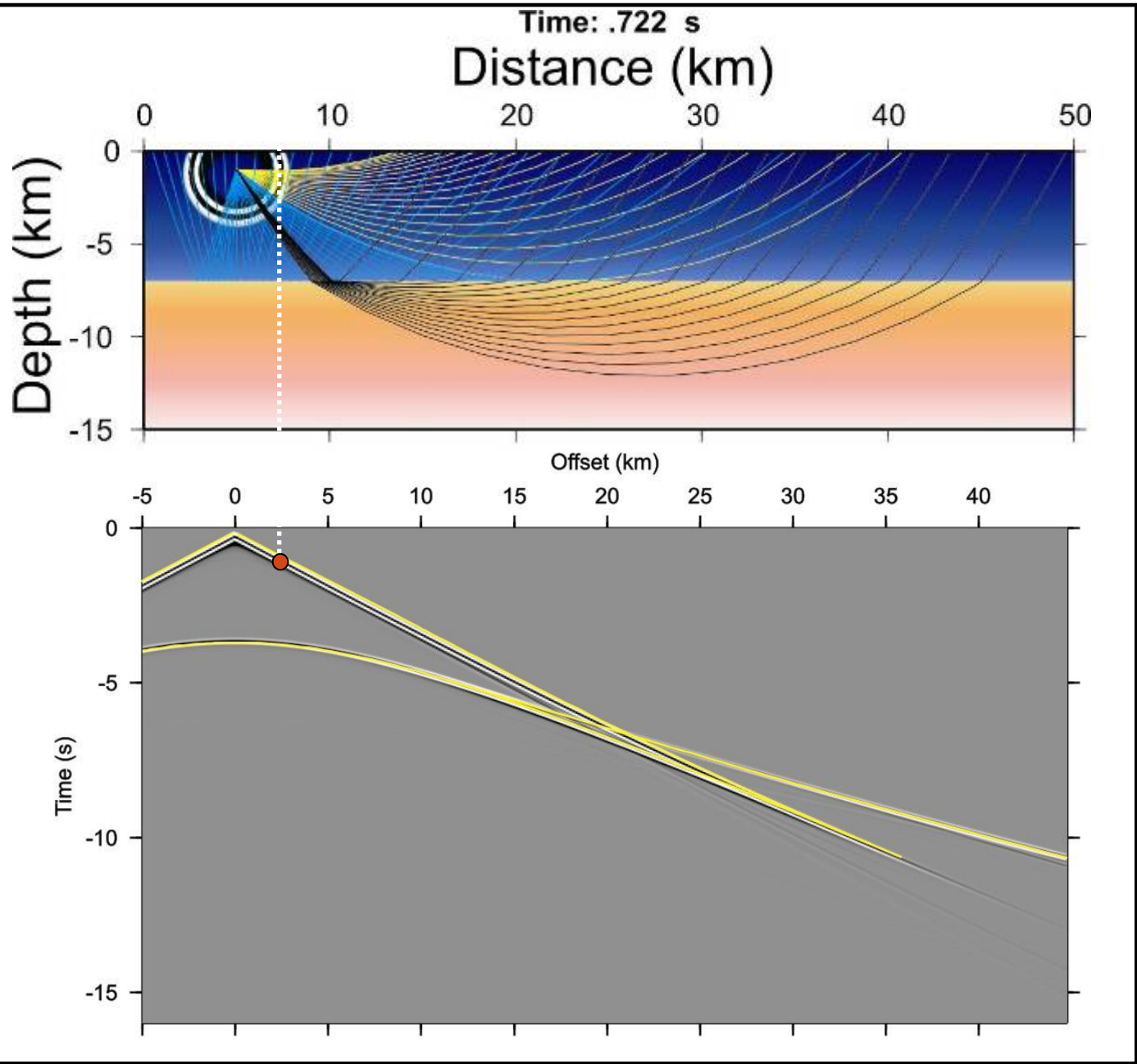
RAY & WAVEFRONT

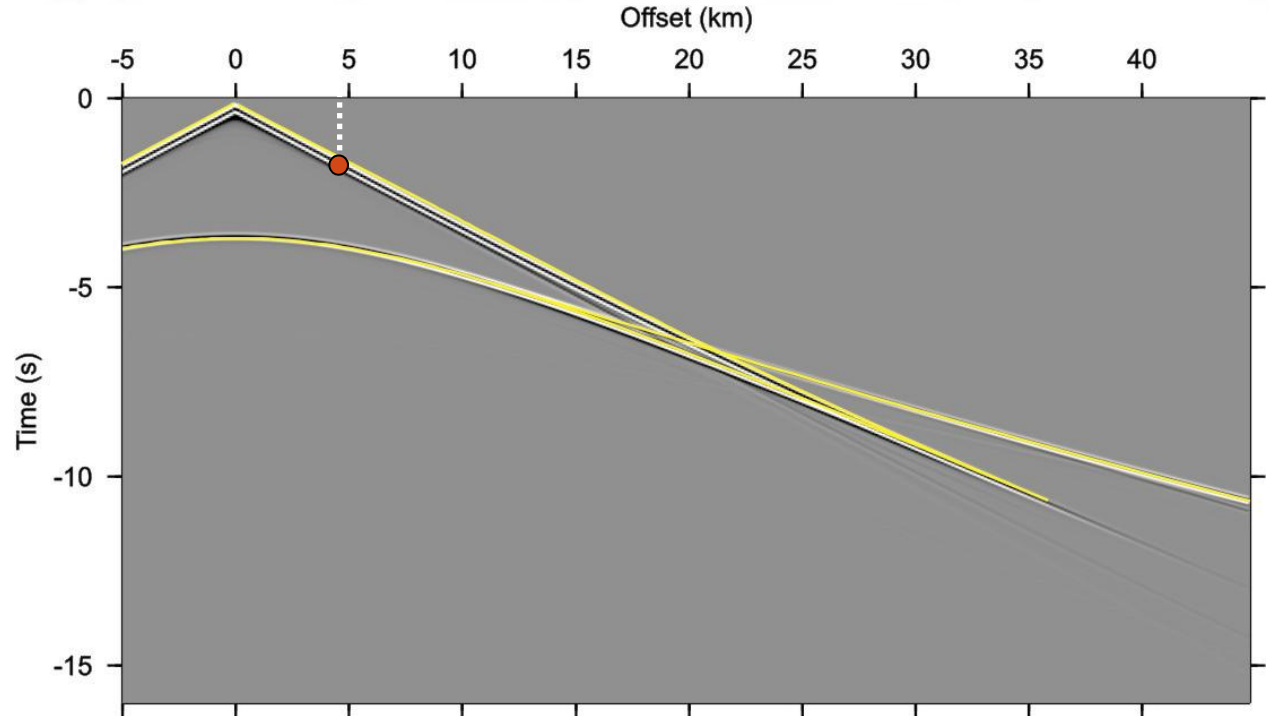
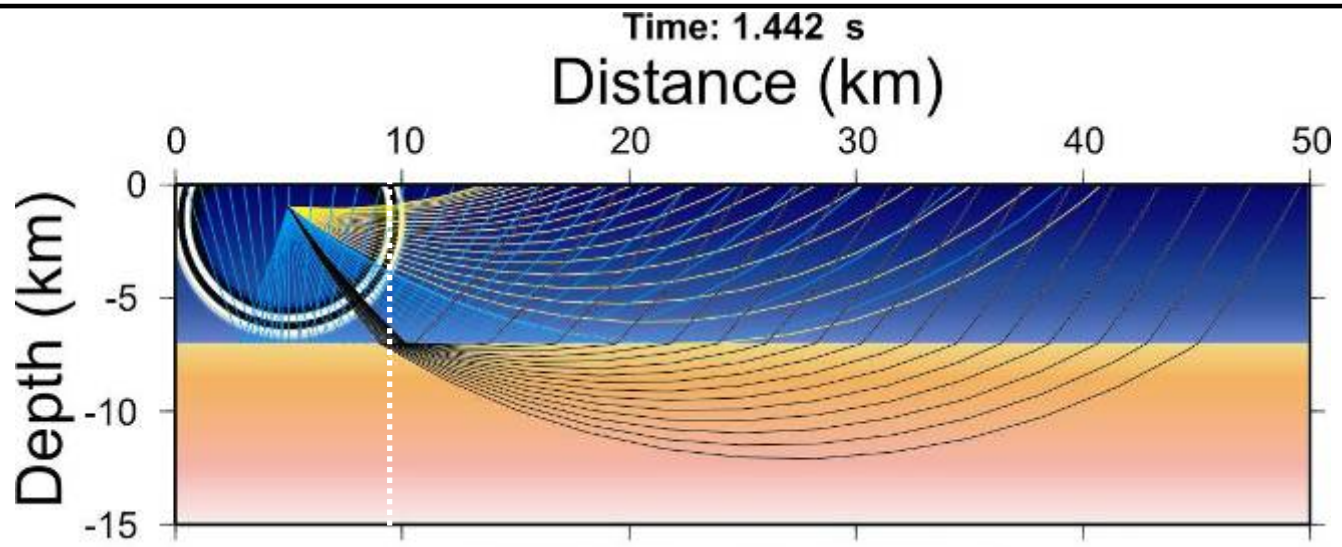
From Stéphane Operto

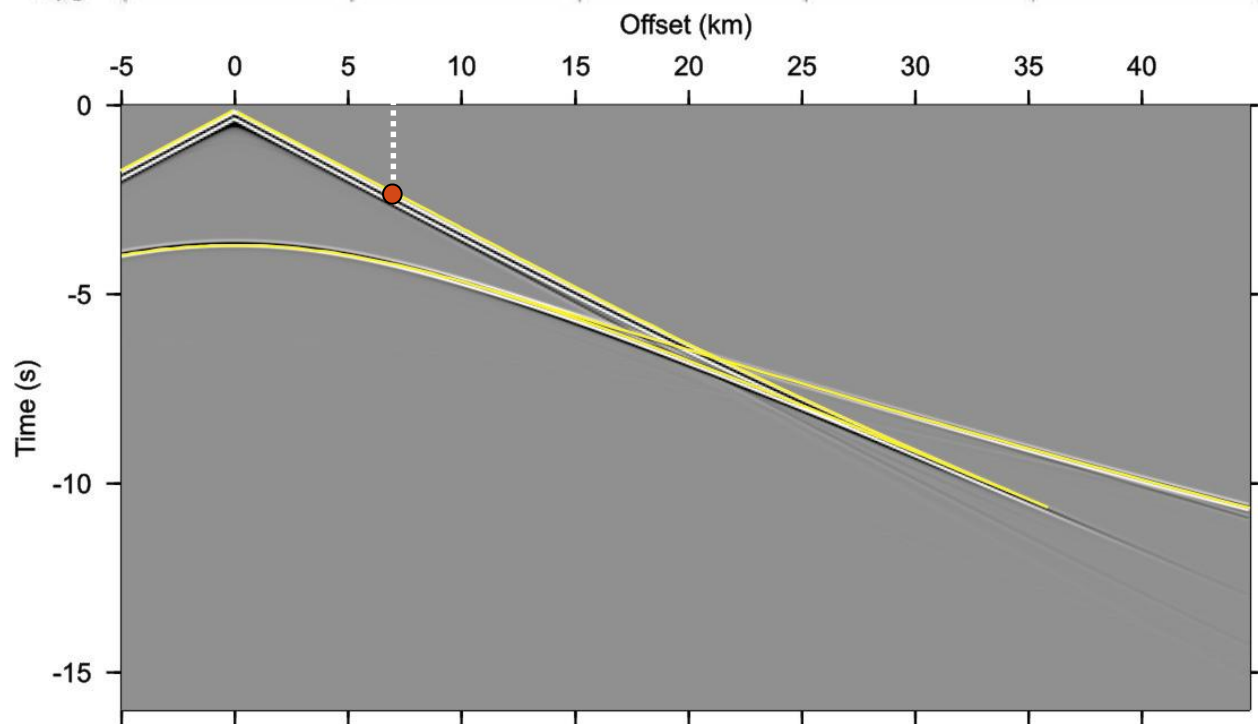
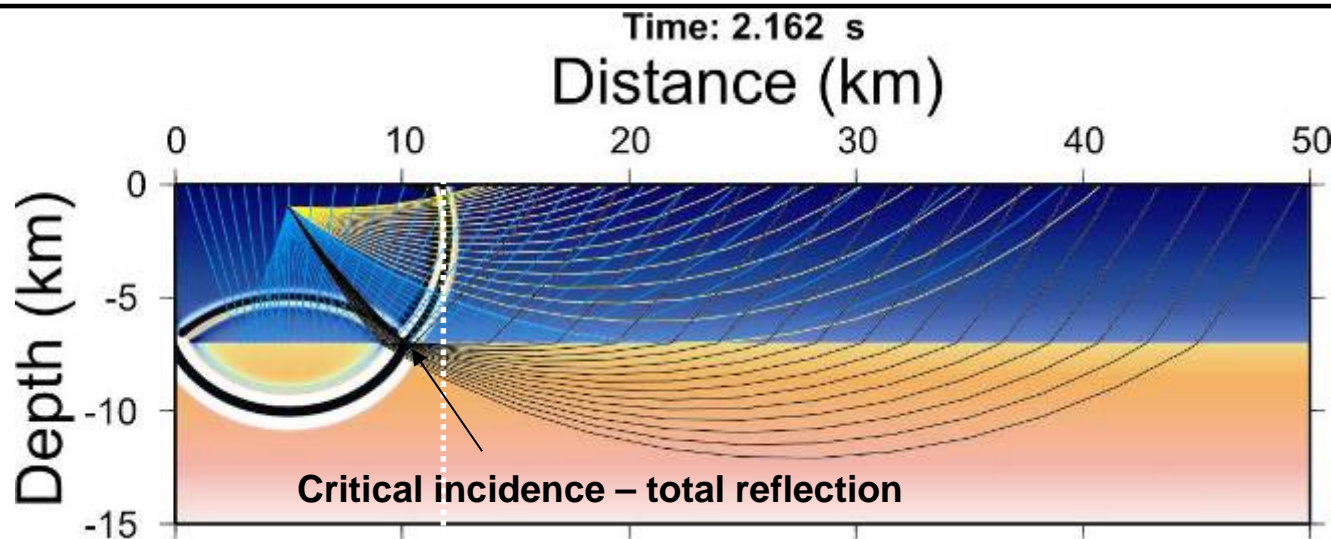


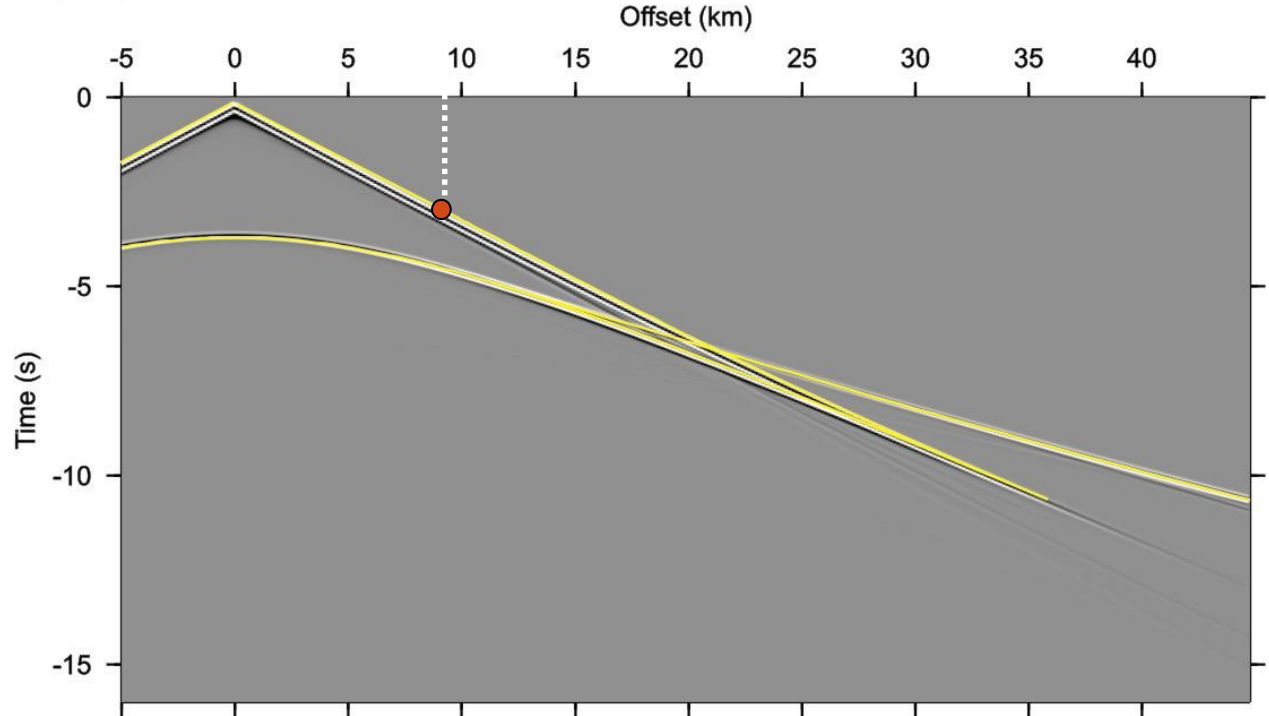
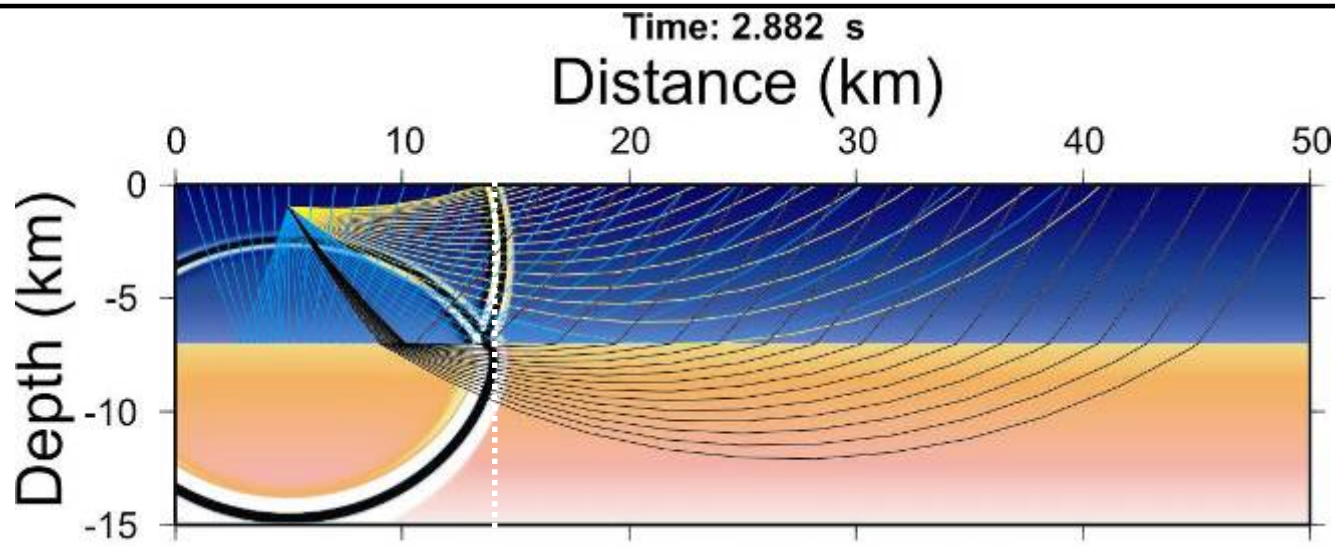
## Anatomy of global-offset seismograms: *Continuous sampling of apertures from transmission to reflection*

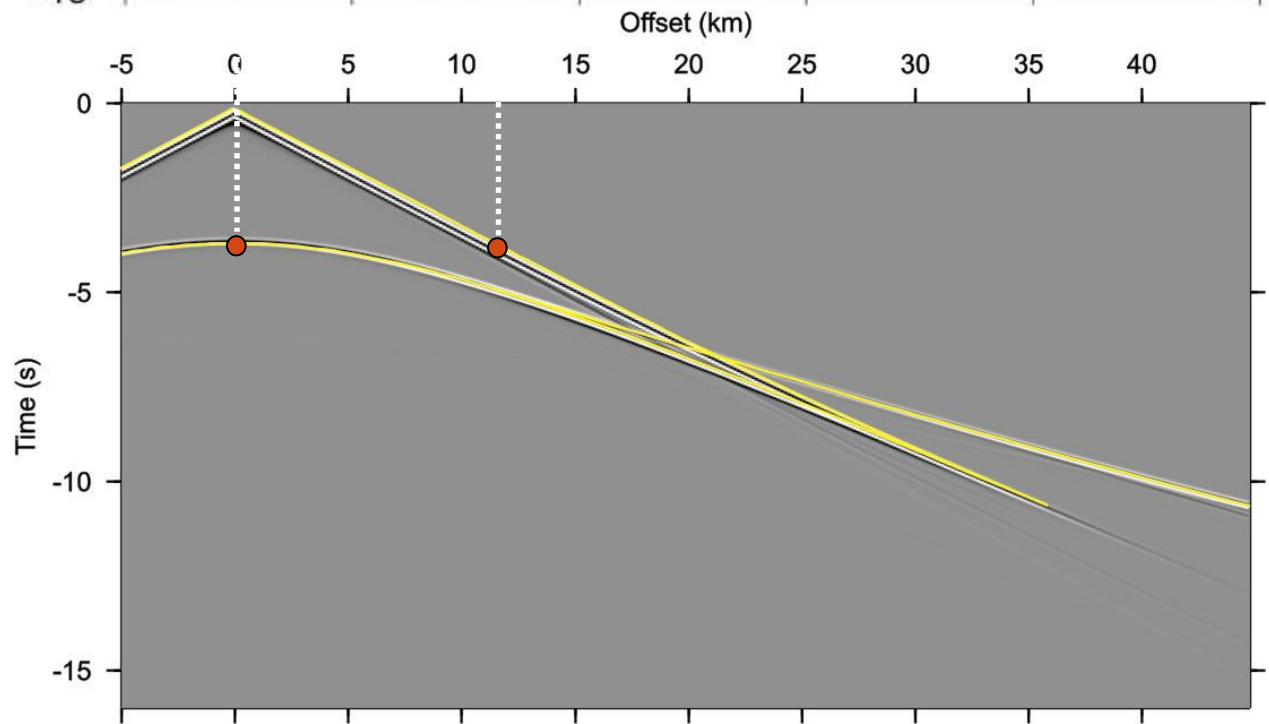
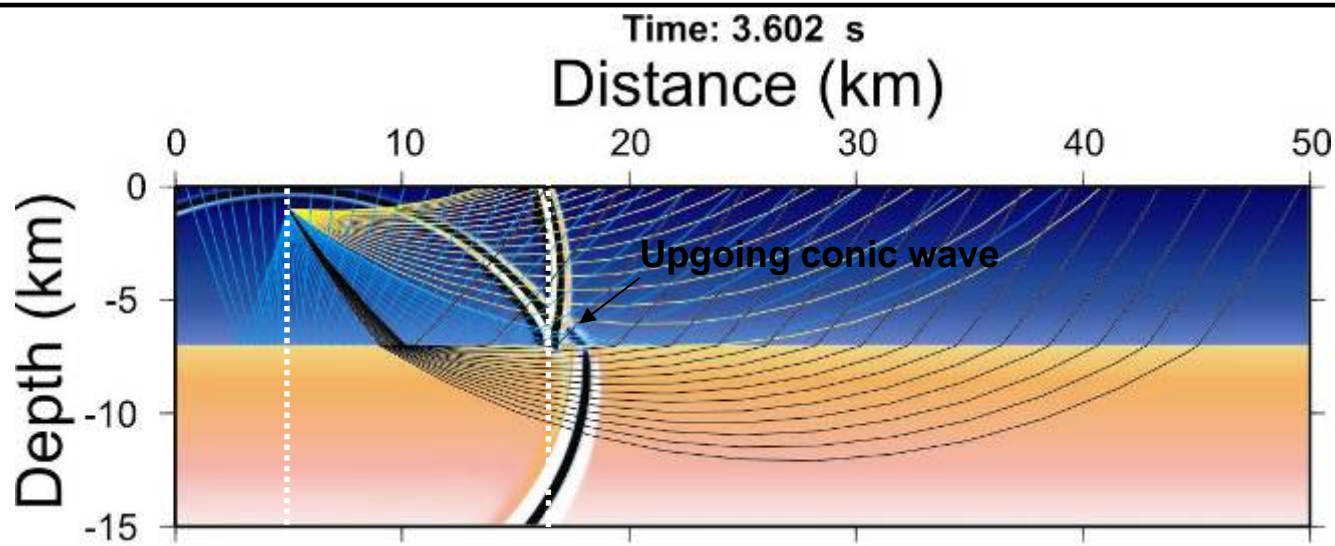


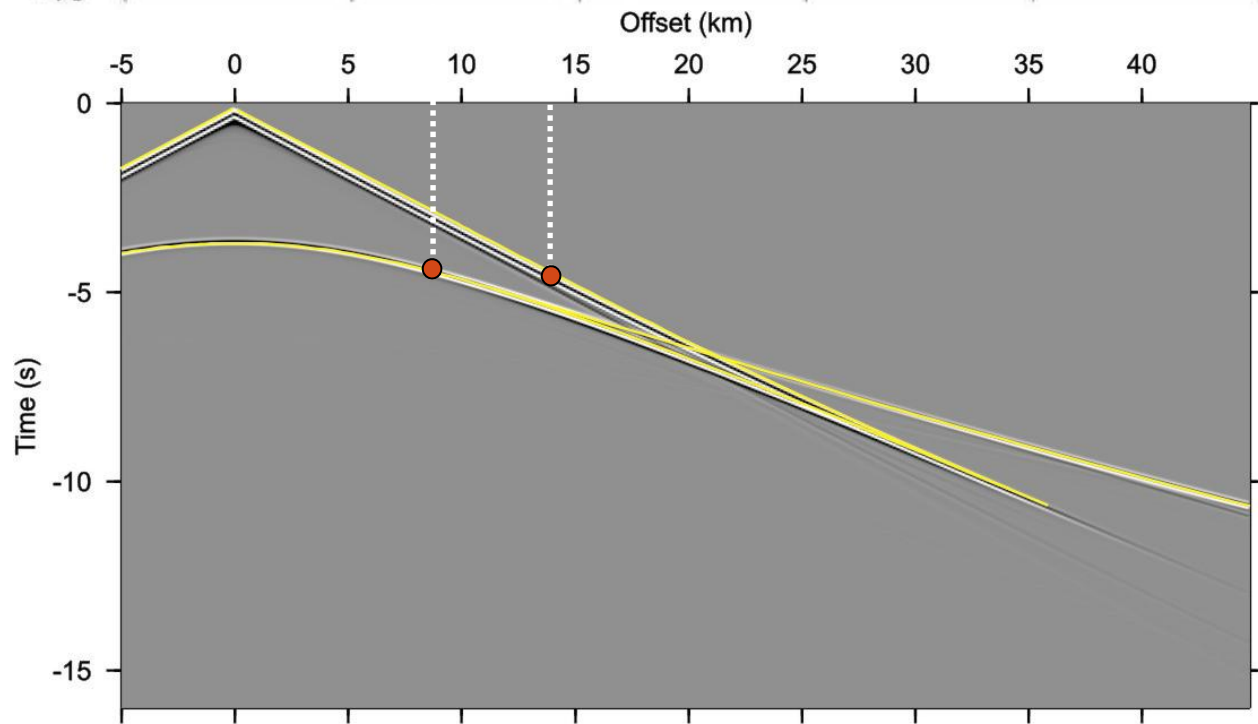
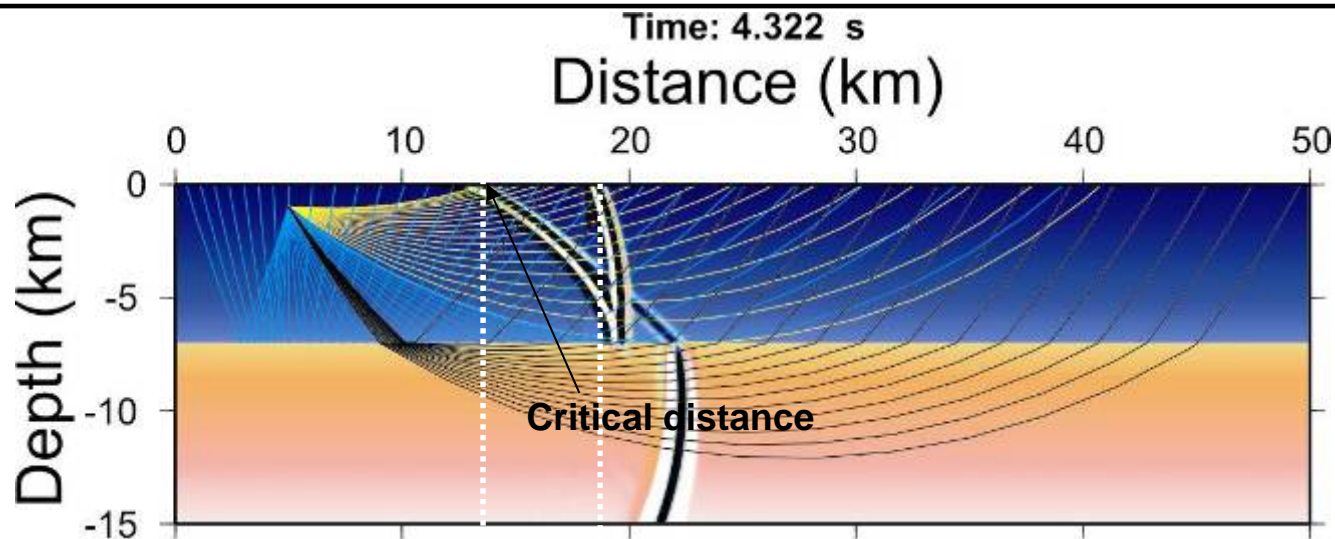


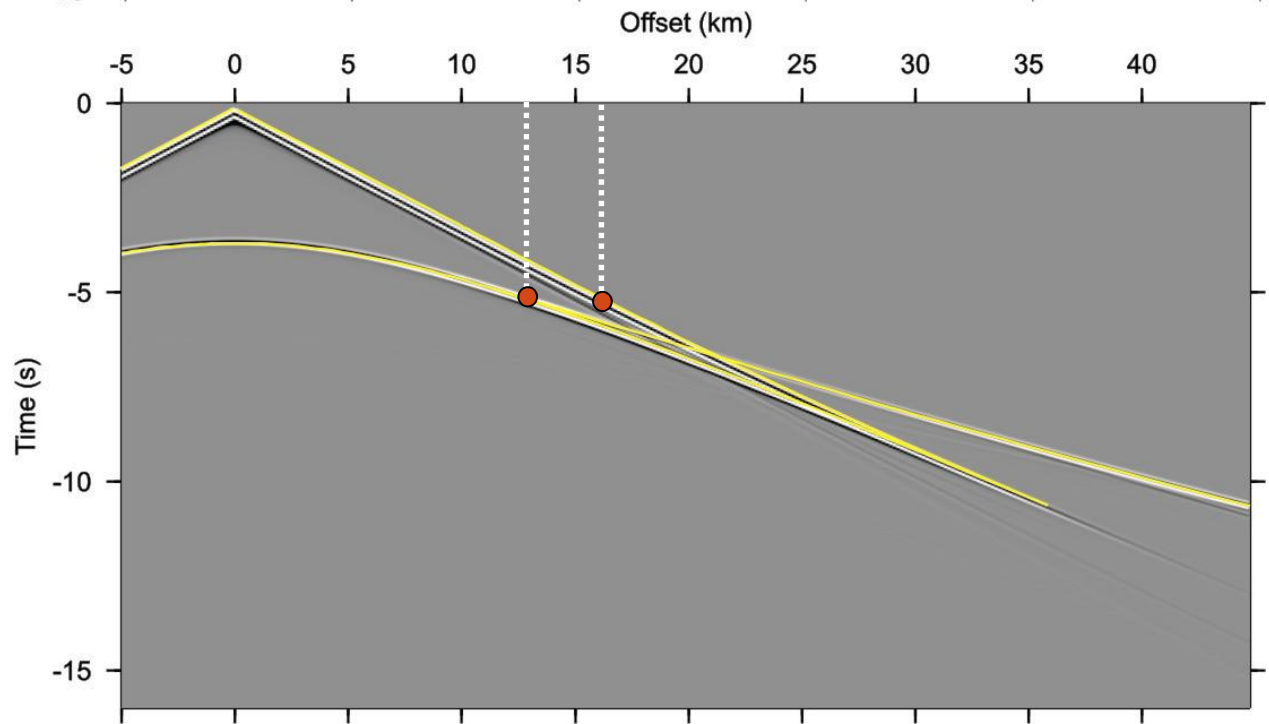
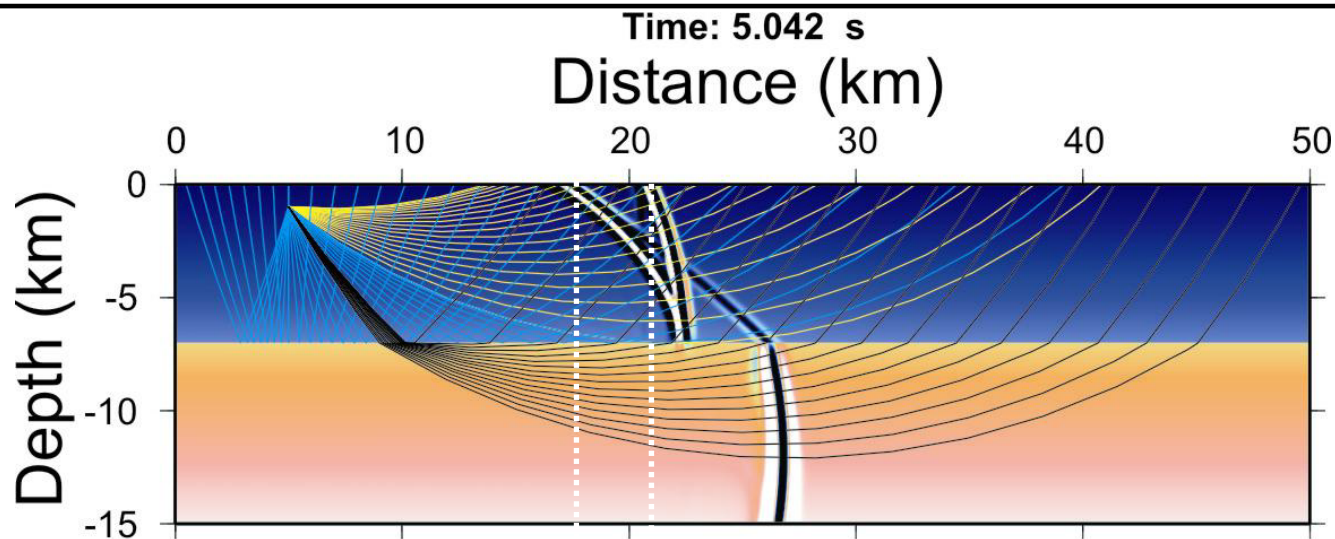




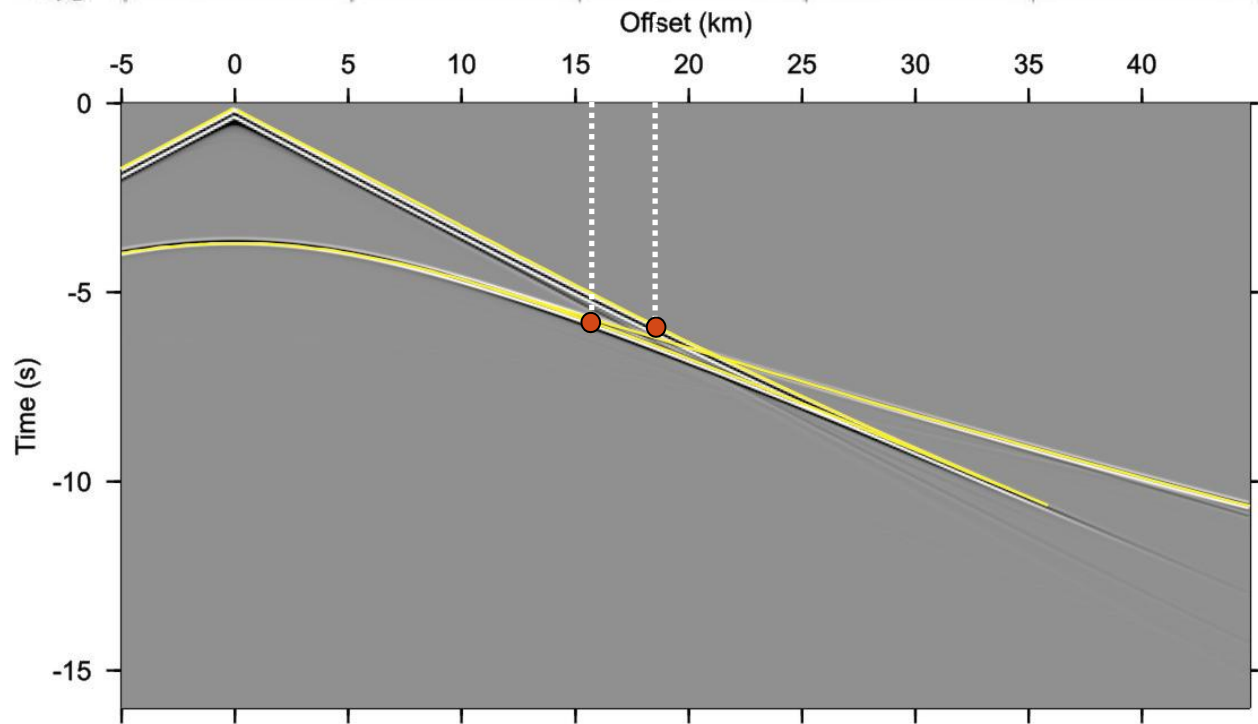
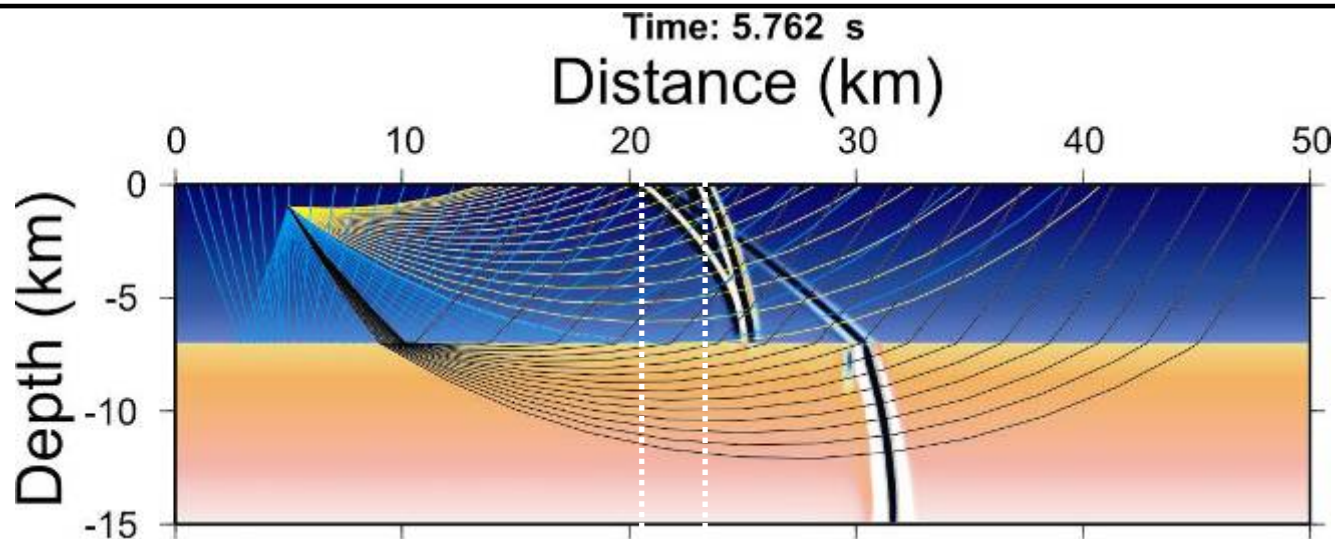


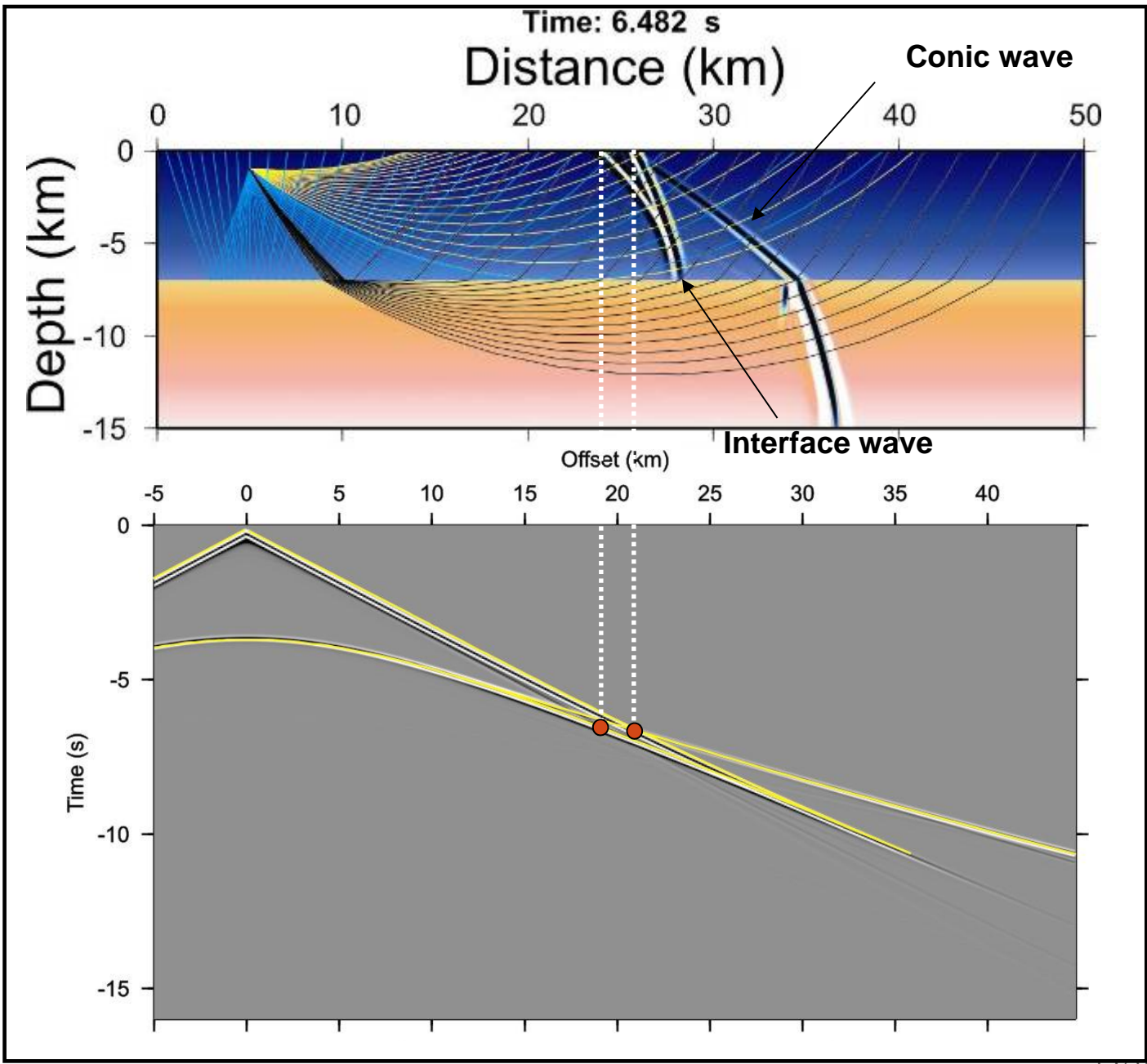


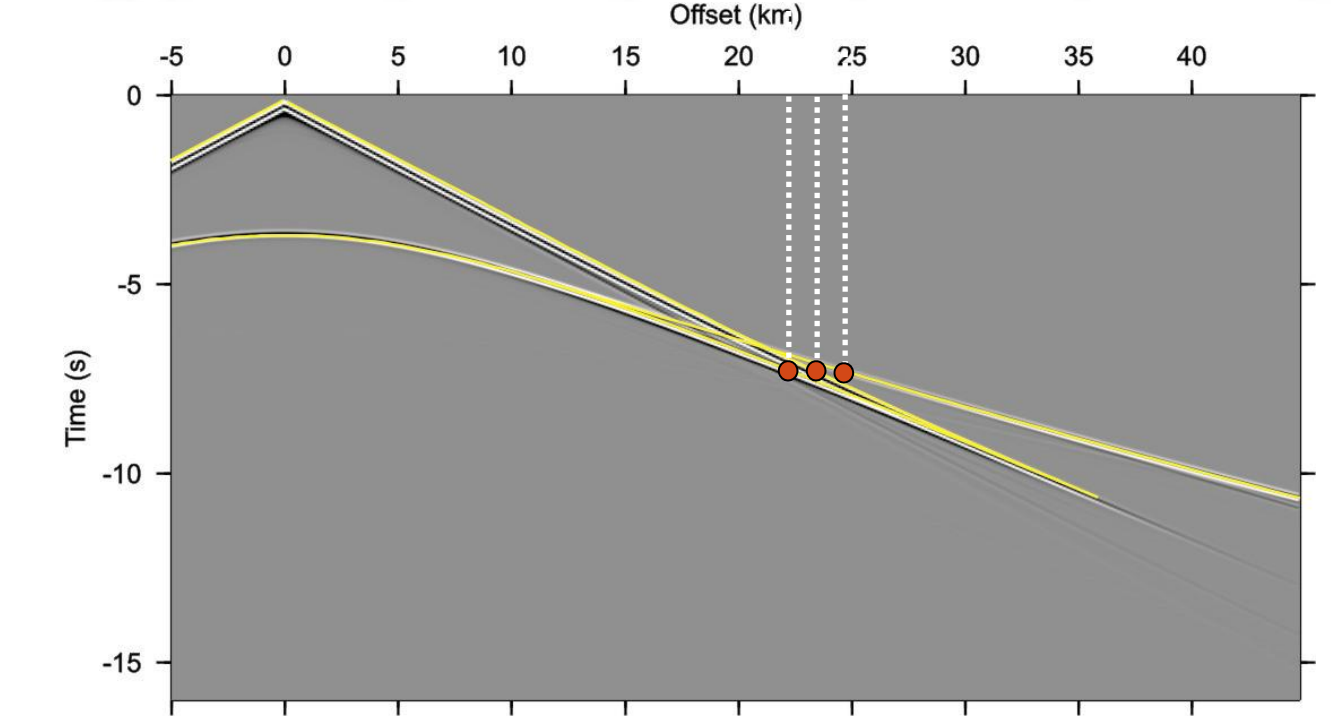
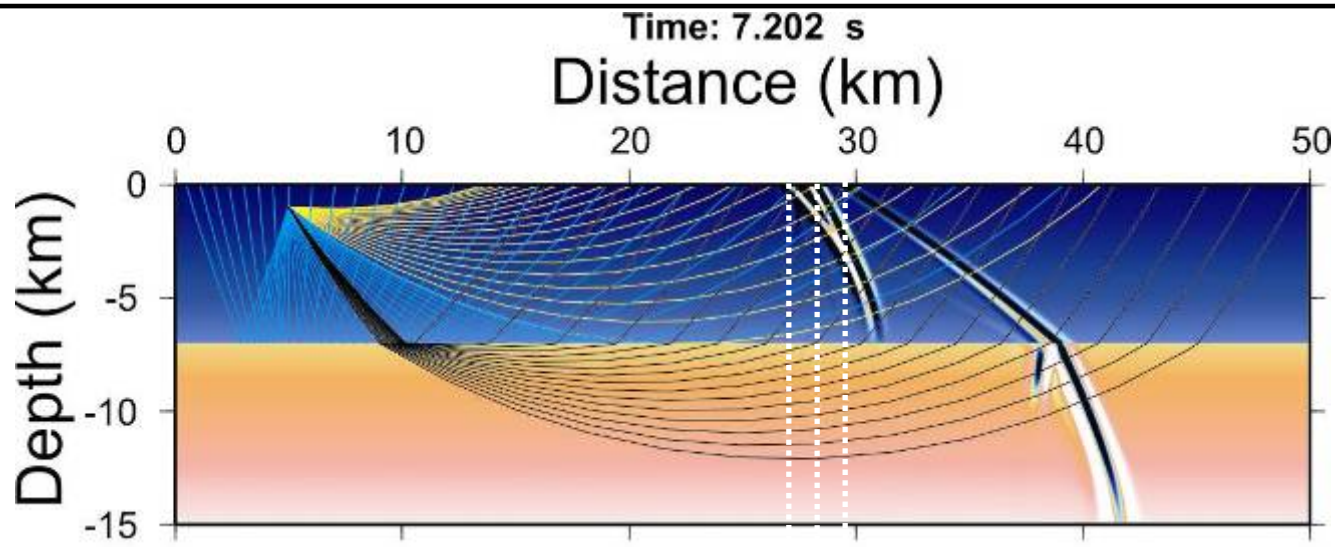


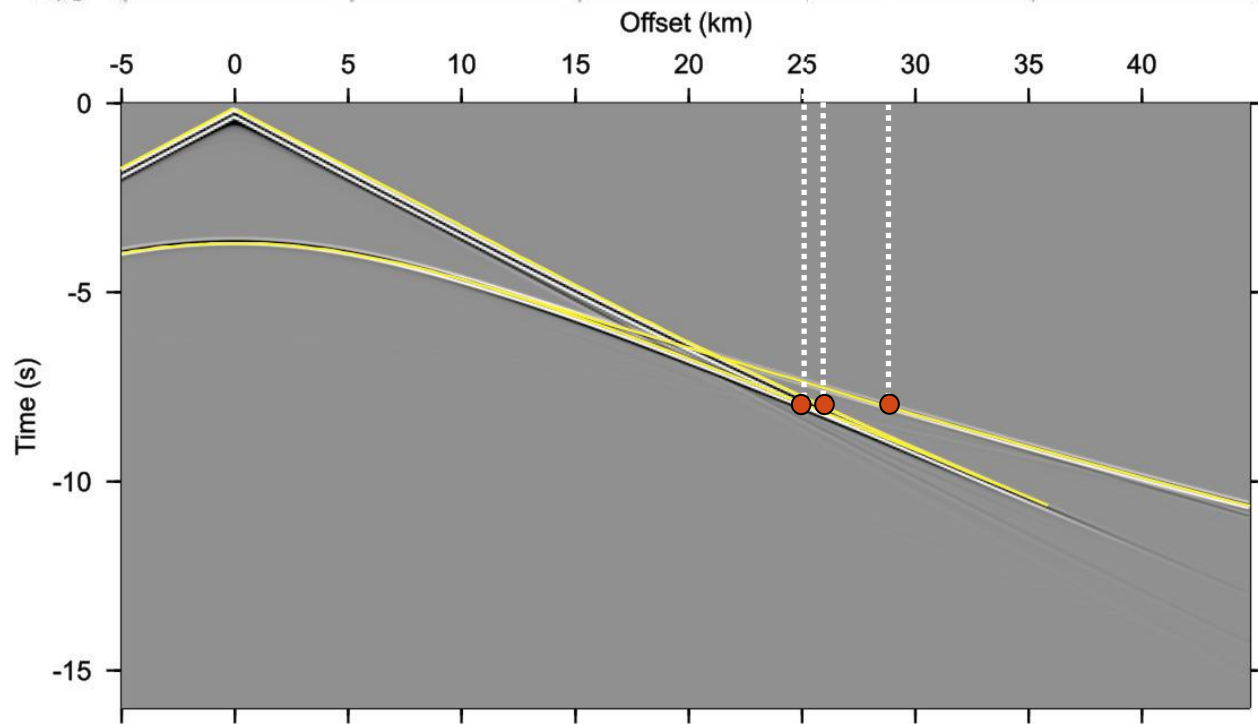
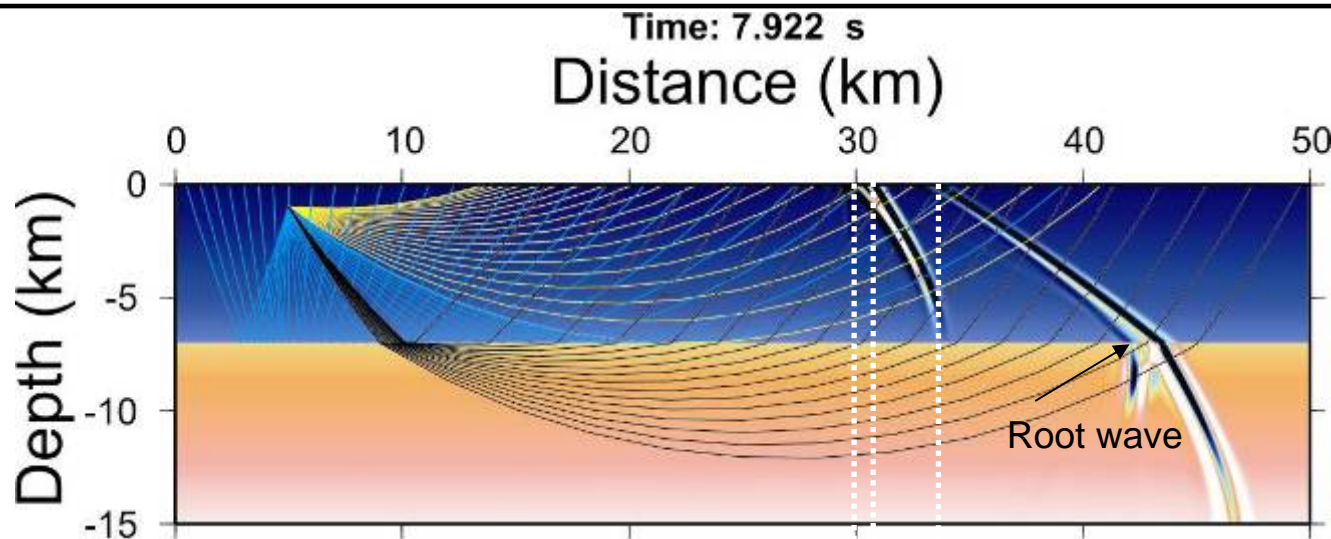


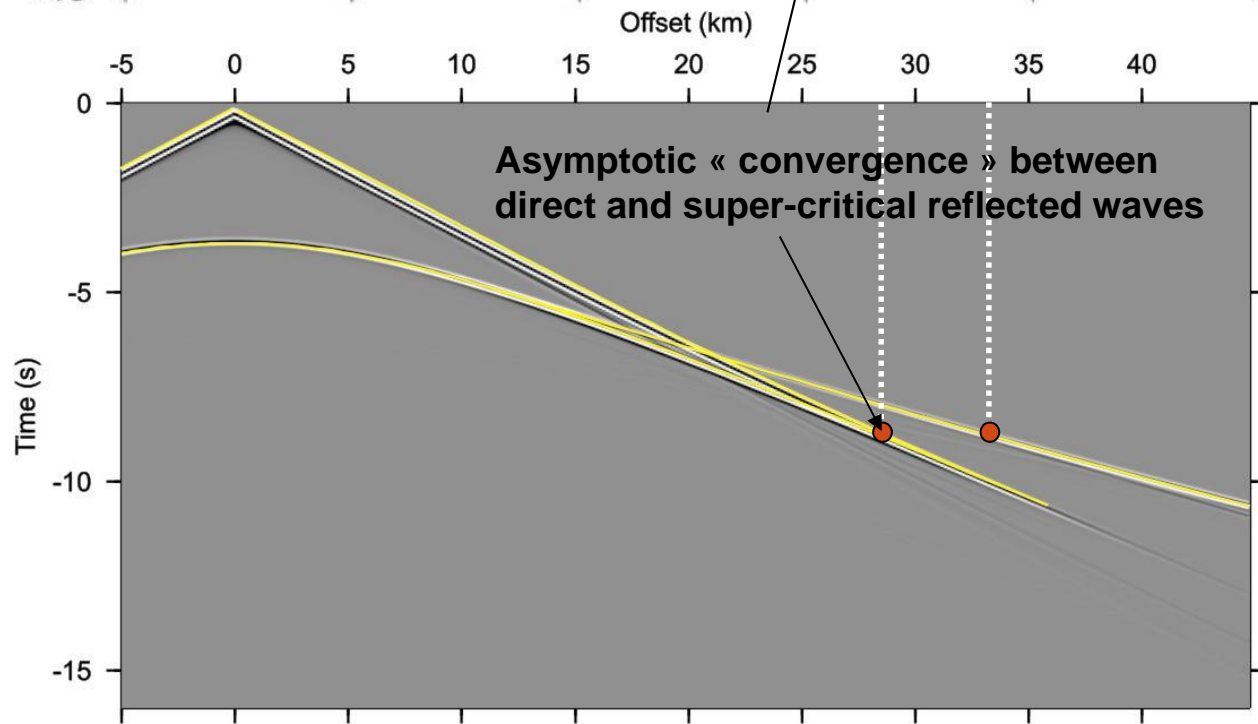
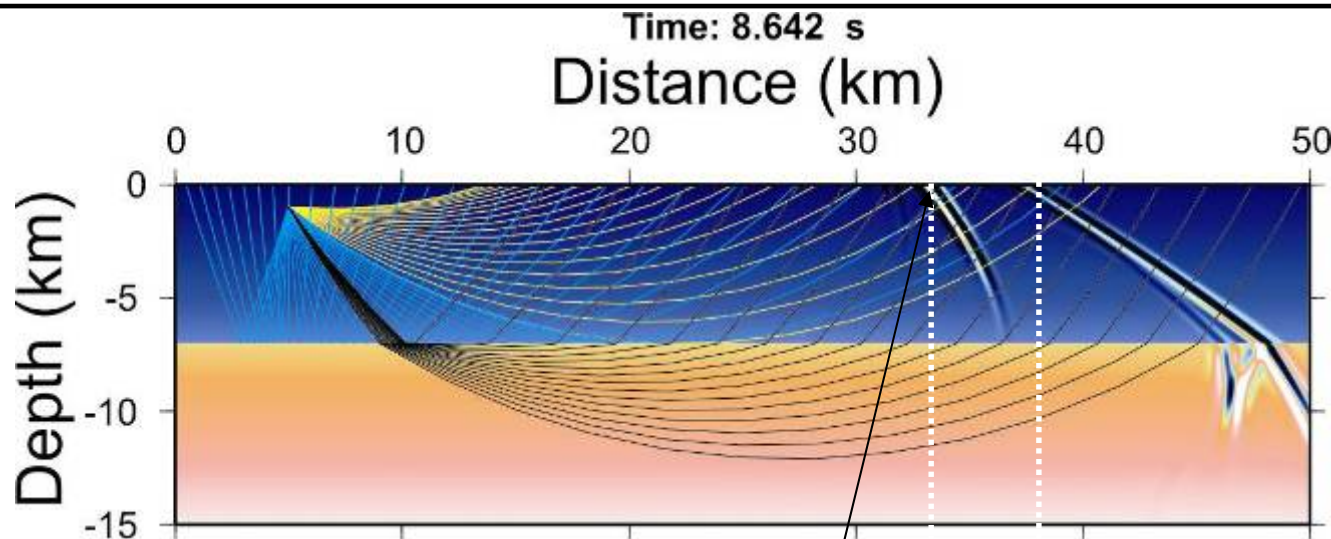


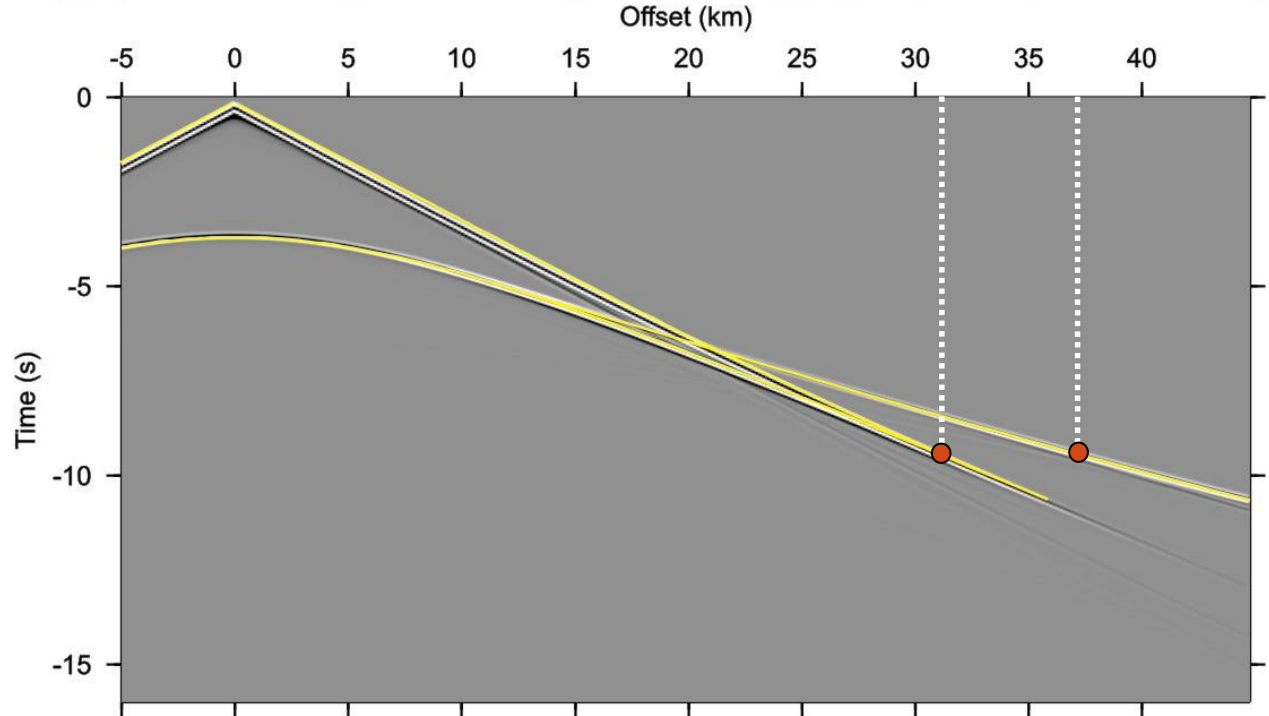
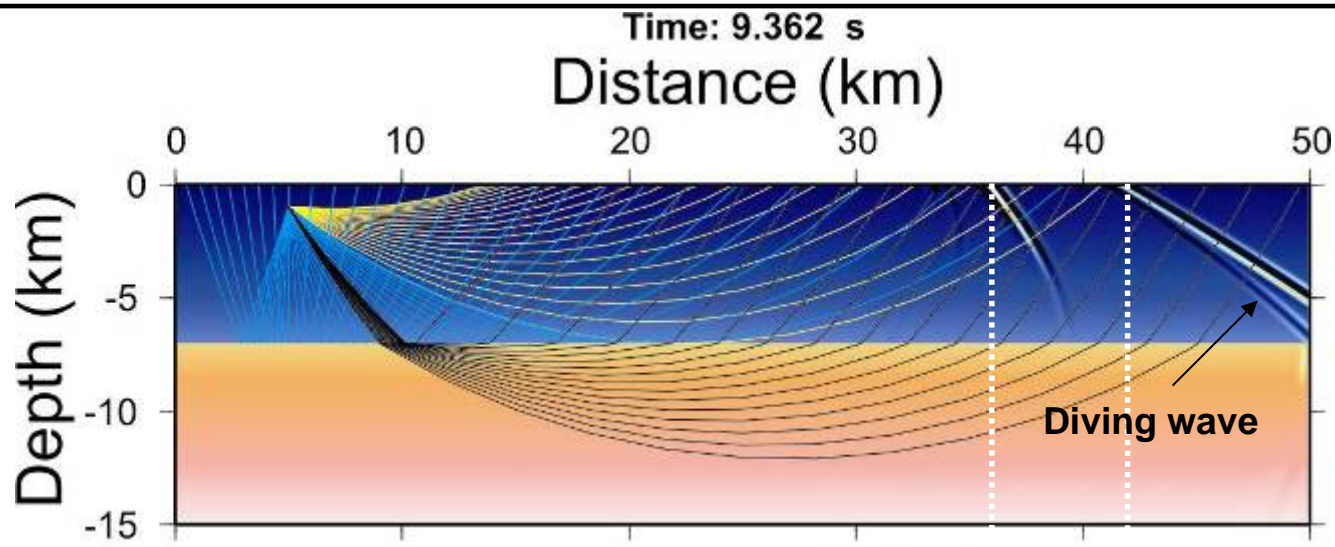




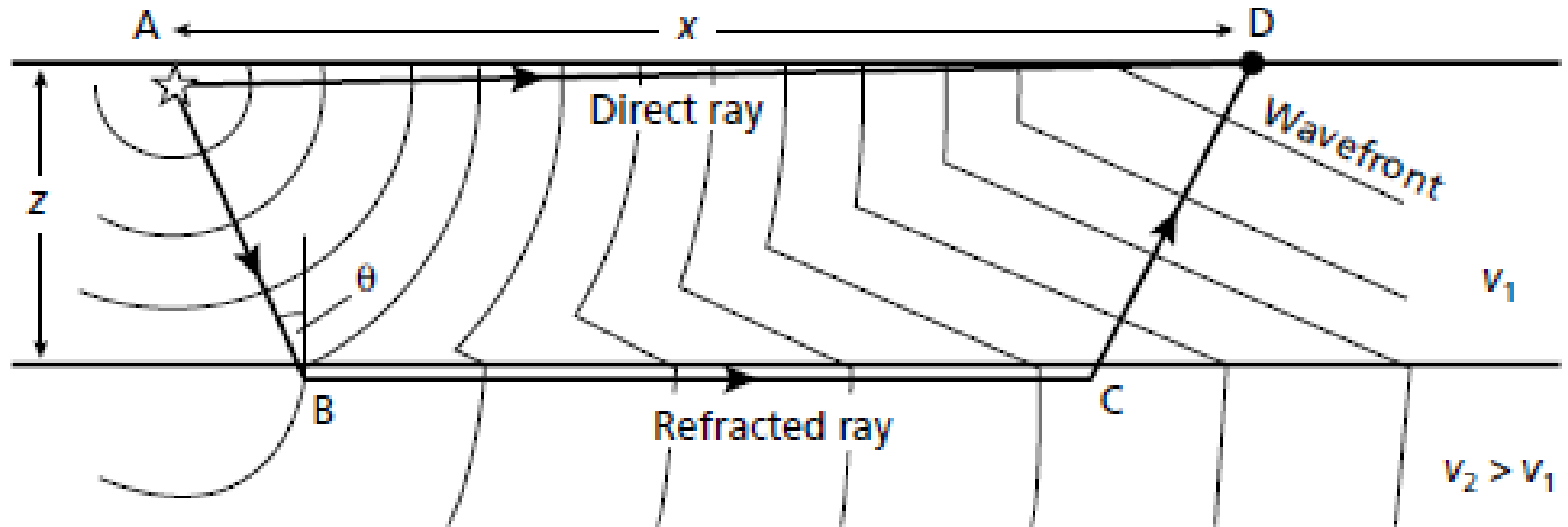






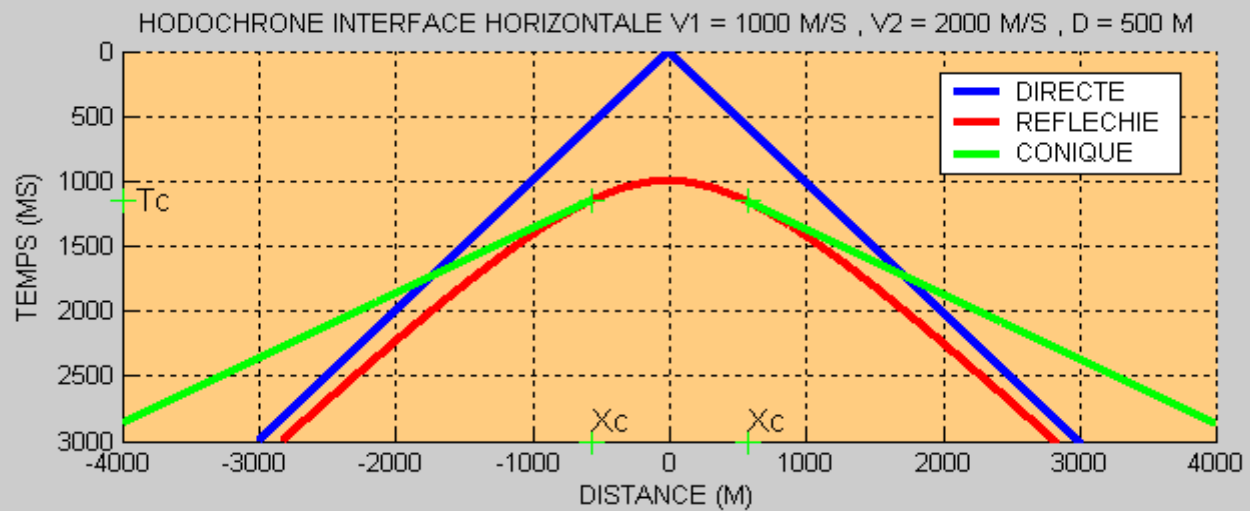
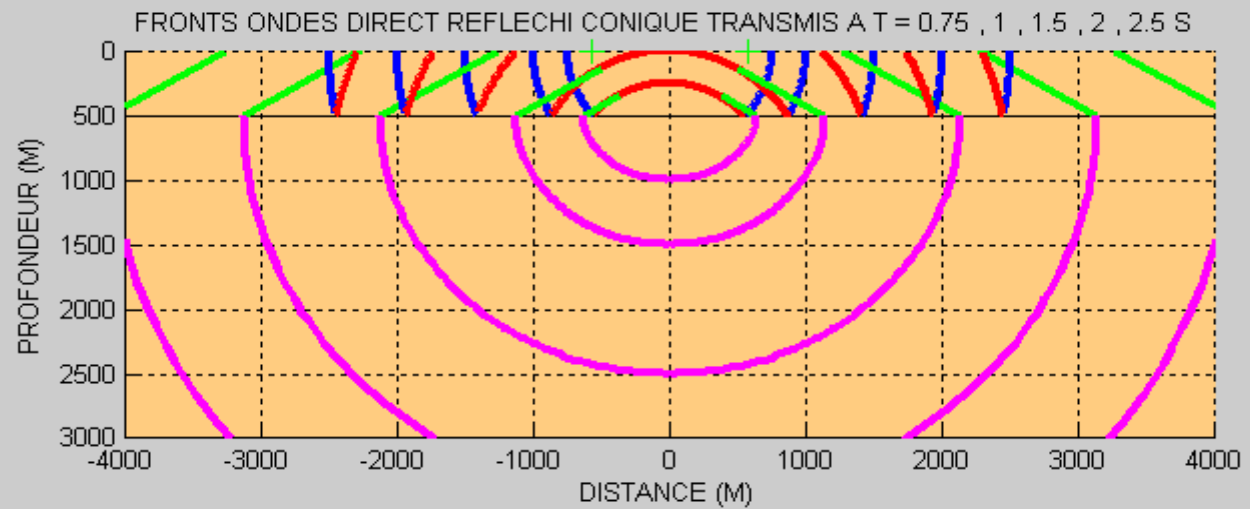


# Rays and Wavefronts



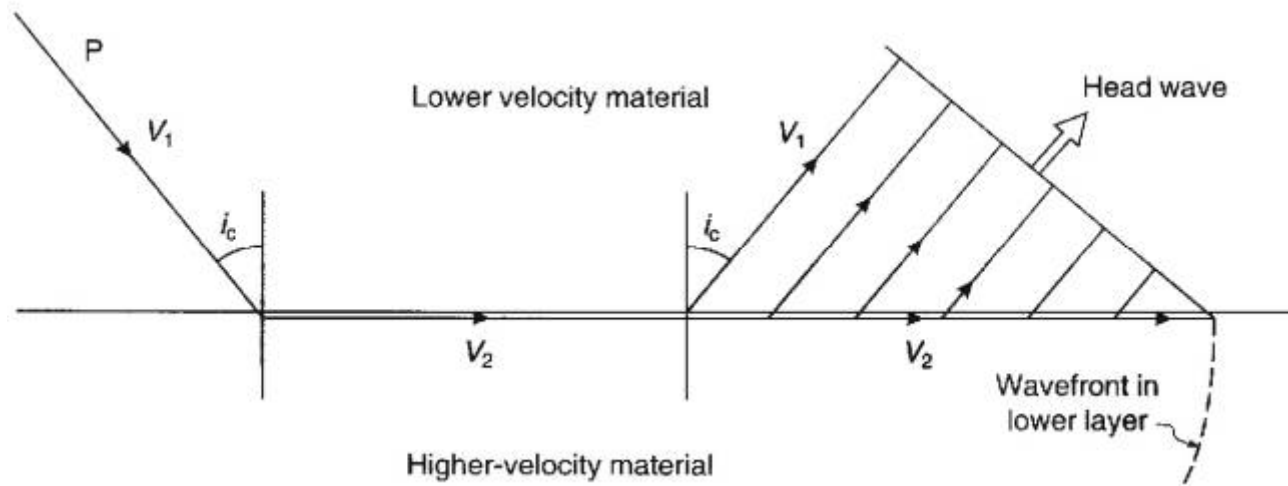
Reflections have been omitted in this cartoon

# Wave Propagation according to Huygens Principle

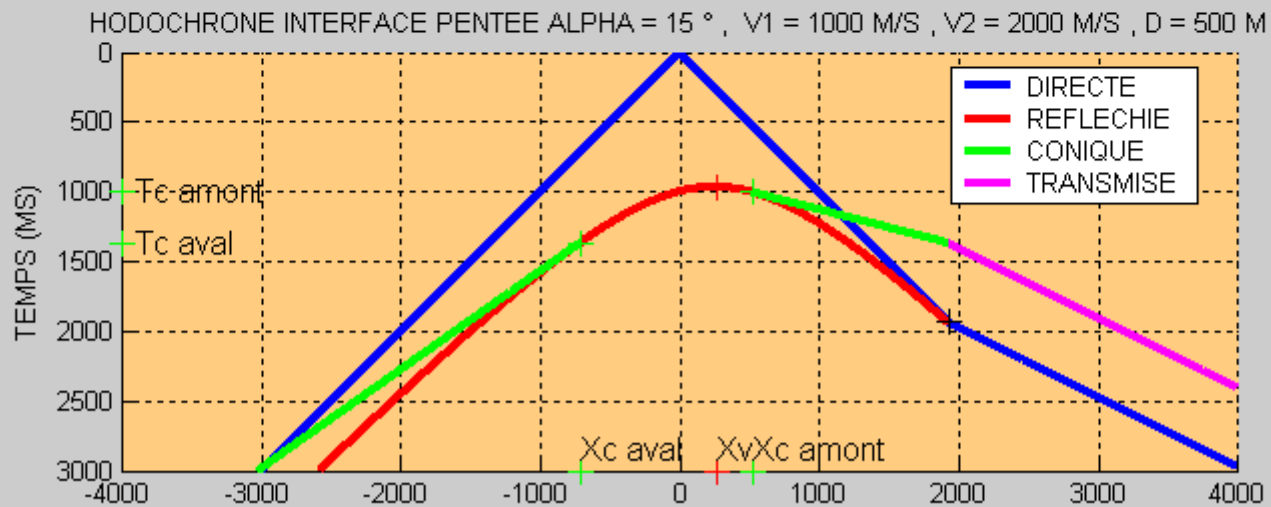
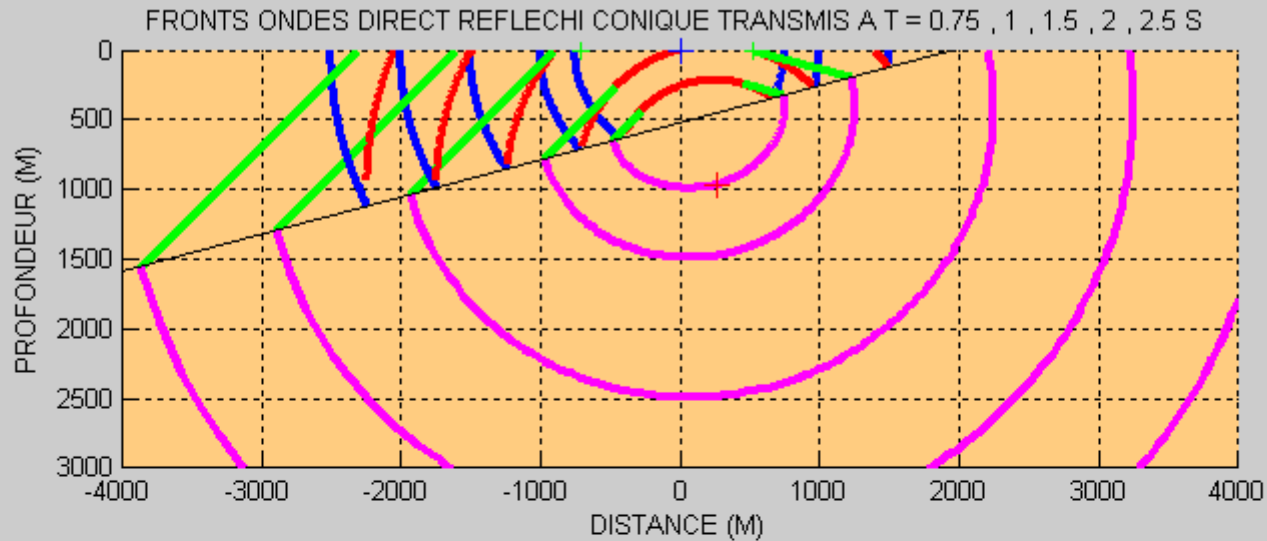




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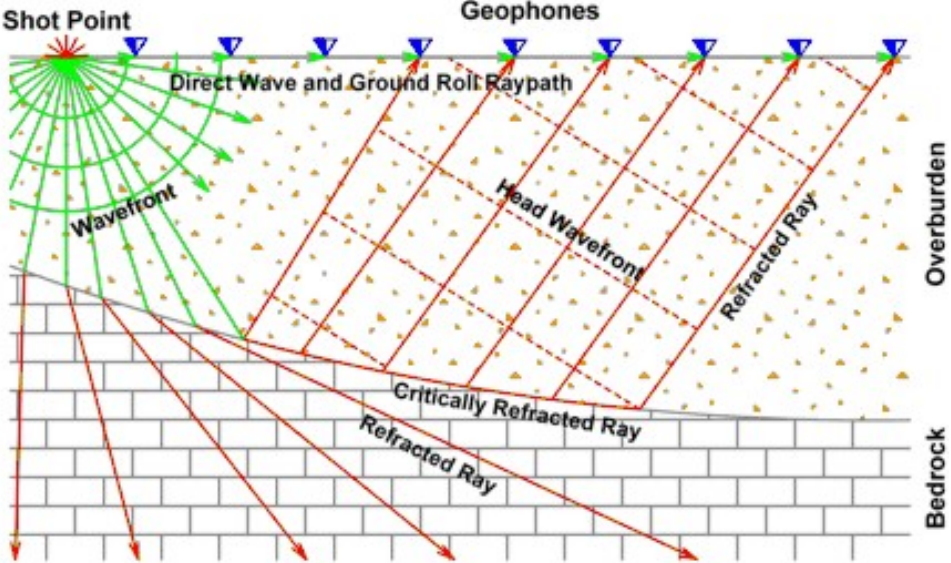


# Wave Propagation according to Huygens Principle

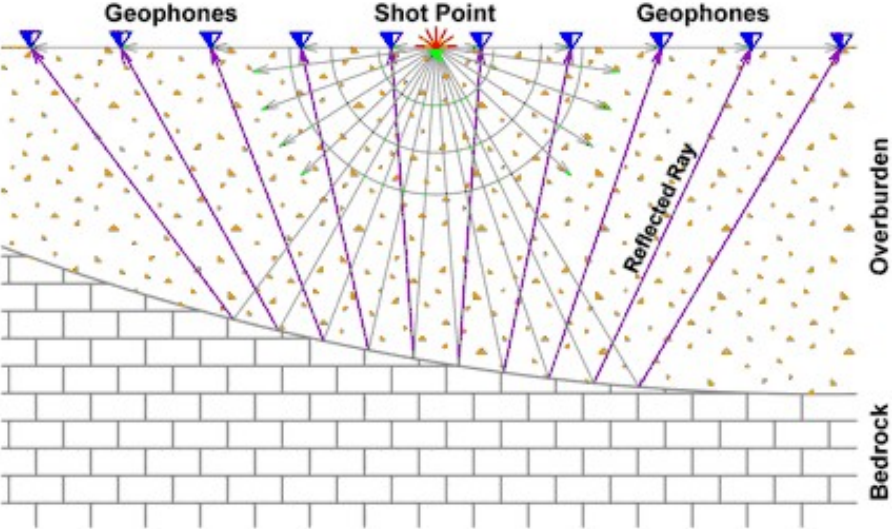


# Seismic Method comparison

## Seismic Refraction Geometry



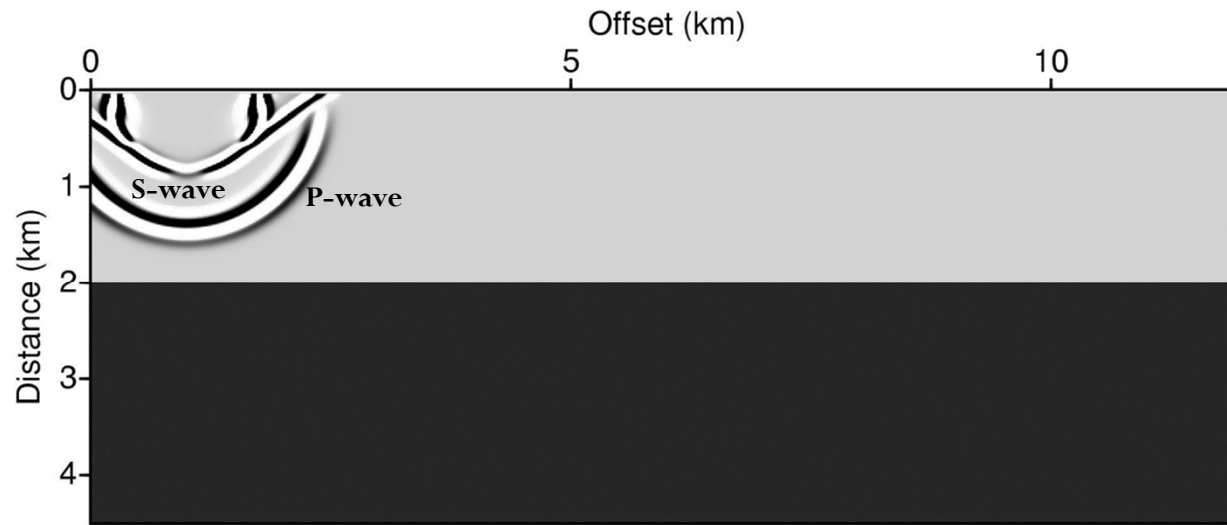
## Seismic Reflection Geometry



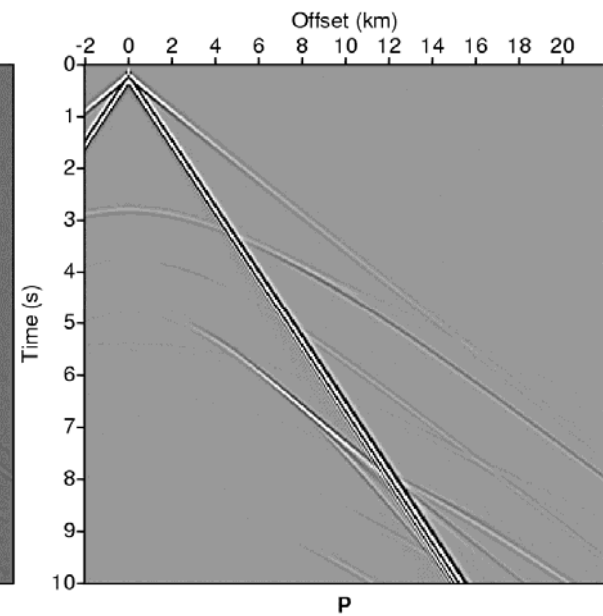
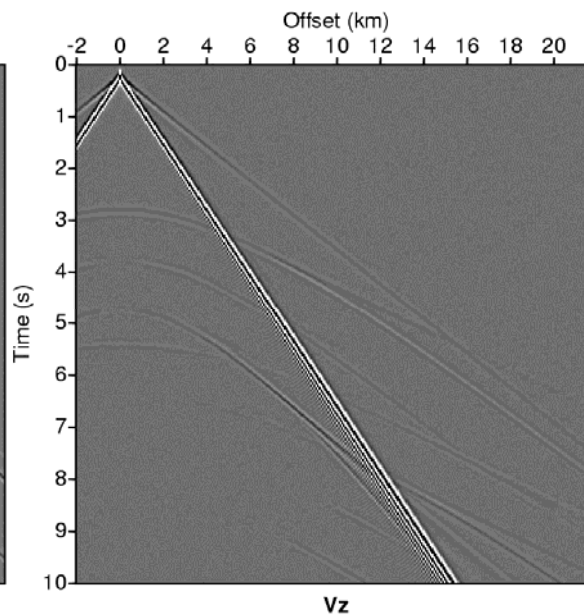
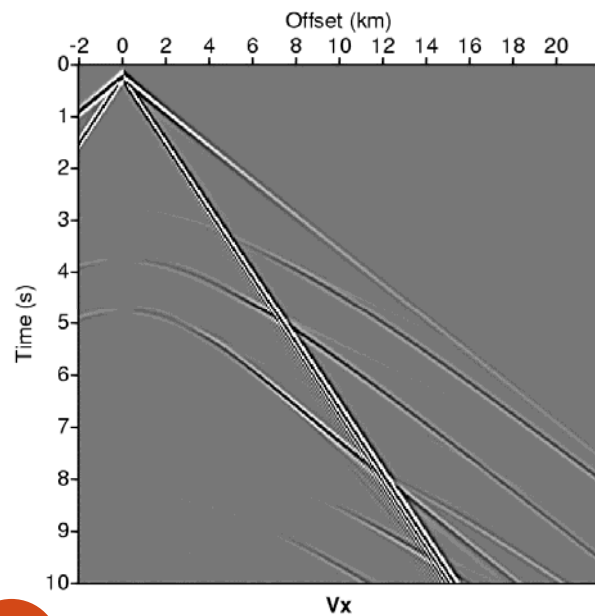
# Seismic Method comparison

	<b>Refraction</b>	<b>Reflection</b>
Typical targets	<b>Near-horizontal density contrasts at depths less than ~100 feet</b>	<b>Horizontal to dipping density contrasts, and laterally restricted targets such as cavities or tunnels at depths greater than ~50 feet</b>
Required Site Conditions	<b>Accessible dimensions greater than ~5x the depth of interest; unpaved greatly preferred</b>	None
Vertical Resolution	<b>10 to 20 percent of depth</b>	<b>5 to 10 percent of depth</b>
Lateral Resolution	<b>~1/2 the geophone spacing</b>	<b>~1/2 the geophone spacing</b>
Effective Practical Survey Depth	<b>1/5 to 1/4 the maximum shot-geophone separation</b>	<b>&gt;50 feet</b>
Relative Costs	N	3N-5N

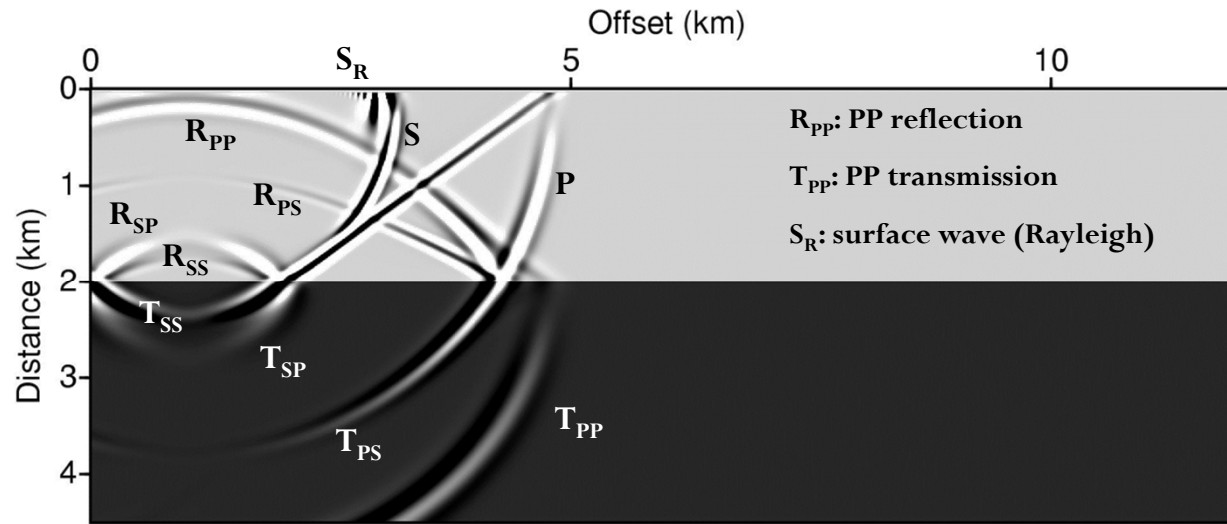
# Propagation in an **elastic** two-layer medium with a free surface



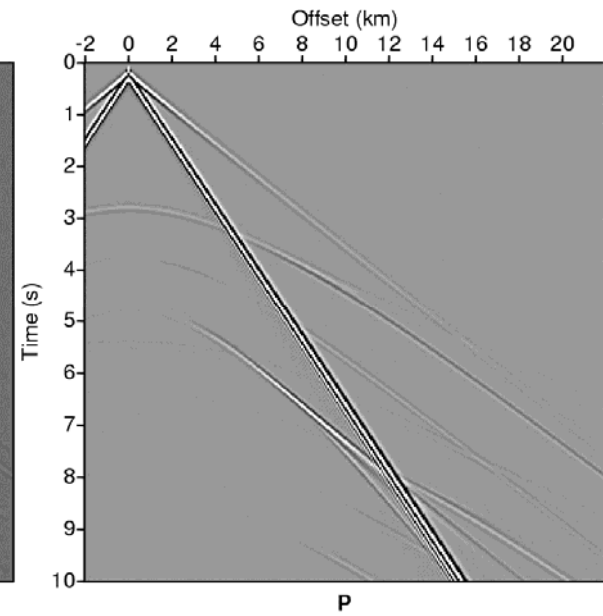
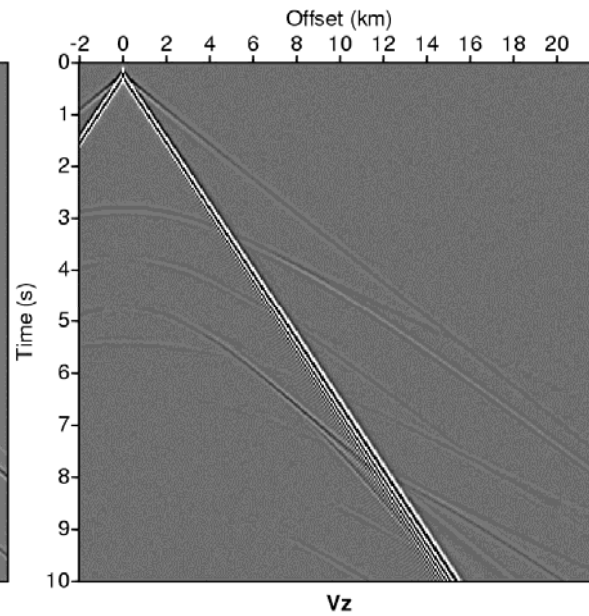
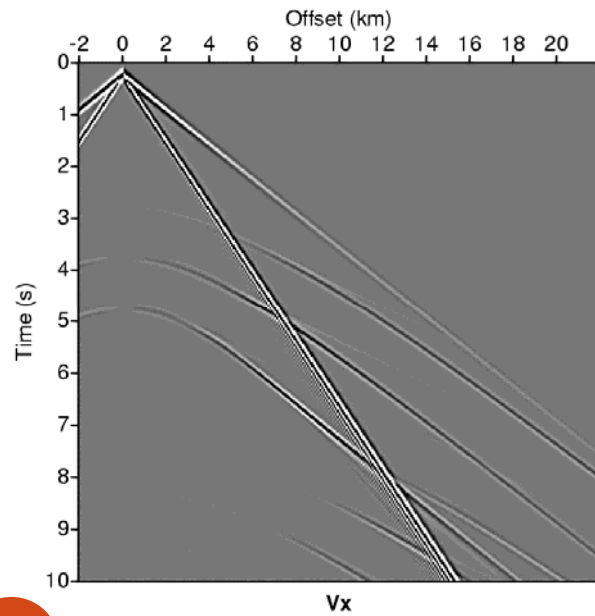
Time 1.6 s



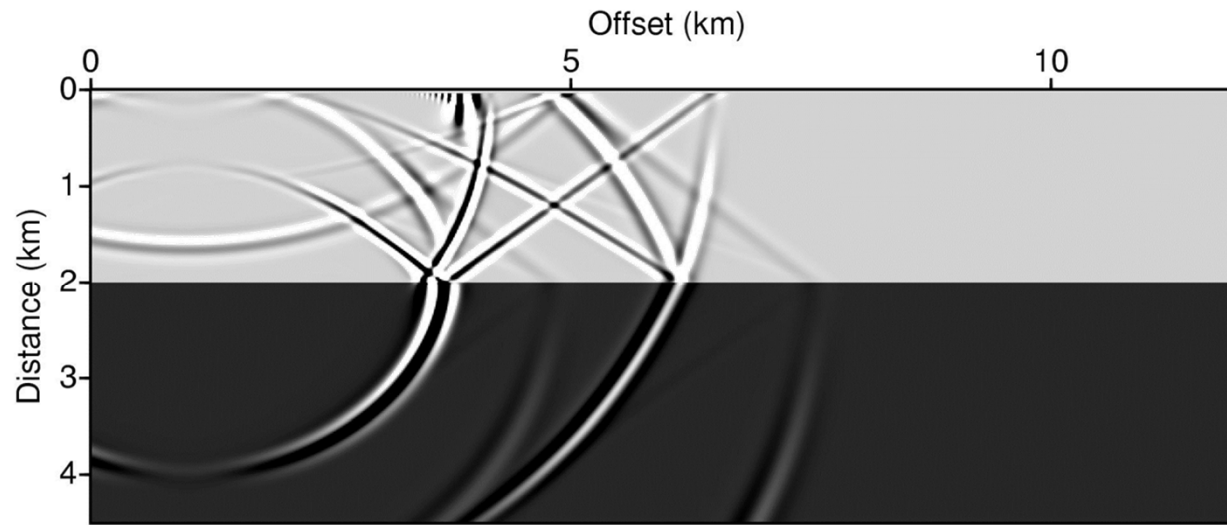
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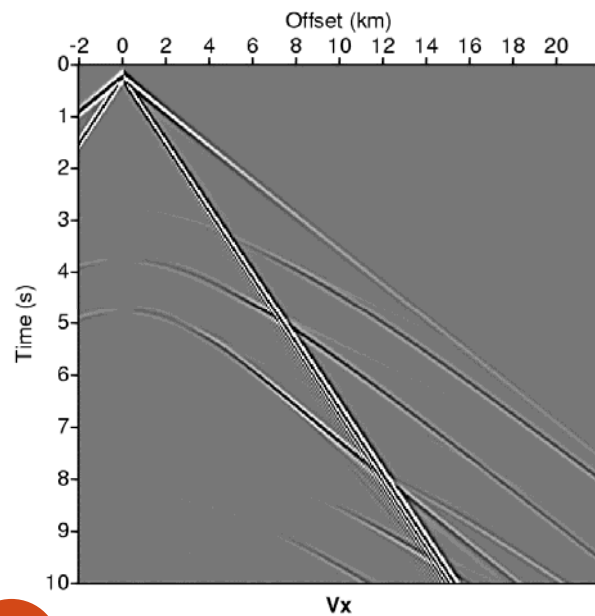
Time 3.6 s



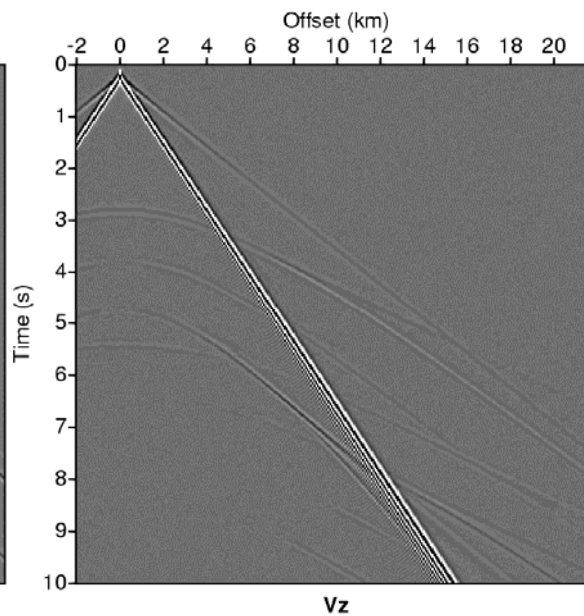
# Propagation in an elastic two-layer medium with a free surface



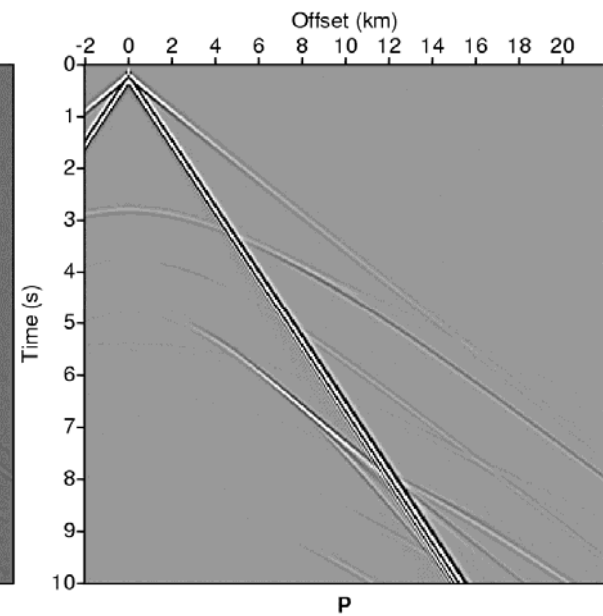
Time 5.0 s



$V_x$

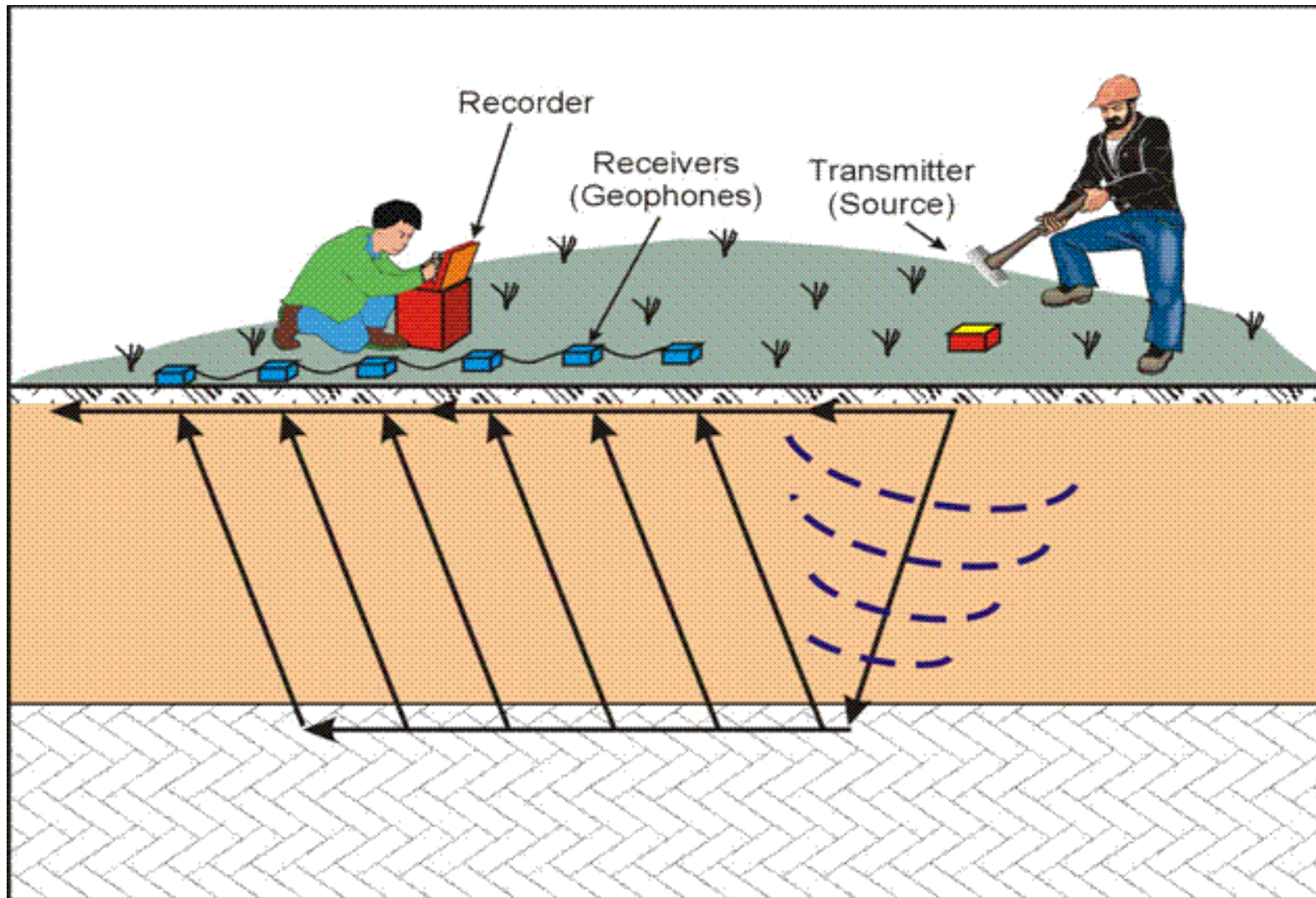


$V_z$



$P$

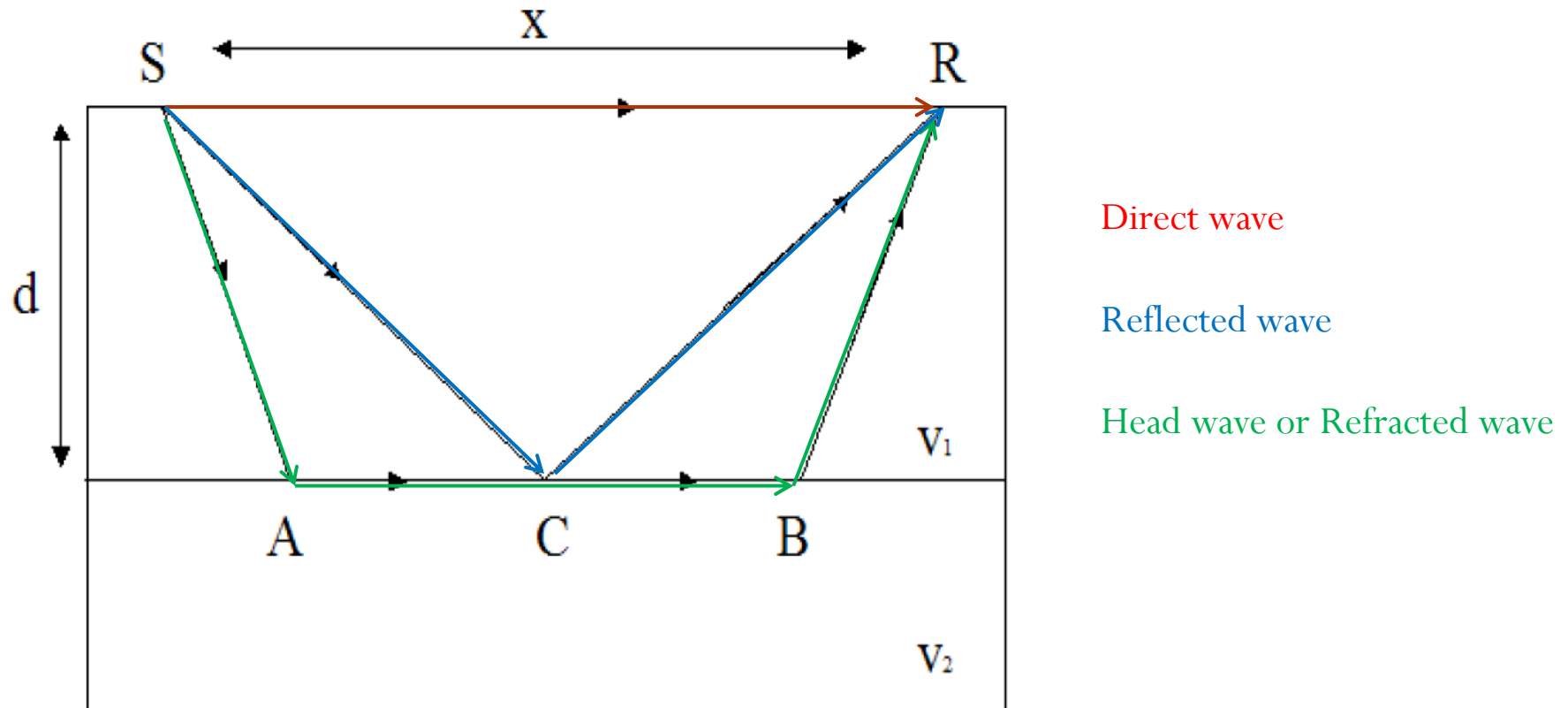
# Two-layered model





# Two-layered model

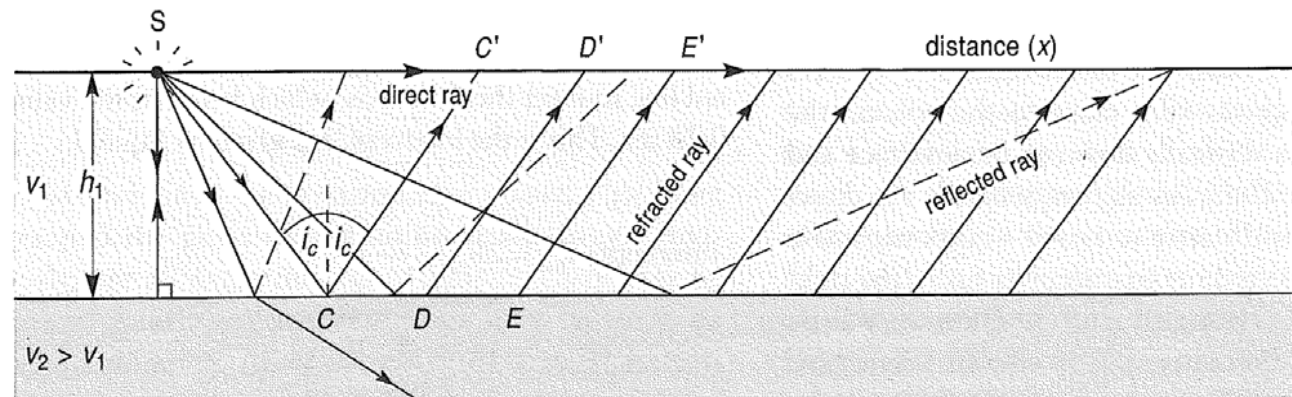
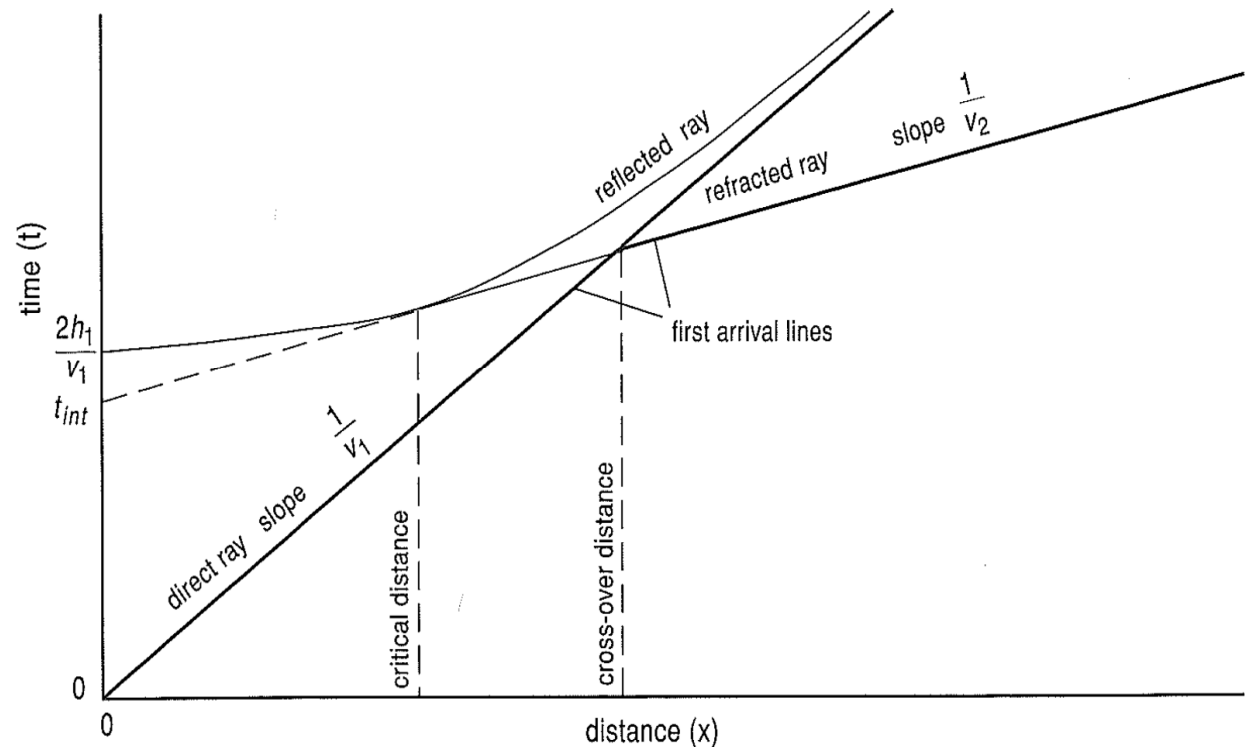
Energy from the source can reach the receiver via different paths



# Time-Distance Diagram (Travel Time curves)

## Think about:

- What would a fast velocity look like on this plot?
- Why is the direct ray a straight line?
- Why must the direct ray plot start at the origin (0,0)?
- Why is the refracted ray a straight line?
- Why does the refracted ray not start at the origin?
- Why does the reflected ray asymptotically approach the direct ray?
- Why is the reflected ray asymptotic with the direct ray?

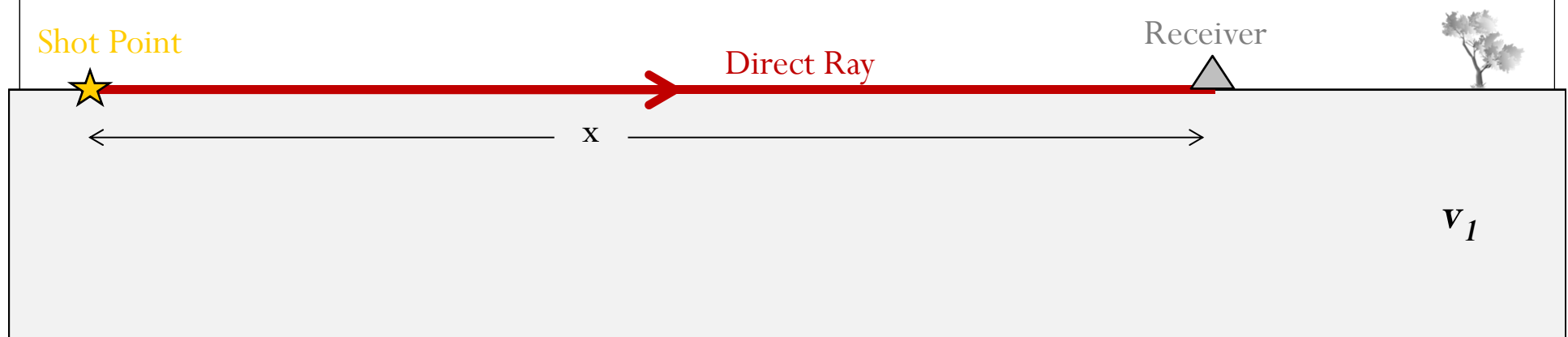
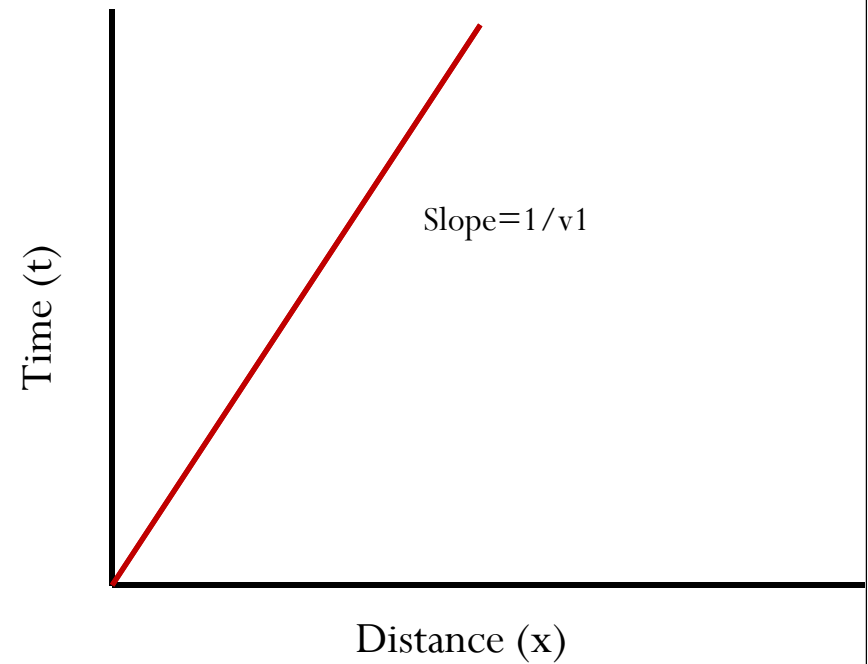


# Two-layered model

## 1. Direct wave

Energy travelling through the top layer, travel-time

The travel-time curve for the direct wave is simply a linear function of the seismic velocity and the shot-point to receiver distance

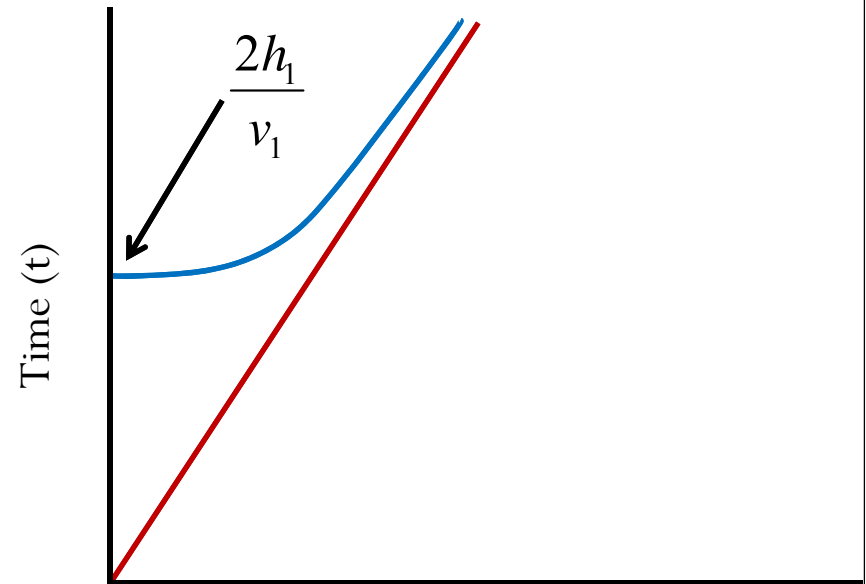


# Two-layered model

1. Direct wave

2. Reflected wave

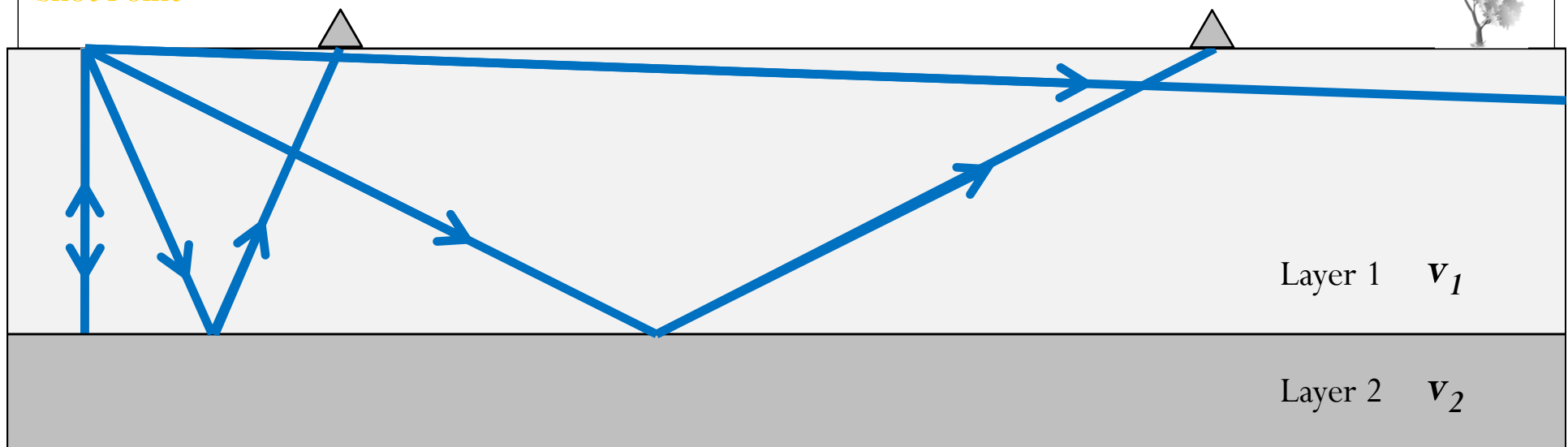
- Energy reflecting off the velocity interface.
- As the angles of incidence and reflection are equal, the wave reflects halfway between source and receiver.
- The reflected ray arrival time is never a first arrival.



Distance (x)

Receiver

Shot Point



## 2. Reflected wave

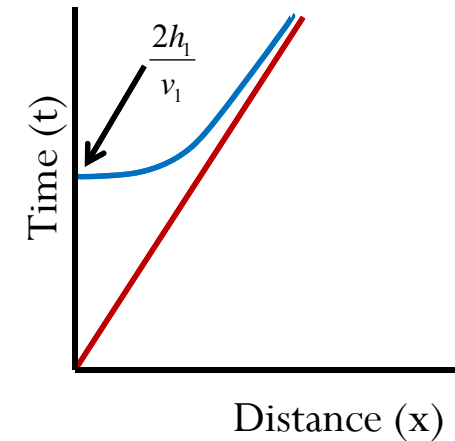
The travel time curve can be found by noting that  $x/2$  and  $h_1$  form two sides of a right triangle, so

$$T_R(x) = \frac{2\sqrt{\left(\frac{x}{2}\right)^2 + h_1^2}}{v_1}$$

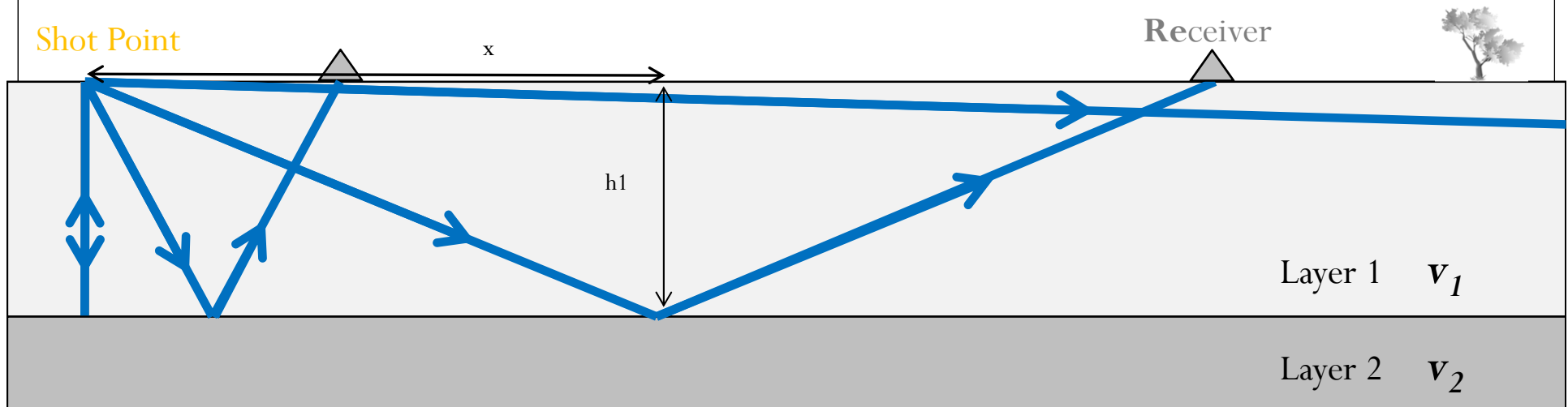
This curve is a hyperbola, it can be written as

$$T_R^2(x) = \frac{x^2}{v_1^2} + 4\frac{h_1^2}{v_1^2}$$

For  $x = 0$  the reflected wave goes straight up and down, with a travel time of  $T_R(0) = 2h_1/v_1$ . At distances much greater than the layer thickness ( $x \gg h_1$ ), the travel time for the reflected wave asymptotically approaches that of the direct wave.



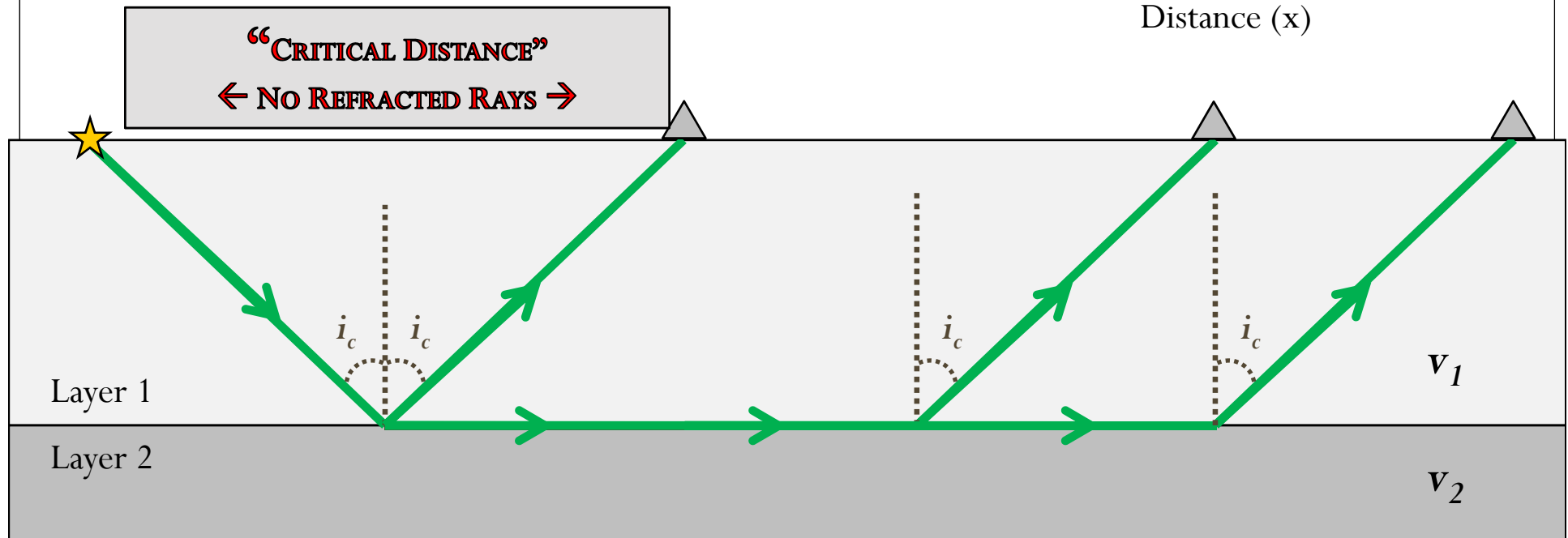
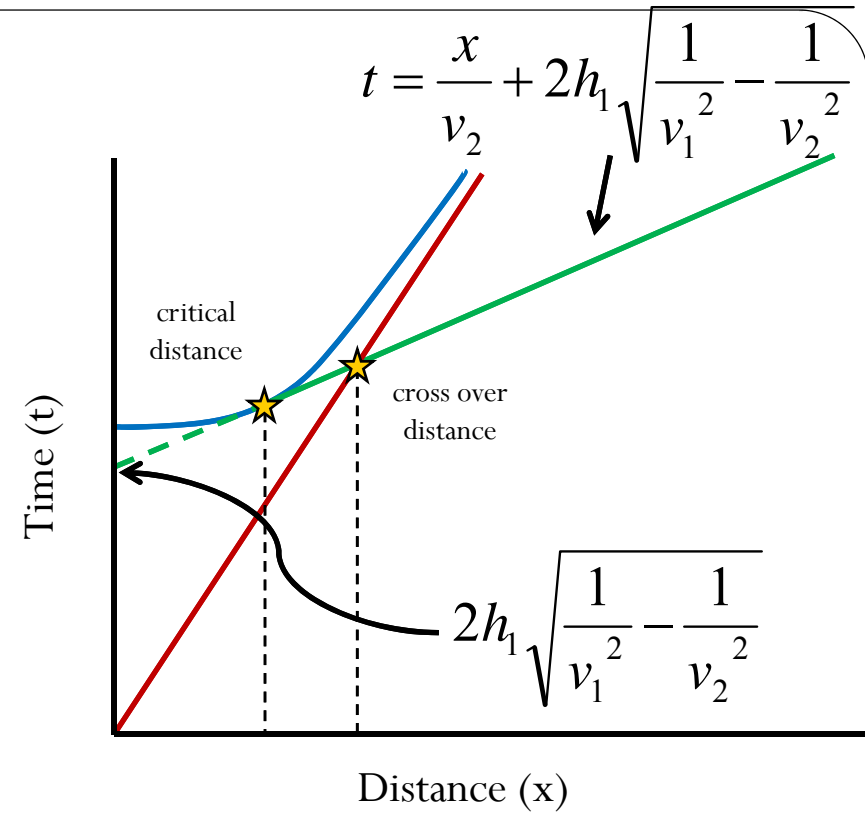
**“INTERCEPT TIME”  
GIVES LAYER THICKNESS**



# Two-layered model

1. Direct wave
2. Reflected wave
3. Head wave or Refracted wave

- Energy refracting across the interface.
- Only arrives after **critical distance**.
- Is first arrival only after **cross over distance**



### 3. Head wave or Refracted wave

The travel time can be computed by assuming that the wave travels down to the interface such that it impinges at critical angle, then travels just below the interface with the velocity of the lower medium, and finally leaves the interface at the critical angle and travels upwards to the surface.

$$T_h(x) = \frac{AB}{V_1} + \frac{BC}{V_2} + \frac{CD}{V_1}$$

Show that:

$$T_h(x) = \frac{x}{V_2} + \frac{2h_1 \cos(i_c)}{V_1}$$

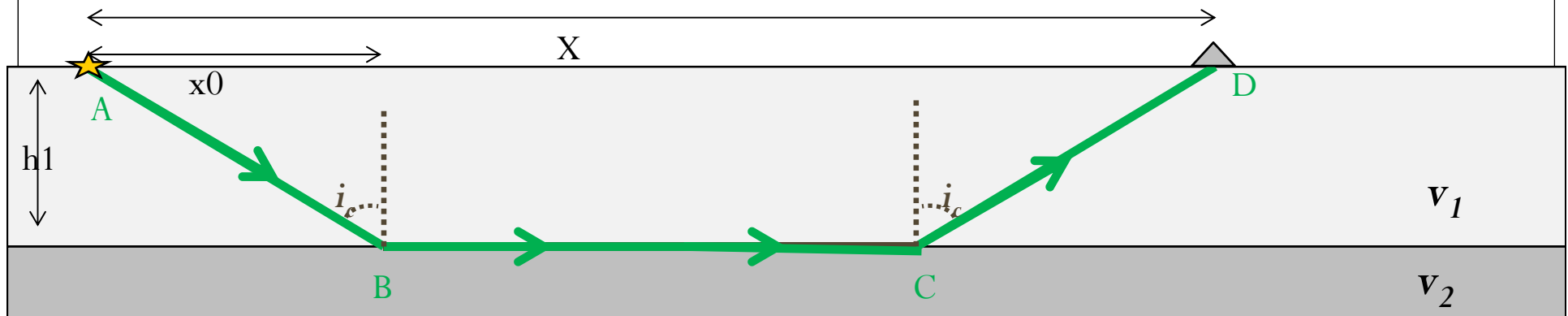
Do it many  
times  
yourself !

Reminder

$$\sin(i_c) = \frac{V_1}{V_2}$$

$$\tan = \frac{\sin}{\cos}$$

$$\cos^2 + \sin^2 = 1$$



### 3. Head wave or Refracted wave

The axis *intercept time* is found by projecting the travel time curve back to  $x = 0$ . The intercept time allows a depth estimation.

$$\tau = 2h_1 \sqrt{\left(\frac{1}{v_1}\right)^2 - \left(\frac{1}{v_2}\right)^2}$$

*Critical distance  $x_c$* : distance beyond which critical incidence first occurs.

$$x_c = 2h_1 \tan(i_c)$$

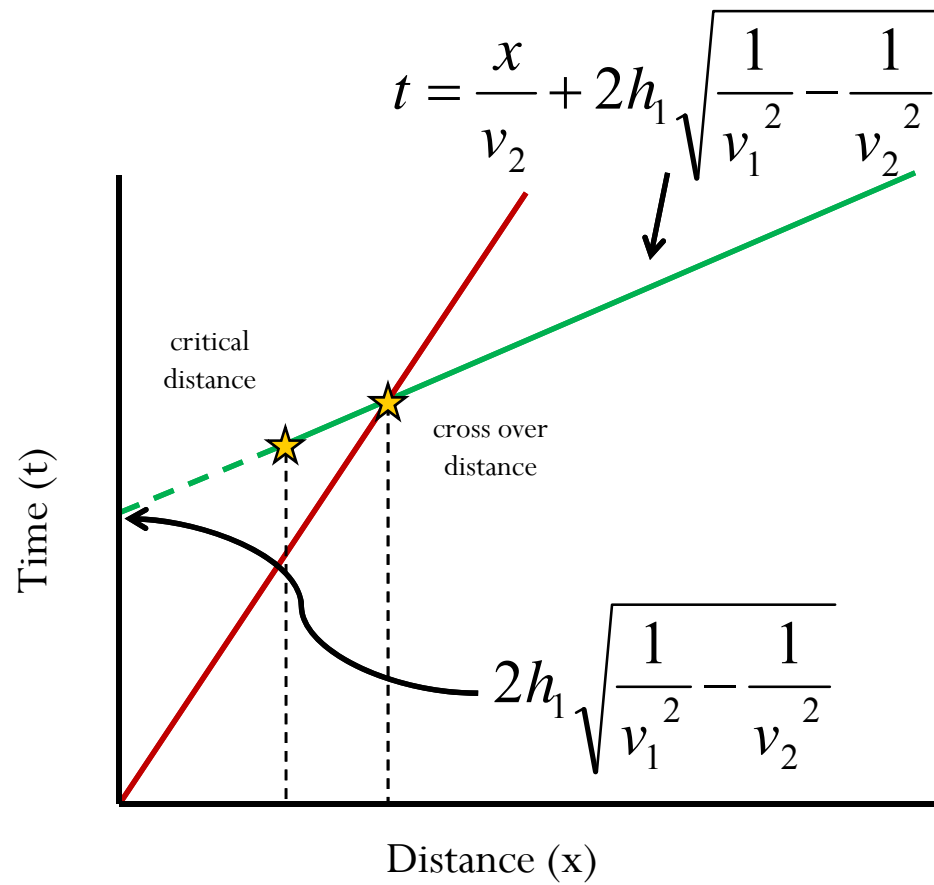
At the critical distance the direct wave arrives before the head wave. At some point, however, the travel time curves cross, and beyond this point the head wave is the first arrival. The *crossover distance,  $x_d$* , where this occurs, is found by setting  $TD(x) = TH(x)$ , which yields:

$$x_d = 2h_1 \sqrt{\frac{v_2 + v_1}{v_2 - v_1}}$$

The crossover distance is of interest to determine the length of the refraction line.



# Travel-time for refracted waves



Reminder:  $\sin(ic) = \frac{\alpha_1}{\alpha_2}$

## Note on Refraction angle

Interesting to notice that the higher the velocity contrast, the smaller the refraction angle.

$$V_1 = 1000 \text{ m/s} \quad \lambda = 11^\circ$$

$$V_2 = 5000 \text{ m/s}$$

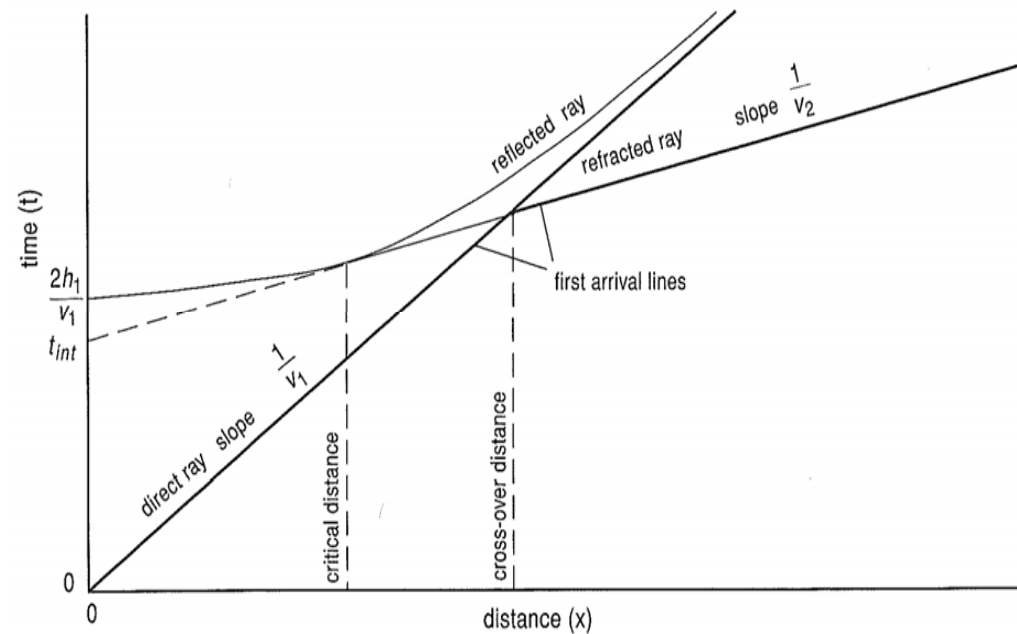
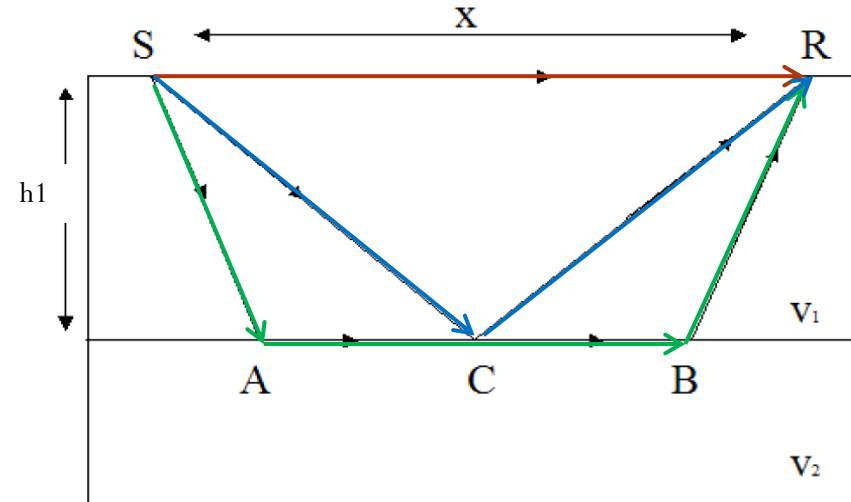
$$V_1 = 1000 \text{ m/s} \quad \lambda = 30^\circ$$

$$V_2 = 2000 \text{ m/s}$$

=> We can only analyse cases with an increasing velocity function with depth

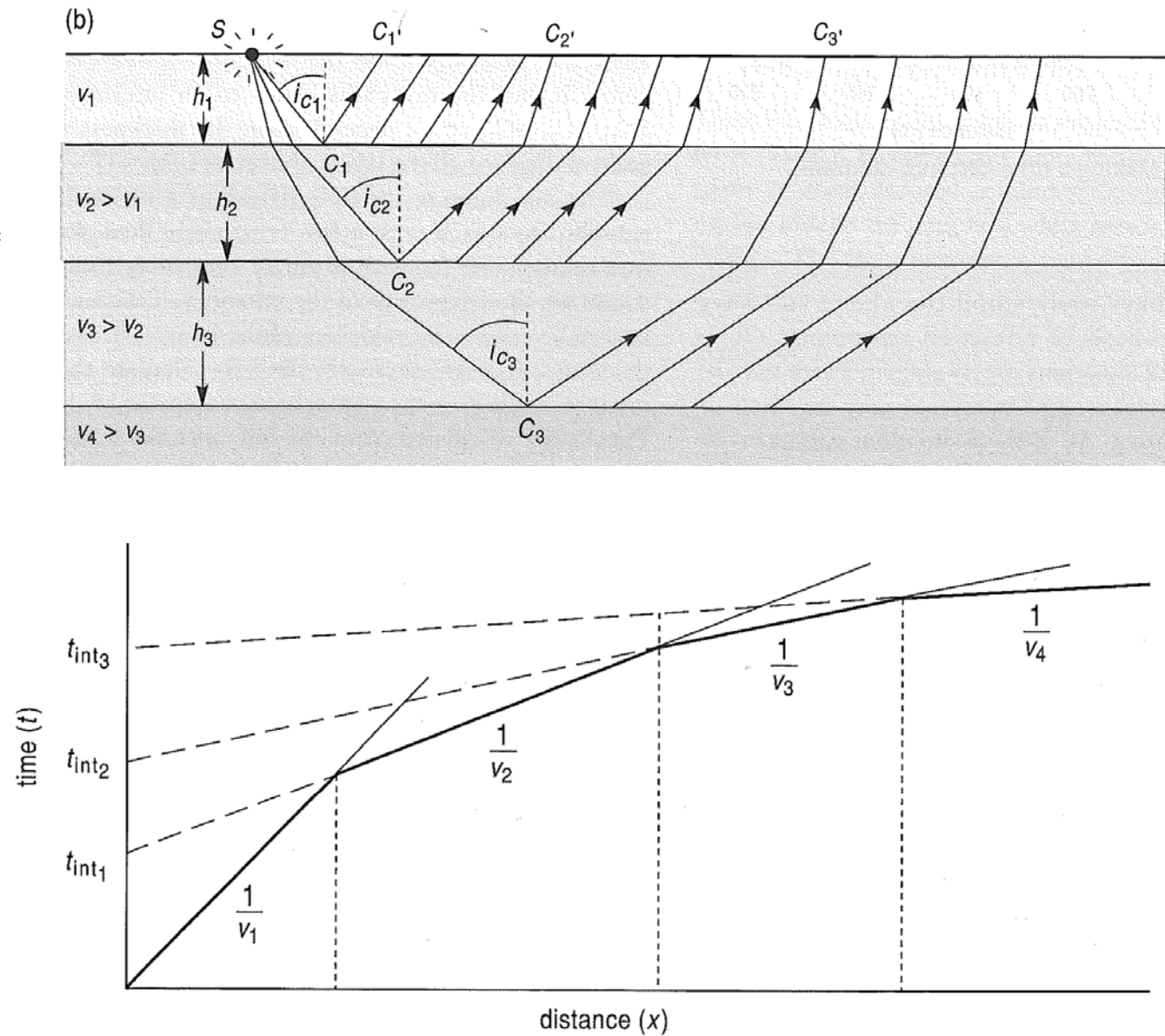
# Summary

- $v_1$  determined from the slope of the direct arrival (straight line passing through the origin)
- $v_2$  determined from the slope of the head wave (straight line first arrival beyond the critical distance)
- Layer thickness  $h_1$  determined from the intercept time of the head wave (already knowing  $v_1$  and  $v_2$ )



# Multiple-layers

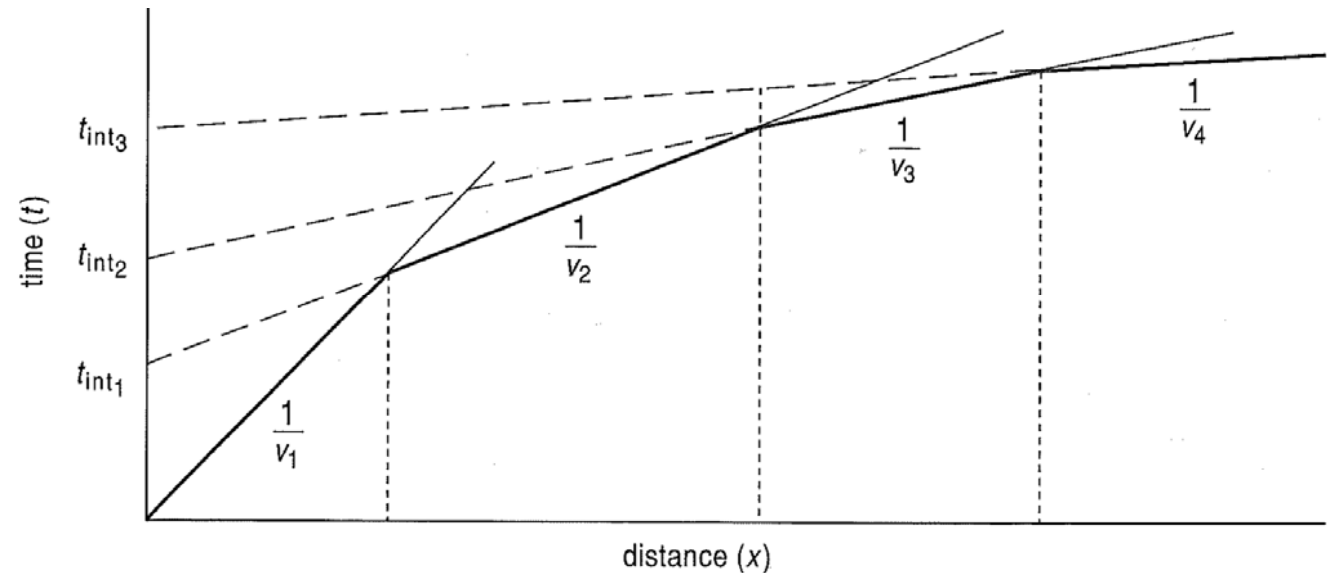
For multiple layered models we can apply the same process to determine layer thickness and velocity sequentially from the top layer to the bottom.



# Multiple-layers

- The layer thicknesses are not as easy to find
- Recall...

$$t = \frac{x}{v_1} + \frac{2h_1 \cos i_c}{v_1}$$



$$t_{\text{int}_1} = \frac{2h_1 \cos i_{c_1}}{v_1}$$

Solve for  $h_1$ ...

$$h_1 = \frac{v_1 t_{\text{int}_1}}{2 \cos i_{c_1}}$$

Now, plug in  $h_1$  and solve the remaining layers one at a time ...

$$t_{\text{int}_2} = \frac{2h_1 \cos i_{c_1}}{v_1} + \frac{2h_2 \cos i_{c_2}}{v_2}$$

**BEWARE!!!**  $h_1, h_2$ , are layer thicknesses, not depth to interfaces. So, depth to bottom of layer 3 / top of layer 4 =  $h_1 + h_2 + h_3$

# Multiple-layers

General formulation

$$T_n = \frac{\Delta}{V_n} + \frac{2H_0 \cos(ic)}{V_0} + 2 \sum_{i=1}^{n-1} \frac{H_i \cos(ic_{i+1})}{V_1}$$

Home works for those interested in mathematical manipulations

Next chapters for the other class