2 COUPLE EXERCE PAR LA PRESSION ASSOCIEE AUX MOUVEMENTS DANS LE NOYAU SUR LE MANTEAU

Si la surface du noyau n'est pas exactement sphérique, les forces de pression peuvent exercer un couple sur le manteau. Imaginons une bosse de la surface du noyau.

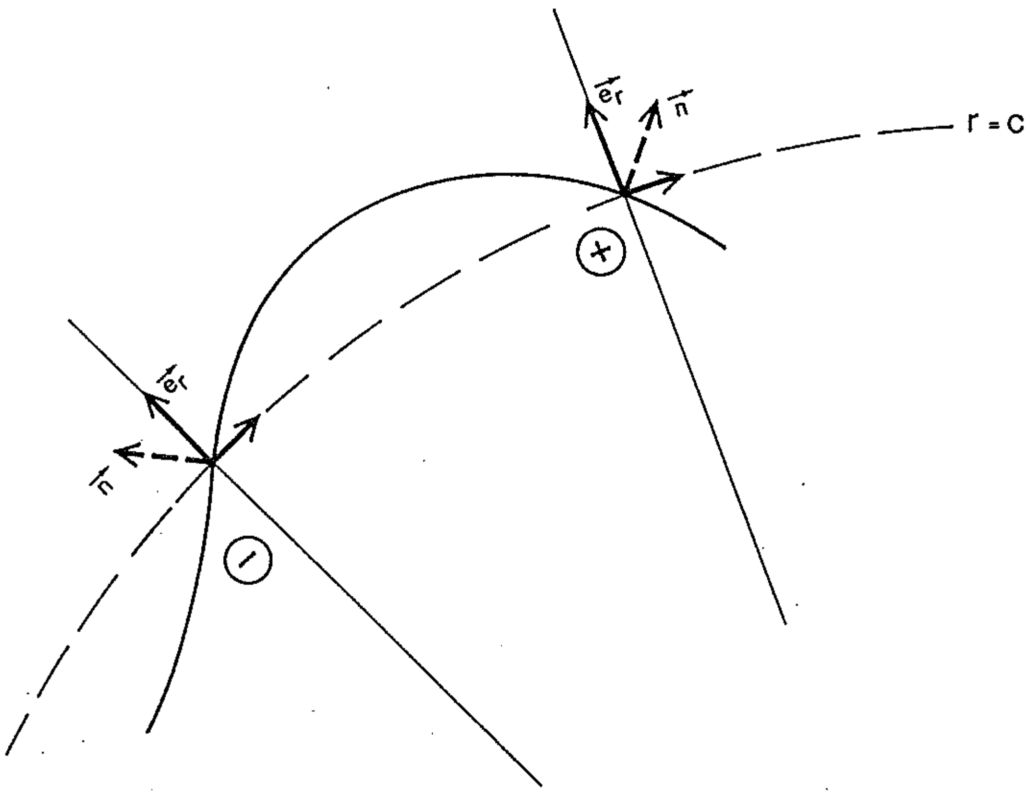

Figure 1: bosse de la surface du noyau

La pression p, à la surface du noyau, exerce une force $p\vec{n}$ ($\vec{n}$ normale sortante) sur le manteau. Le noyau n'étant pas exactement sphérique,

$$\vec{n} \neq \vec{e}_r$$

La composante de $p\vec{n}$ suivant $\vec{e}_\phi$(représentée en gras sur la figure 1) exerce un couple axial sur le manteau. Seules les variations latérales de la pression doivent être prises en compte; elles sont représentées ici par les symboles ⊕ et ⊖ (noter l'effet du changement de signe sur $((p\vec{n}.\vec{e}_\phi)\vec{e}_\phi)$. Suivant la position relative du champ de pression p et de la topographie du noyau h, les forces $p\vec{n}$ exercées par la pression en différents endroits de la surface du noyau se renforcent ou se détruisent mutuellement. Hide (1969) a étudié l'ordre de grandeur de ce couple en supposant que la force de pression $-\vec{\nabla}p$ a une amplitude comparable à l'accélération de Coriolis. La même idée (équilibre entre les composantes tangentielles de l'accélération de Coriolis et du gradient de pression) sert de point de départ au calcul de mouvements tangentiellement géostrophiques à la surface du noyau (I.2.3.3). Ces derniers peuvent s'exprimer comme fonction de la seule pression. Mais réciproquement, les formules (27) et (28) de (I.2) permettent de calculer la pression, une fois un mouvement tangentiellement géostrophique à la surface du noyau déterminé par l'inversion de la variation séculaire du champ magnétique. L'hypothèse de mouvements tangentiellement géostrophiques est indispensable à tout calcul de pression et donc de couple exercé par les forces de pression.

2.1 Couple de pression associé à un mouvement tangentiellement géostrophique

Nous étudions ce couple dans un article "The topographic torque associated with a tangentially geostrophic motion at the core surface and inferences on the flow inside the core" reproduit dans ce mémoire (II.3) et auquel je me réfère comme à l'article 1. Le calcul du couple de pression

$$\Gamma_p = -\vec{k}.\int\int\int \vec{r}\wedge\vec{\nabla}p = \vec{k}.\int\int p\vec{n}\wedge\vec{r} = \int\int p\frac{\partial h}{\partial\phi}dS \qquad (1)$$

ne présente aucune difficulté, une fois la topographie h et le champ de pression p connus. L'équilibre tangentiellement géostrophique à la surface du noyau se prolonge à l'intérieur par une équation du type (équation (11) de l'article 1):

$$2\rho(\vec{\Omega}\wedge\vec{u}) = -\vec{\nabla}p + G\vec{e}_r + \vec{F} \qquad (2)$$

$$\vec{u}.\vec{e}_r = 0$$

$$\vec{\nabla}.(\rho\vec{u}) = 0$$

où $G\vec{e}_r$ principal terme de gravité; $\vec{F} = \vec{0}$ à la surface du noyau; rien n'interdit de supposer que les forces $\vec{F}$ n'exercent aucun couple sur les cylindres $C(s)$ centrés sur l'axe de rotation de la Terre ou simplement qu'il existe un rayon s_0 (s rayon cylindrique) tel que:

$$\vec{k}.\int\int_{s=s_0}\vec{r}\wedge\vec{F}\,dS = 0$$

Mais nous avons remarqué en (I.2.5) que l'accélération de Coriolis ne donne lieu à aucune variation du moment cinétique des cylindres $C(s)$. Dans le second membre de (2), seule la force de pression $-\vec{\nabla}p$ exerce un couple sur le manteau et sur les différents cylindres $C(s)$; aucun terme dans le premier membre ne vient équilibrer ce couple: il y a là une contradiction.

Notre article (paragraphe 3.2) montre qu'en fait un couple de Coriolis fictif provenant de ($(\vec{u}.\vec{n}) \neq 0$) (puisque nous avons imposé $(\vec{u}.\vec{e}_r) = 0$) vient compenser le couple de pression. Dans une seconde étape (paragraphe 3.3), nous calculons le mouvement satisfaisant à la condition $\vec{u}.\vec{n} = 0$: comme les couples de volume étudiés en I.2.5, le couple de pression se traduit par une accélération $\partial\vec{v}/\partial t$ de la rotation en bloc des cylindres $C(s)$. Nous avons ainsi vérifié que nous pouvions utiliser la pression calculée en géométrie sphérique pour calculer le couple de pression s'exerçant sur une frontière non sphérique.

2.2 Couple de gravité

Dans notre article (1), nous introduisons un second couple lié également à la topographie du noyau: le couple de gravité. Si la frontière noyau-manteau n'est pas axisymétrique, les surfaces équipotentielles du champ de gravité et les surfaces d'égale densité ne le sont pas non plus: les écarts à l'axisymétrie sont décrits par l'équation d'équilibre hydrostatique (paragraphe (2.3) de l'article 1); nous pouvons traiter les écarts à l'axisymétrie comme des écarts à la sphéricité. Les termes hydrostatiques $(U_s(r)+U'_s(r,\theta,\phi),\rho_s(r)+\rho'_s(r,\theta,\phi))$ sont très grands devant l'hétérogénéité de densité $\rho_1(r,\theta,\phi)$ (et le potentiel $U_1(r,\theta,\phi)$) liés aux mouvements dans le noyau. La force de gravité intervenant dans l'équation du mouvement s'écrit

$$\rho_1\vec{\nabla}(U_s+U'_s)+(\rho_s+\rho'_s)\vec{\nabla}U_1=$$
$$(\rho_1\vec{\nabla}U_s-U_1\vec{\nabla}\rho_s)+\rho_1\vec{\nabla}U'_s-U_1\vec{\nabla}\rho'_s+\vec{\nabla}(U_1\rho_s+U_1\rho'_s)$$

Nous remarquons au passage que la pression calculée avec l'approximation tangentiellement géostrophique, n'est pas la pression fluide p_f mais $p_f-U_1\rho_s$. Nous appelons couple de gravité de volume, le couple (équation (14) de l'article (1))

$$\Gamma_{gb}=\vec{k}.\int\int\int\vec{r}\wedge(\rho_1\vec{\nabla}U'_s-U_1\vec{\nabla}\rho'_s)$$

L'amplitude de ce couple peut se comparer à celle du couple de pression

$$\frac{\Gamma_{gb}}{\Gamma_p}\approx\frac{|h_U|}{|h|}$$

où h_U, valeur typique de l'écart d'une équipotentielle du champ de gravité à une surface axisymétrique.

2.3 Calculs numériques du couple de pression

Dans un second article "Core-mantle boundary shape: constraints inferred from the pressure torque acting between the core and the mantle", nous nous intéressons aux calculs numériques du couple de pression. Le premier article nous a appris que les forces de pression accélèrent chaque anneau de façon indépendante: au lieu de considérer tout de suite le couple total sur le noyau, nous pouvons étudier les couples exercés sur chaque cylindre et les comparer à l'accélération des mouvements toroïdaux zonaux (c'est à dire, dans le cadre de notre théorie, à l'accélération des rotations rigides des cylindres $C(s)$) calculée à partir des modèles de variation séculaire du champ magnétique.

Nous avons utilisé pour cette application numérique la topographie de Morelli et Dziewonski (1987). Que l'on considère le couple total sur le manteau (et donc sur le noyau) ou le couple sur chaque anneau cylindrique, les valeurs obtenues sont cent fois plus importantes que les observations (couple nécessaire pour produire les irrégularités de la longueur du jour, variation des mouvements toroïdaux zonaux). Nous nous sommes alors attachés à comprendre l'origine de cette différence d'ordre de grandeur. La figure (1) montre clairement que certaines positions relatives du champ de pression à la surface du noyau par rapport à la frontière noyau manteau correspondent à des couples de pression nuls. Certaines de ces configurations sont stables: après un petit écart du champ de pression à une situation d'équilibre, les forces de pression exercent un couple de rappel. Nous avons montré que les oscillations des anneaux cylindriques $C(s)$ dont seraient ainsi responsables les forces de pression ont des constantes de temps telles qu'elles pourraient participer à la variation séculaire du champ magnétique observée.

Le champ de pression à la surface du noyau est mieux déterminé que la topographie de la frontière noyau-manteau. Connaissant ce champ de pression, il est possible de calculer une topographie de la frontière noyau-manteau proche d'une topographie connue et telle que le couple de pression s'exerçant sur chaque cylindre $C(s)$ est nul. Nous avons calculé une telle topographie à partir du modèle de Morelli et Dziewonski (1987). Enfin, nous proposons une technique de calcul pour évaluer une "distance" entre une topographie donnée et l'ensemble des topographies satisfaisantes (c'est à dire associées à un couple nul sur chaque cylindre).

Dans ce modèle, les forces de pression associées à la partie principale du champ de vitesse, stationnaire, n'exercent aucun couple sur le manteau. Ainsi il permet de rendre compte de la différence entre les temps caractéristiques du couple à l'origine des irrégularités décennales de la longueur du jour d'une part et de la variation séculaire du champ magnétique (c'est à dire grosso-modo des mouvements dans le noyau, mais voir I.2.2) d'autre part.

2.4 Excitation de l'oscillation de Chandler

Hinderer et al.(1987) proposent deux mécanismes d'excitation de l'oscillation libre de Chandler (voir la description de ce mouvement propre de la Terre en I.1) basés l'un comme l'autre sur l'action de la pression associée aux mouvements dans le noyau sur le manteau. En effet cette excitation n'a toujours pas trouvé d'explication définitive. Plusieurs mécanismes ont été envisagés. L'effet des tremblements de terre est trop faible (Chao et Gross, 1987; Souriau et Cazenave, 1985). Les mouvements dans l'atmosphère sont responsables d'une oscillation forcée annuelle (et semestrielle) de l'axe de rotation de la Terre. La période de l'oscillation de Chandler (435j.) n'étant pas très différente d'un an, on doit s'attendre à un rôle important de l'atmosphère (et des océans) dans le processus d'excitation de cette oscillation. Pourtant, Wahr (1983) ne peut expliquer que 20 à 25% de l'excitation observée par des mouvements dans les parties fluides externes de la Terre. Il fallait donc envisager le rôle des mouvements dans le noyau, susceptibles eux aussi de varier sur des temps caractéristiques proches de la période de résonance; la figure (11) de Gavoret et al. (1986) montre un exemple de variations rapides (quelques années) du champ magnétique d'origine interne; il est raisonnable de penser que les importantes variations du champ magnétique d'origine externe (et les filtrages destinés à en débarrasser les données) cachent des variations d'origine interne qui seraient associées à des mouvements à la surface du noyau variant en quelques mois.

La pression à la surface du noyau peut d'une part déformer le manteau -et par suite modifier son produit d'inertie- et d'autre part exercer un couple équatorial sur le manteau. Hinderer et al. (1987) considèrent l'effet des termes tesseraux (p_2^{1c}, p_2^{1s}) dans un développement en harmoniques de la pression. Seuls ces termes peuvent modifier le moment d'inertie de telle façon que l'oscillation de Chandler soit excitée; de plus, cette pression peut exercer un couple en agissant sur la frontière elliptique du noyau. L'expression (1.8) d'Hinderer et al. (1990) illustre l'effet d'une variation brutale de la pression à la surface du noyau; la pression est modélisée comme une fonction de Heaviside

$$p = P_0 H(t)$$

Il y a alors excitation de l'oscillation chandlerienne

Si à $t = 0$ $\qquad m(0) = 0$

$t > 0$ $\qquad m(t) = \dfrac{A(1 - e^{i\sigma_{cw}t})K P_0 H(t)}{A^m \rho^c c^2 \Omega \sigma_{cw}}$

où le vecteur rotation de la Terre s'écrit (voir I.1)

$$\vec{\Omega}(t) = \Omega \begin{pmatrix} m_1(t) \\ m_2(t) \\ 1 + m_3(t) \end{pmatrix} \qquad m(t) = m_1(t) + m_2(t)$$

A, et A_m moments d'inertie équatoriaux de la Terre et du manteau, σ_{cw} fréquence de l'oscillation chandlerienne, et ρ_c densité moyenne du noyau; le coefficient K s'écrit comme la somme d'un terme de déformation et d'un terme de couple. Ces deux mécanismes ont tendance à s'opposer. Hinderer et al. (1990) proposent

$$K = 2\,10^{-4}$$

alors $P_0 = 200 Pa$ correspond à une variation m de l'amplitude de l'oscillation

$$m \approx 0.3\,10^{-7} rd$$

Je discute plus loin la signification de ces valeurs numériques

Hinderer et al. (1990) examinent les termes de pression (différents de p_2^1) qui peuvent eux aussi exercer un couple équatorial en agissant sur la topographie du noyau; l' ellipticité du noyau et la déformation du manteau ne jouent plus aucun rôle ici. Cette étude est particulièrement importante si, comme le proposent Gire et Le Mouël (1990), le mouvement est symétrique par rapport au centre de la Terre car alors $p_2^1 = 0$. L'expression (3.5) d'Hinderer et al. (1990) donne le couple équatorial $C(t)$ en fonction de la pression $p(t)$ et de la topographie h. Hinderer et al.(1990) montrent que si l'amplitude de la topographie est aussi élevée que le proposent Morelli et Dziewonski (1987), le saut de pression (différente de $p_2^1 = 0$) nécessaire pour produire un déplacement donné du pôle de rotation instantané est comparable au saut de $p_2^1 = 0$ qui aurait le même effet.

Evaluons maintenant l'efficacité de ce mécanisme. La pression à la surface du noyau (voir fig.4 dans I.2), associée aux mouvements tangentiellement géostrophiques ne dépasse pas 10^3 Pa. Le mouvement semble stable sur plusieurs dizaines d'années et il est difficile d'envisager un saut brutal de pression de plus de 100 pascals; il correspondrait à un déplacement du pôle instantané de rotation d'environ 2 10^{-8} rad. En 20 ans, de 1940 à 1960, l'amplitude de l'oscillation de Chandler a augmenté de 10^{-6} rad. (Guinot, 1982) (voir I.1). Si l'on suppose N

percussions distribuées au hasard, l'amplitude de l'oscillation croît typiquement comme $\sqrt{N}$ ("marche au hasard"). L'excitation observée entre 1940 et 1960 correspond ainsi à plus de 2000 percussions. Ce chiffre doit nous paraître d'autant plus irréaliste que les fluctuations très rapides (quelques semaines à quelques mois) et temporaires (modélisées comme une fonction créneau) de la pression sont d'autant moins efficaces qu'elles sont plus courtes. Soit

$$p = (H(t) - H(t - t_0))P_0$$

alors $\quad 0 < t < t_0 \qquad m(t) = m\left(1 - e^{i\sigma_{cw}t}\right)$

$\qquad t_0 < t \qquad m(t) = m\left(e^{i\sigma_{cw}(t-t_0)} - e^{i\sigma_{cw}t}\right)$

où m déplacement du pôle instantané produit par un saut de pression P_0. Si $t_0\sigma_{cw} \ll 1$

$$m(t) \approx -im\sigma_{cw}t_0 e^{i\sigma_{cw}t}$$

Il y a une relation linéaire entre la durée du créneau et l'amplitude de l'accroissement de l'oscillation.

Les fluctuations de la pression associées aux mouvements tangentiellement géostrophiques à la surface du noyau, tels que l'on peut les déduire de l'étude de la variation séculaire du champ magnétique, ne semblent pas pouvoir être tenues responsables de l'excitation de l'oscillation de Chandler. Cependant l'étude de la variation séculaire du champ magnétique n'est pas très appropriée à ce travail sur l'oscillation de Chandler car elle ne nous donne accès qu'aux mouvements de temps caractéristiques longs devant la période de Chandler.

3. ETUDE DU COUPLE TOPOGRAPHIQUE ASSOCIE A DES MOUVEMENTS TANGENTIELLEMENT GEOSTROPHIQUES A LA SURFACE DU NOYAU; CONSEQUENCES SUR L'ORGANISATION DU MOUVEMENT A L'INTERIEUR DU NOYAU

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THE TOPOGRAPHIC TORQUE ASSOCIATED WITH A TANGENTIALLY GEOSTROPHIC MOTION AT THE CORE SURFACE AND INFERENCES ON THE FLOW INSIDE THE CORE

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In the last few years seismologists have proposed core-mantle topographies. At the same time much effort has been devoted by geomagneticians to calculate the fluid flow (and the related pressure field) at the top of the core, based on the observation of the secular variation of the geomagnetic field. A "topographic torque", which results from the action of the pressure field at the core surface, has long been invoked to allow for exchanges of angular momentum between the core and the mantle. In this paper, we show that this torque can be computed if forces at the top of the core are in geostrophic balance. The deep nature of this topographic torque can be understood only if one goes beyond the case of a pseudo-static equilibrium and considers explicitly the acceleration term in the equation of motion. We show that the pressure field acts in such a way as to accelerate a zonal flow consisting of cylindrical annuli. These annuli rotate like rigid bodies, with an angular velocity which depends on the distance to the rotation axis. Furthermore, we show that a gravity torque may also act on these same cylinders.

KEY WORDS: Earth's rotation, Earth's core, core-mantle boundary, topographic effects, differential rotation.

INTRODUCTION

The decade variations of the length of the day (l.o.d.) have been since long attributed to exchanges of angular momentum between the mantle and the fluid core. Some knowledge of the flow inside this fluid core is required to evaluate the time changes of its angular momentum. The secular variation of the geomagnetic field is generally thought to be due to the advection of the lines of force of the main field by the fluid motion at the top of the core. Secular variation (S.V.) data allow one to compute this flow provided that some additional assumption is made; the most likely one is that the flow is tangentially geostrophic at the core surface. Dynamical considerations lead to suppose that the part of the flow which carries the angular momentum of the core (with respect to the rotation axis) is organized in cylindrical shells, which rotate like rigid bodies about the axis of rotation. It is then possible to compute the angular momentum of the core, knowing the flow only at the core surface (Jault *et al.*, 1988). We showed that the time changes of the angular momentum of the core do balance the time changes of the angular momentum of the mantle which are inferred from l.o.d. data. More specifically, we showed that this balance holds over the last twenty years, for which reasonably good S.V. models exist. In the present paper we analyse the mechanism which allows the transfer of angular momentum from the core to the mantle. The pressure field at the core surface may be estimated if it is assumed to be associated with a tangentially geostrophic motion. It exerts a torque on the core-mantle boundary (CMB) as soon as this surface presents departures from axisymmetry. In that case the equipotential surfaces of the gravity field also present departures from axisymmetry, and the buoyancy force itself exerts a torque on the core. We show, extending the work of Anufriyev and Braginsky (1977), who dealt with the effect of the pressure torque associated with zonal flows, that the effect of both the pressure torque and the torque exerted by the buoyancy forces, except for the particular case studied by Anufriyev and Braginski, is indeed to accelerate a zonal flow consisting of rigidly rotating cylindrical annuli. In order to derive this result, it is necessary to resolve the geostrophic degeneracy, *i.e.* to reintroduce the acceleration term in the momentum equation. In other words, the true torque budget, which is hidden when one uses the tangentially geostrophic approximation, must be made explicit. The torques which are exerted by the buoyancy and pressure forces on the mantle can account for both the exchange of angular momentum between the core and the mantle and the time varying differential annular rotation inside the core on a decade timescale; the Lorentz force opposes the relative motion of the mantle with respect to the core on one hand and of the different cylindrical shells inside the core on the other hand (Braginsky, 1970).

1. TORQUE BUDGET IN THE EARTH'S CORE

Let us first review the different accelerations and forces which can contribute to the axial torque budget. We neglect the influence of the inner-core and suppose that the core-mantle boundary is rigid.

The torque budget (with respect to the axis of rotation of the mantle, supposed to be fixed in an inertial space) may be written as

$$\frac{d\sigma_c}{dt}=\Gamma, \tag{1}$$

with

$$\frac{d\sigma_c}{dt}=\mathbf{k}\cdot\iiint \mathbf{r}\times\left[\frac{\partial}{\partial t}(\rho\mathbf{u})+\nabla\cdot(\rho\mathbf{u}\mathbf{u})+2\rho\boldsymbol{\Omega}\times\mathbf{u}+\rho\frac{d\boldsymbol{\Omega}}{dt}\times\mathbf{r}+\rho\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r})\right]dv,$$

and

$$\Gamma=\mathbf{k}\cdot\iiint \mathbf{r}\times[\mathbf{F}_g+\mathbf{F}_e+\mathbf{F}_v-\nabla p_f]\,dv;$$

r denotes the position vector, **u** is the velocity in a coordinate system rotating with the mantle at the angular velocity **Ω** (Ω**k**), p_f the fluid pressure, ρ the density, σ_c the core angular momentum. $\mathbf{F}_g$, $\mathbf{F}_e$ and $\mathbf{F}_v$ are the gravity, electromagnetic and viscous forces.

The density heterogeneity ρ_1, responsible for the buoyancy force and changing with time with the time constant T of the secular variation, is very small when compared with the mean density ρ, which is constant in time ($\rho_1/\rho \simeq 10^{-9}$). (See Section 2.2.) Then

$$(\partial_1/\partial t \ll \nabla\cdot\rho\mathbf{u}), \qquad \rho_1+\rho\simeq\rho.$$

The equation for conservation of mass then reduces to

$$\nabla\cdot(\rho\mathbf{u})=0.$$

1.1. The moments of the different accelerations

We express the moment of the Coriolis acceleration, $\Gamma_{\text{cor.}}$, in a cylindrical coordinate system (s, ϕ, z):

$$\Gamma_{\text{cor.}}=\iiint 2\mathbf{r}\times\rho(\mathbf{\Omega}\times\mathbf{u})\cdot\mathbf{k}\,dv$$

$$=2\Omega\int\left(\iint_{\Sigma_s}\rho\mathbf{u}\cdot d\mathbf{S}\right)s\,ds, \tag{2}$$

where Σ_s is the lateral surface (inside the core) of the cylinder of radius s (Figure 1). But

$$\iint_{\Sigma_s}\rho\mathbf{u}\cdot d\mathbf{S}=-\iint_{\Sigma_s'}\rho\mathbf{u}\cdot\mathbf{n}\,dS,$$

where Σ_s' consists of the two caps which close the cylinder of radius s at the core surface. There $\mathbf{u}\cdot\mathbf{n}=0$, and $\Gamma_{\text{cor.}}=0$ (**n** being the outward unit vector normal to the core surface). This expresses the absence of any flow across the core-mantle boundary: the total moment of the convective acceleration is zero too. It is readily checked that the moment of the centrifugal acceleration is zero; from now on, we omit the centrifugal acceleration and the pressure term is modified to include it. Then, only two acceleration terms contribute to the final budget:

$$\iiint\mathbf{r}\times\rho\left(\frac{d\Omega}{dt}\times\mathbf{r}\right)\cdot\mathbf{k}\,dv=I_c\frac{d\Omega}{dt},$$

where I_c is the moment of inertia of the core, and

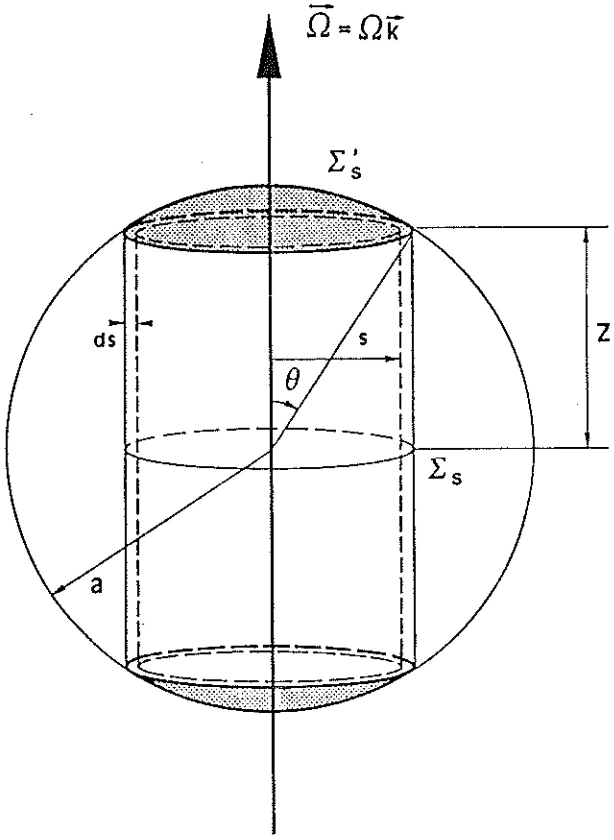

Figure 1 Sketch of the various surfaces and parameters used in the text.

$$\mathbf{k}\cdot\iiint \mathbf{r}\times\frac{\partial}{\partial t}(\rho\mathbf{u})\,dv=\frac{d\sigma_{\text{rel.}}}{dt}.$$

Therefore

$$\frac{d\sigma_c}{dt}=I_c\frac{d\Omega}{dt}+\frac{d\sigma_{\text{rel.}}}{dt}.$$

Exchanges of angular momentum between the core and the mantle are expressed through the identity (I_m being the moment of inertia of the mantle):

$$\frac{d\sigma_{\text{rel.}}}{dt}=-(I_m+I_c)\frac{d\Omega}{dt}.$$

If we suppose that $\partial(\rho\mathbf{u})/\partial t=0$, then $d\sigma_{\text{rel.}}/dt=0$ and $d\Omega/dt=0$. In order to evaluate the torques between the core and the mantle, it is necessary to take into account the inertial acceleration term $\partial(\rho \mathbf{u})/\partial t$. This conclusion, already made clear in the work of Anufriyev and Braginsky (1977), seems obvious here because we consider the core as a closed body. It can be forgotten in studies based on the computation of a mean stress between two infinite media (Moffatt, 1978). Moffatt calculated the effect of bumps at the core-mantle interface by perturbing a basic state characterized by tangential and uniform velocity and magnetic fields and obtained a non-zero mean tangential stress although using steady-state equations. However, his expression (54) for this mean stress can be shown to be proportional to the electromagnetic stress, while the mantle is supposed, in his model, to be insulating. So, the physical meaning of such a computation is doubtful; considering the more usual situation where the toroidal field is zero in the insulating mantle leads to a zero stress. We find it important to check that the intervening forces, leading to non-zero torques, are actually balanced by the inertial term.

1.2. The Torques

The present work is mostly concerned with the right hand side of Eq. (1), i.e. the torques acting on the core. Viscous coupling is generally supposed to be negligible on the decade timescale and a recent estimation of the core viscosity (Poirier, 1988) supports this point of view.

The electromagnetic coupling has been widely discussed (Rochester, 1960; Roberts, 1972; Stix and Roberts, 1984). It relies on the conductivity of the deep mantle of which we have a very poor knowledge; (a recent experiment by Li and Jeanloz (1987) would even suggests that the deep mantle may be almost insulating). Qualitative considerations lead us to believe that the electromagnetic coupling cannot account for the exchanges of angular momentum between the core and the mantle over the past twenty years.

The varying part, on the decade timescale, of the electromagnetic torque Γ_e tends to oppose any relative motion between the core and the mantle, especially the toroidal zonal motions (Stix and Roberts, 1984). More precisely, the changes in the torque $\Gamma_{e,m}$, acting on the mantle, are proportional to the changes in the latitude-dependent angular rotation of the core surface relative to the mantle and predominantly to the mean westward rotation t_1^0 (Stix and Roberts, 1984):

$$d\Gamma_{e,m} = -A\, dt_1^0, \qquad (A>0),$$

t_1^0 being counted positively westward.

Now, the westward drift t_1^0 of the core relative to the mantle is believed to have increased since 1970 (Gire *et al.*, 1984):

$$dt_1^0/dt > 0, \quad \text{so that} \quad d\Gamma_{e,m}/dt < 0.$$

Since

$$\Gamma_{e,m} = I_m (d\Omega/dt)_e,$$

(e standing for electromagnetic), one would expect:

$$(d^2\Omega/dt^2)_e < 0.$$

But Earth rotation data (see e.g. the figure 3 of Jault *et al.*, 1988) clearly suggest (o standing for observed):

$$(d^2\Omega/dt^2)_o > 0, \quad \text{since 1970.}$$

Therefore it seems difficult to associate the decade changes in the length of the day with the electromagnetic torque induced by the motions of the core surface responsible for the geomagnetic secular variation. It stimulates us to investigate other kinds of coupling.

There are two other forces capable of producing a coupling between the core and the mantle. The pressure force is responsible for the so-called "topographic torque" acting at the core surface Σ:

$$\Gamma_{p_f} = -\mathbf{k}\cdot\iiint_{\text{core}} \mathbf{r}\times\nabla p_f\, dv = \mathbf{k}\cdot\iint_{\Sigma} p_f \mathbf{n}\times\mathbf{r}\, dS. \tag{3}$$

The buoyancy force is responsible for the gravity torque:

$$\Gamma_g = \mathbf{k}\cdot\iiint_{\text{core}} \mathbf{r}\times\rho\nabla U\, dv. \tag{4}$$

If the core-mantle boundary (CMB) presents departures from axisymmetry (bumps about 100 m high) the pressure torque may account for the observed variations in the length of the day (Hide, 1977; Hide, 1969) and thus it deserves our attention.

2. THE DATA AVAILABLE TO COMPUTE THE PRESSURE AND GRAVITY TORQUES

Equation (3) can be used to compute the 'topographic torque' (e.g. Speith *et al.*, 1986); two sets of data are needed, namely topographies of the CMB and surface pressure fields. However, the data necessary to compute the gravity torque are out of reach.

2.1. CMB topographies

Seismic tomography using body waves (e.g. Morelli and Dziewonski, 1987; Creager and Jordan, 1986), free oscillations (Giardini *et al.*, 1987), geoid anomalies (Hager *et al.*, 1985), Earth's nutation data (Gwinn *et al.*, 1986), as well as the Earth's gravity coefficients C_2^1 and S_2^1 (Wahr, 1987a) have all been used to try and constrain the core-mantle boundary shape. Although these different techniques have not yet yielded a coherent picture of the topography at the core-mantle boundary, they give rise to the hope of a better accuracy in the coming years. Currently, they suggest to envision in our models large horizontal scale departures of the CMB from a hydrostatic ellipsoid with altitudes h in the range 100 m–10 km.

2.2. *Tangential geostrophy, the pressure at the CMB*

The geomagnetic secular variation data allow us to compute, assuming a tangentially geostrophic balance, the pressure p at the top of the core. As the motion responsible for the secular variation changes with a time constant T of the order of a few decades, the ratio of the inertial acceleration to the Coriolis acceleration is of the order of 10^{-4}; the hypothesis of a tangentially geostrophic motion at the CMB then requires principally neglecting the Lorentz force at the CMB. This latter assumption, which has already been thoroughly discussed (Le Mouël *et al.*, 1985; Backus and Le Mouël, 1986) requires that the gradient of the unknown toroidal field Q is not too large at the top of the core (the ratio of the Lorentz force to the Coriolis acceleration is of the order of $10^6\ \partial Q/\partial r$ (expressed in Tesla. m^{-1})).

We approximate the CMB with a sphere and suppose the buoyancy force to be radial; the tangentially geostrophic balance is written, at the core surface:

$$2\rho(\boldsymbol{\Omega}\times\mathbf{u}_0)_H=-\nabla_H p_0,$$

where p_0 and $\mathbf{u}_0$ are the pressure and the velocity; H is the horizontal component. Then, in the neighbourhood of the core surface:

$$2\rho(\boldsymbol{\Omega}\times\mathbf{u}_0)=-\nabla p_0+A_0\mathbf{e}_r,\quad \nabla\cdot(\rho\mathbf{u}_0)=0,\quad \mathbf{u}_0\cdot\mathbf{e}_r=0; \tag{6}$$

A_0 is the buoyancy term; $\mathbf{e}_r$ is the unit radial vector. By crossing $\mathbf{e}_r$ with vector equation (6), we obtain (θ is the colatitude) (Le Mouël *et al.*, 1987):

$$\rho\mathbf{u}_0=\frac{1}{\cos\theta}\left(\frac{1}{2\Omega}\mathbf{e}_r\times\nabla p_0+\rho_{u_{r,0}}\mathbf{k}\right). \tag{7}$$

Writing $\nabla\cdot(\rho\mathbf{u}_0)=0$, it follows that

$$\frac{p_0}{2\Omega}=r^2\cos\theta\int\frac{\partial}{\partial z}(\rho u_{r,0})\,d\phi-r\sin^2\theta\int\rho u_{r,0}\,d\phi+\pi(r,\theta),$$

π being an unknown function. At the core surface ($r=a, u_{r,0}=0$)

$$p_0 = 2\Omega \cos^2\theta\, a^2 \int \frac{\partial}{\partial r}(\rho u_{r,0})\, d\phi + 2\Omega\pi(a,\theta). \tag{8}$$

In the same way $\rho u_{\theta,0}$, $\rho u_{\phi,0}$ and A_0 can be expressed as functionals of $\rho u_{r,0}$ and its radial derivatives only. The scalars $\rho u_{r,0}$, p_0, $\rho u_{\phi,0}$, $\rho u_{\theta,0}$ and A_0 can be expanded in surface harmonics (zonal terms of $\rho u_{r,0}$ are forbidden); the coefficients of the expansions of p_0, $\rho u_{\phi,0}$, $\rho u_{\theta,0}$ and A_0 are derived from the coefficients of the expansion of $\rho u_{r,0}$. We obtain in this way a set of tangentially geostrophic flows $\mathbf{u}_0$. Zonal toroidal motions which are associated with a zonal pressure appear as an integration constant in the problem (see (8)).

Coming back to the data, it can be shown that the tangentially geostrophic flow at the CMB which generates the observed secular variation from the observed main field—neglecting diffusion (Roberts and Scott, 1965)—is determined in a reasonably unique way (Backus and Le Mouël, 1986). Such an inversion provides the first terms (i.e. low degree or large scale) of the expansion of $\partial(\rho u_{r,0})/\partial r$ and of the pressure p_0 at the core surface (Le Mouël *et al.*, 1985; Gire and Le Mouël, 1989). From the data it is not possible to compute the buoyancy term A_0, but the hypothesis $A_0 = 0$ in the neighbourhood of the core surface would imply that the motion $\mathbf{u}_0$ is toroidal zonal at the core surface and such an assumption is not consistent with s.v. data (Gire and Le Mouël, 1989). If there is a tangentially geostrophic balance at the core surface, the buoyancy force has to be comparable with the Coriolis acceleration. Let us then consider the form of this buoyancy term. The spherical quantities U_s and ρ_s are the main parts of the gravity potential U and of the density ρ; we have neglected the non-spherical parts to write the tangentially geostrophic balance (6). Then, in the neighbourhood of the core surface:

$$2\rho(\Omega \times \mathbf{u}_0) = -\nabla p_{f0} + \rho_s \nabla U_1 + \rho_1 \nabla U_s; \tag{9}$$

ρ_1 is the heterogeneity of density linked to the motion (of the order of $10^{-9}\rho_s$ since the buoyancy force is comparable to the Coriolis force), and U_1 the corresponding gravity potential p_{f0} is the fluid pressure. Equation (6) is obtained from Eq. (9) through the transformations:

$$p_0 = p_{f0} - \rho_s U_1, \quad A_0 = \rho_1\, \partial U_s/\partial r - U_1\, \partial \rho_s/\partial r.$$

The pressure p_0 computed from s.v. data is then different from the fluid pressure p_{f0}.

Using the pressure p_0 and a model of the topography of the core surface, we can compute a non-zero pressure torque (3). This pressure torque includes some gravity terms, since p_0 is different from the fluid pressure p_{f0}; we will come back (Section 3) on this intricate situation. Another important observation is the following: in order to obtain the pressure field at the top of the core, the inertial acceleration has been neglected since a tangentially geostrophic balance has been considered. But we have seen (Section 1) that the torque budget then reduces to zero; we will resolve this inconsistency in the following (Section 3.2).

2.3. *The Data Available to Compute the Gravity Torque*

Bumps at the core-mantle boundary (and possible density heterogeneities in the lower mantle) induce departures of the equipotential and equidensity surfaces inside the fluid core from axisymmetry (The reference state in an hydrostatic Earth). The non-axisymmetrical gravity field and density act on the density anomalies ρ_1 and the gravity potential U_1 linked to the motion. A non-zero gravity torque results (4). To compute this torque, we have to know the deviations U'_h and ρ'_h of the gravity potential and of the density from the axisymmetrical quantities U_h, ρ_h. The equations describing the hydrostatic equilibrium in the core are:

$$\nabla(\Pi_h+\Pi'_h)=(\rho_h+\rho'_h)\nabla(U_h+U'_h), \quad \nabla^2 U'_h=-4\pi G\rho'_h, \quad \rho'_h=f(\Pi'_h). \tag{10}$$

The dependence f of ρ'_h on Π'_h can be inferred from the dependence of ρ_h on Π_h. It comes out that bumps with an altitude of the order of 1 km are associated with density heterogeneities ρ'_h of the order of 10^{-5} ρ_h: these density heterogeneities are actually the main cause for possible lateral variations of seismic velocities in the fluid core (Wahr, 1987b) (Conversely, knowing these lateral variations and the corresponding density anomalies might allow us to infer the non-axisymmetrical potential U'_h through (10).) The hydrostatic equations allow us to describe the decrease, from the core surface to the interior of the core, of the deviation from an ellipsoid of the gravity field level surfaces; but the deviation at the core surface is far from being known. The buoyancy term A_0 cannot either be computed, at the core surface, knowing only $(\partial(\rho u_{r,0})/\partial r)_{r=a}$ obtained through inversion of the geomagnetic secular variation. It can be shown that $A_{0,r=a}$ depends also on $(\partial^2(\rho u_{r,0})/\partial^2 r)_{r=a}$. Moreover, in order to compute the gravity torque, we have to know ρ_1 and U_1 not only at the core surface but also in the whole volume of the core. Therefore, we are not able to compute the gravity torque, but we will observe that the order of magnitude of this torque may be comparable to the order of magnitude of the pressure torque (Section 4).

3. RESPONSE OF THE CORE TO THE GRAVITY AND PRESSURE TORQUES

As said in Section 2.2, the main part $\mathbf{u}_0$ of the large scale motion at the core surface can be inferred, in a spherical approximation, from secular variation data; it obeys the following equations in the core (9):

$$2\rho(\boldsymbol{\Omega}\times\mathbf{u}_0)=-\nabla p_0+\rho_1\frac{\partial U_s}{\partial r}\mathbf{e}_r-U_1\frac{\partial \rho_s}{\partial r}\mathbf{e}_r+\mathbf{F}, \quad p_{f0}=p_0+\rho_s U_1,$$
$$\mathbf{u}_0\cdot\mathbf{e}_r=0 \text{ at the core surface}, \quad \nabla\cdot(\rho\mathbf{u}_0)=0; \tag{11}$$

F represents the Lorentz force which we neglect in the neighbourhood of the core surface. We have seen in Section 1 that the moment of the Coriolis force on any cylinder of radius s is zero. In the spherical (or axisymmetrical, see Remark) approximation, the buoyancy and pressure forces exert no torques. Hence the Lorentz force F satisfies the Taylor constraint (Taylor, 1963):

$$\mathbf{k}\cdot\iint_{\Sigma_s} \mathbf{r}\times\mathbf{F}\,dS=0 \tag{12}$$

Σ_s is the lateral surface of the cylinder of radius s (see Figure 1).

We will study separately different departures from the equilibrium (11, 12) which induce non-zero torques on each cylinder of radius s associated with non-zero acceleration terms $\rho\,\partial\mathbf{u}/\partial t$ and $\rho\,d\Omega/dt\times\mathbf{r}$. A number of papers have been devoted to the restoring and dissipative parts played by the electromagnetic torque acting on each cylinder of radius s. In particular, it has been shown that Alfvén waves (Braginsky, 1970) propagate through the core as soon as the Taylor constraint (12) is not exactly obeyed for each cylinder. However, we have argued in Section 1 that the gravity and pressure torques could be predominant on the decade timescale; we will then first concentrate on the departures from the equilibrium (11) which give rise to these torques.

We have pointed out (Section 2.2) that the expression Γ_p for the pressure torque (using the pressure p_0 computed within the tangentially geostrophic hypothesis) includes gravity terms:

$$\Gamma_p=\mathbf{k}\cdot\iint_{r=a+h} p_0\mathbf{n}\times\mathbf{r}\,dS=\mathbf{k}\cdot\iint_{r=a} p_0\mathbf{n}\times\mathbf{r}\,dS, \tag{13}$$

in the first order.

Since $p_0=p_{f0}-\rho_s U_1$,

$$\Gamma_p=\mathbf{k}\cdot\iint_{r=a}(p_{f0}-\rho_s U_1)\mathbf{n}\times\mathbf{r}\,dS$$

$$=\Gamma_{pf}-\mathbf{k}\cdot\iint_{r=a}\rho_s U_1\mathbf{n}\times\mathbf{r}\,dS.$$

But the gravity torque Γ_g may be written as

$$\Gamma_g=\mathbf{k}\cdot\iiint\mathbf{r}\times[\rho_1\nabla(U_s+U_s')+(\rho_s+\rho_s')\nabla U_1]\,dv,$$

and then

$$\Gamma_{pf}+\Gamma_g=\Gamma_p+\Gamma_{gb},$$

where

$$\Gamma_{gb}=\mathbf{k}\cdot\iiint\mathbf{r}\times(\rho_1\nabla U_s'+\rho_s'\nabla U_1)\,dv; \tag{14}$$

Γ_p originates from the deviations of the outward normal $\mathbf{n}$ to the CMB from $\mathbf{e}_r$ and will be referred to as the pressure torque; Γ_{gb} originates from the deviations of the gravity potential and of the density distribution from spherical symmetry in the whole volume of the core. It can be called the gravity body torque, or simply the gravity torque. We will consider separately the action on the core and on the mantle of these two torques, then we will come back on the role of the Lorentz forces at the end of our discussion (Section 5).

Remark We assumed spherical symmetry in Section 2.2 to compute the large scale and zeroth-order motion $\mathbf{u}_0$ as well as the pressure p_0 at the core surface. On the other hand, we considered in Sections 2.1 and 2.3 departures from axisymmetry of CMB topography, equipotential surfaces, and equidensity surfaces. In a hydrostatic model of the Earth, these surfaces are actually ellipsoids, described by the Clairaut equations. In particular, in a hydrostatic Earth, the ellipsoidal core-mantle boundary is also an equipotential surface of the gravity field: the buoyancy force is orthogonal to the tangent plane to the core surface and the tangentially geostrophic balance is not modified.

Equations (13) and (14) for the pressure and gravity torques can be transformed into

$$\Gamma_p = a^2 \iint_{r=a} p_0 \frac{\partial h}{\partial \phi} \sin\theta \, d\theta \, d\phi, \tag{15}$$

$$\Gamma_{gb} = \iiint \left(\rho_1 \frac{\partial U_s'}{\partial \phi} - U_1 \frac{\partial \rho_s'}{\partial \phi} \right) dv, \tag{16}$$

These expressions make it clear that the axisymmetrical parts of the deviations h, U_s', and ρ_s' from the spherical quantities a, U_s and ρ_s induce neither pressure torque nor gravity torque. We are thus entitled to use indifferently departures from axisymmetry or from spherical symmetry to compute pressure and gravity torques.

3.1. Response of the Core to the Gravity Body Torque

The part of this torque acting on the interior of the bumps is negligible when compared to the whole torque. We can then assume that the core surface is spherical and that only the level surfaces of the gravity field and the equidensity surfaces are distorted. In other words, we add the perturbations U_s' and ρ_s' to the spherical potential U_s and density ρ_s. We are looking for a motion $\mathbf{u}_1$ which obeys the full set of equations:

$$\rho \frac{\partial \mathbf{u}_1}{\partial t} + 2\rho \boldsymbol{\Omega} \times \mathbf{u}_1 + \rho \frac{d\boldsymbol{\Omega}_1}{dt} \times \mathbf{r} = -\nabla p_1 + \rho_1 \frac{\partial U_s}{\partial r} \mathbf{e}_r + \rho_1 \nabla U_s' - U_1 \frac{\partial \rho_s}{\partial r} \mathbf{e}_r - U_1 \nabla \rho_s' + \mathbf{F},$$

$$p_{f1}=p_1+\rho_s U_1, \quad \mathbf{u}_1\cdot\mathbf{e}_r=0, \quad \nabla\cdot(\rho\mathbf{u}_1)=0, \tag{17}$$

$\mathbf{F}=0$, in the neighbourhood of the core surface

$$\mathbf{k}\cdot\iint_{\Sigma_s}\mathbf{r}\times\mathbf{F}\,dS=0.$$

The gravity torque is

$$\Gamma_{gb}=\mathbf{k}\cdot\iiint\mathbf{r}\times(\rho_1\nabla U_s'-U_1\nabla\rho_s')\,dv$$

$$=\int_0^a\left(\mathbf{k}\cdot\iint_{\Sigma_s}\mathbf{r}\times(\rho_1\nabla U_s'-U_1\nabla\rho_s')\,dS\right)ds, \tag{18}$$

$$\Gamma_{gb}=\int_0^a\Gamma_s\,ds;$$

$\Gamma_s\,ds$ is the torque exerted on the cylinder of radius s and width ds. Then

$$\mathbf{k}\cdot\iint_{\Sigma_s}\mathbf{r}\times\left[\rho\frac{\partial\mathbf{u}_1}{\partial t}+\rho\frac{\partial\mathbf{\Omega}_1}{dt}\times\mathbf{r}\right]dS=\mathbf{k}\cdot\iint_{\Sigma_s}\mathbf{r}\times(\rho_1\nabla U_s'-U_1\nabla\rho_s')\,dS=\Gamma_s. \tag{19}$$

And since, at this step, the gravity torque alone acts on the mantle:

$$-I_m\frac{d\Omega_1}{dt}=\Gamma_{gb}=\int_0^a\Gamma_s\,ds. \tag{20}$$

The Coriolis acceleration is much larger than the inertial one (see Section 2.2) and the only motions $\mathbf{u}_1$ for which the corresponding Coriolis force $2\rho(\mathbf{\Omega}\times\mathbf{u}_1)$ can be balanced by a gradient, the pressure force, are rotations $\mathbf{t}_1$ organized in quasi-rigid cylindrical annuli ($\partial(\rho\mathbf{t}_1)/\partial z=0$). The growth of these motions is not prevented by an unbalanced increase of the corresponding Coriolis force. Then, in (17), $\rho\partial\mathbf{u}_1/\partial t$ reduces to $\rho\partial\mathbf{t}_1/\partial t$, where $\mathbf{t}_1$ is a rotation organized in quasi-rigid cylindrical annuli.

Let

$$\mathbf{F}_1=\rho\frac{\partial\mathbf{t}_1}{\partial t}+\rho\frac{\partial\mathbf{\Omega}_1}{dt}\times\mathbf{r}=F_1\phi$$

and

$$\Gamma_s(s)=\mathbf{k}\cdot\iint_{\Sigma_s}(\mathbf{r}\times\mathbf{F}_1)\,dS$$

Then the system

$$2\rho\Omega\times\mathbf{u}_1=-\nabla p_1+\rho_1\frac{\partial U_s}{\partial r}\mathbf{e}_r+\rho_1\nabla U_s'-U_1\frac{\partial\rho_s}{\partial r}\mathbf{e}_r-U_1\nabla\rho_s'-\mathbf{F}_1+\mathbf{F},$$

$$\nabla\cdot(\rho\mathbf{u}_1)=0,\quad p_{f1}=p_1+\rho_s U_1,\quad \mathbf{u}_1\cdot\mathbf{e}_r=0, \tag{21}$$

is of the kind

$$2\rho(\Omega\times\mathbf{u})=-\nabla p+\mathbf{F}',\quad \nabla\cdot(\rho\mathbf{u})=0,\quad \mathbf{k}\cdot\iint_{\Sigma_s}\mathbf{r}\times\mathbf{F}'\,dS=0,\quad \mathbf{u}\cdot\mathbf{e}_r=0. \tag{22}$$

Taylor (1963) studied this kind of system (22) (with ρ uniform) and showed that it admits a 'quasi-stationary' solution (Smylie *et al.* (1984) studied the case of a non-uniform ρ). Again annular rotations $\mathbf{t}$ ($\partial(\rho\mathbf{t})/\partial z=0$) appear as a degree of freedom of the solution: if $\mathbf{u}$ is a solution if the system (22), $\mathbf{u}+\mathbf{t}$ is also a solution (p being replaced by $p+p(\mathbf{t})$). Let $\mathbf{v}_1$ be a particular solution of (21). Equation (21) governs the steady solution $\mathbf{v}_1$ whereas (19) together with (20) determine the time changes of the annular rotation $\mathbf{t}_1$, and $\mathbf{u}_1=\mathbf{v}_1+\mathbf{t}_1$ is a solution of the whole system (17).

Since $U_s'\ll U_s$ and $\rho_s'\ll\rho_s$,

$$p_1-p_0\ll p_0 \quad\text{and}\quad |\mathbf{v}_1-\mathbf{u}_0|\ll|\mathbf{u}_0|.$$

The first order perturbation, induced by the gravity torque, reduces to the time changing annular solution $\mathbf{t}_1$, $p(\mathbf{t}_1)$ and $\mathbf{u}_1=\mathbf{u}_0+\mathbf{t}_1$.

The gravity torque is an illustration of the more general case of a body torque. We will now check that the pressure torque is equivalent to a body torque.

3.2. *The Fictitious Coriolis Torque, Balancing the Pressure Torque*

Let us now consider a topography on the core-mantle boundary ($r=a+h$). All the equations which govern the fluid motion, except for the non-penetration condition $\rho\mathbf{u}\cdot\mathbf{n}=0$, are satisfied by the solution $\mathbf{u}_0$ calculated in the spherical geometry. In the bumps $u_r=(r-a)\;\partial u_{r,0}/\partial r|_{r=a}$. We can compute independently the motion induced by the pressure acting on the CMB topography and the zonal motion $\mathbf{t}_1$ induced by the gravity torque since the zonal motions will be shown to generate no pressure torques. We can compute the pressure torque from the formula (15) and, substituting (8) into (15), we obtain

$$\Gamma_p=-2\Omega a^4\iint_{r=a}\left.\frac{\partial(\rho u_{r,0})}{\partial r}\right|_{r=a}h\cos^2\theta\sin\theta\,d\theta\,d\phi.$$

We emphasized earlier that there is no net Coriolis torque, because there is no flow through the core boundary. But here we have artificially introduced such a flow and we will see that, as a consequence, a Coriolis torque appears, which balances the pressure torque. Let us recall an expression of the Coriolis torque (see (2)):

$$\Gamma_{cor.,1} = -2\Omega \int s\left(\iint_{\Sigma_s} \rho \mathbf{u}_0 \cdot \mathbf{n}\, dS\right) ds. \tag{23}$$

Only the zonal terms of the scalar function $\rho \mathbf{u}_0 \cdot \mathbf{n}$ contribute to the torque and, among them, only those which are symmetrical about the equator. The flux $\Phi(s)$ through each couple of caps Σ_s' is calculated using (7):

$$\rho \mathbf{u}_0 \cdot \mathbf{n} = \rho u_{r,0} + \frac{1}{2\Omega \cos\theta}(\mathbf{e}_r \times \nabla p_0)\cdot \mathbf{n}.$$

We obtain

$$\Phi(s) = \iint_{\Sigma_s} \rho \mathbf{u}_0 \cdot \mathbf{n}\, dS$$

$$= \frac{1}{2\Omega}\iint_I \left(\frac{1}{\cos\theta}\right)\left(\frac{\partial h}{\partial\theta}\frac{\partial p_0}{\partial\phi} - \frac{\partial p_0}{\partial\theta}\frac{\partial h}{\partial\phi}\right) d\theta\, d\phi + \iint_I \frac{\partial}{\partial r}(\rho u_{r,0})\, h a^2 \sin\theta\, d\theta\, d\phi, \tag{24}$$

where

$$I = \{\theta, 0 \leqq \theta \leqq \theta_0\} \cup \{\theta, \pi - \theta_0 \leqq \theta \leqq \pi\},$$

and

$$\theta_0 = \sin^{-1}(s/a).$$

The first term of the right-hand side of (24) may be written

$$\frac{1}{2\Omega}\int_0^{2\pi}\left\{\frac{1}{\cos\theta_0}\left[h(\theta_0,\phi)\frac{\partial p_0(\theta_0,\phi)}{\partial\phi} + h(\pi-\theta_0,\phi)\frac{\partial p_0(\pi-\theta_0,\phi)}{\partial\phi}\right]\right.$$

$$\left. - \left[\int_0^{\theta_0} + \int_{\pi-\theta_0}^{\pi}\right]\frac{h\sin\theta}{\cos^2\theta}\frac{\partial p_0(\theta,\phi)}{\partial\phi}\, d\theta\right\} d\phi.$$

Substituting (8) in the second term finally gives

$$\Phi(s) = \frac{1}{2\Omega}\int_0^{2\pi}\left\{\frac{1}{\cos\theta_0}\left[h(\theta_0,\phi)\frac{\partial p_0(\theta_0,\phi)}{\partial\phi} + h(\pi-\theta_0,\phi)\frac{\partial p_0(\pi-\theta_0,\phi)}{\partial\phi}\right] d\phi, \right. \tag{25}$$

$$\Gamma_{\text{cor.},1} = -a^2 \iint h \frac{\partial p_0}{\partial \phi} \sin\theta \, d\theta \, d\phi, \tag{26}$$

see (15).

The fictitious flow through the caps Σ_s' is fed by a flow through the lateral surfaces of the internal cylinders, which implies changes of angular momentum of the cylinder and of the core as a whole. This phenomenon is reminiscent of the Ekman succion phenomenon. But here the flow through the core boundary is artificial.

3.3. Response of the Core to the Pressure Torque

We have to "close" the problem by finding a solution $\mathbf{u}_2$ such that

$$\mathbf{u}_2 \cdot \mathbf{n} = -\mathbf{u}_0 \cdot \mathbf{n}, \quad \text{at the CMB},$$

$$\frac{\partial}{\partial t}(\rho \mathbf{u}_2) + 2\rho \boldsymbol{\Omega} \times \mathbf{u}_2 + \rho \frac{d\boldsymbol{\Omega}_2}{dt} \times \mathbf{r} = -\nabla p_2, \tag{27}$$

$$\nabla \cdot (\rho \mathbf{u}_2) = 0.$$

($\mathbf{u}_0$ being continued inside the bumps; see Section 3.2). We can solve this last problem supposing a spherical configuration (because $\mathbf{u}_2 \cdot \mathbf{n} \simeq \mathbf{u}_2 \cdot \mathbf{e}_r$).

We extend in this section the work of Anufriyev and Braginsky (1977), keeping most of their notations and using the same way of reasoning. These authors showed that, when the velocity $\mathbf{u}_0$ is zonal and toroidal (along the parallels), a solution of the simplified system

$$\mathbf{u}_2 \cdot \mathbf{e}_r = -\mathbf{u}_0 \cdot \mathbf{n}, \quad 2\rho(\Omega \times \mathbf{u}_2) = -\nabla p_2, \quad \nabla \cdot (\rho \mathbf{u}_2) = 0, \tag{28}$$

can be found.

In this particular case $\mathbf{u}_2$ has no contribution to the angular momentum budget; the exchanges of angular momentum between the core and the mantle appear only as a second order effect, when the Lorentz forces are taken into account. Indeed, when the primary velocity $\mathbf{u}_0$ is zonal. $\mathbf{u}_0 \cdot \mathbf{n}$ has no zonal part and there is no associated pressure torque (formulae 23 and 26). (Anufriyev and Braginsky also visualized the perturbation of the zonal flow induced by the bumps; they stressed that the phenomena encountered in rotating annuli such as Proudman–Taylor columns for example are modified in rotating spheres: a column of fluid can change its length by changing its distance from the rotation axis.)

Let us first review the different primary velocity fields $\mathbf{u}_0$ that, like the zonal toroidal ones considered by Anufriyev and Braginsky, give rise to perturbations $\mathbf{u}_2$ which verify the system (28): the pressure associated with such a velocity field $\mathbf{u}_0$

exerts no torque on the CMB topography since there is no change of angular momentum associated with its counterpart $\mathbf{u}_2$.

We use cylindrical coordinates (s, ϕ, z) and write

$$z_1 = (a^2 - s^2)^{1/2} \quad \text{and} \quad -\mathbf{u}_0 \cdot \mathbf{n} = g(s, \phi, \pm).$$

The sign $\pm$ indicates which hemisphere is being considered (+ for North and − for South). With these notations, the boundary condition is written

$$\pm z_1 u_{z,2} + s u_{s,2} = a g(s, \phi, \pm). \tag{29}$$

Following Anufriyev and Braginsky, we notice that the solutions of (27) (and in particular of (28) and (29)) can be parted into two classes: one symmetrical and the other antisymmetrical about the equator. If g is symmetrical, $u_{s,2}$, p_2 and $u_{\phi,2}$ are symmetrical, and $u_{z,2}$ is antisymmetrical. If g is antisymmetrical, $u_{s,2}$, P_2 and $u_{\phi,2}$ are antisymmetrical, and $u_{z,2}$ is symmetrical. Let us examine the different possible cases (the problem is linear):

a) *g is antisymmetrical*

A solution of the Eqs (28) and (29) is

$$u_{\phi,2} = u_{s,2} = p_2 = 0, \qquad z_1 u_{z,2}(s, \phi) = a g(s, \phi, +).$$

b) *g is symmetrical and non-zonal.*

We get

$$\partial(\rho u_{z,2})/\partial z = 0, \tag{30}$$

and then $u_{z,2} = 0$,

$$u_{s,2}(s, \phi) = (a/s) g(s, \phi), \qquad \text{at the core surface,}$$

and since $\partial(\rho u_{s,2})/\partial z = 0$

$$\rho u_{s,2}(s, \phi) = (a/s) \rho(s, z_1) g(s, \phi), \qquad \text{inside the core.}$$

Then, from (28), p_2 is given by

$$-2\rho\Omega u_{s,2} = (1/s)\, \partial p_2/\partial\phi. \tag{31}$$

As $\int_0^{2\pi} g\, d\phi = 0$ (no zonal term in g)

$$p_2(s, \phi, z) = -2\rho(s, z_1)\Omega a \int_0^{\phi} g\, d\phi, \qquad \text{and} \qquad u_{\phi,2} = \frac{1}{2\rho\Omega} \partial p_2/\partial s.$$

c) g is symmetrical and zonal ($g=g(s)$)

The system (28) and (29) has no solution since the two constraints (30) and (31) are no longer compatible when $\int_0^{2\pi} g\,d\phi \neq 0$. Since they both rely on the hypothesis $\rho\partial u_{\phi,2}/\partial t=0$, we must give up this condition, re-introduce the inertial term, and study the full system (27). The equations of motion and boundary conditions being now axisymmetrical, we assume

$$\partial(\rho \mathbf{u}_2)/\partial\phi=0, \qquad \partial p_2/\partial\phi=0.$$

The former constraints (30) and (31) are now replaced by the equations

$$s\frac{\partial\rho}{\partial s}\frac{d\Omega_2}{\partial t}+2\rho\frac{d\Omega_2}{\partial t}+\frac{\partial}{\partial t}\left[\frac{1}{s}\frac{\partial}{\partial s}(\rho s u_{\phi,2})\right]=2\Omega\frac{\partial(\rho u_{z,2})}{\partial z}, \tag{32}$$

(which is the z component of the vorticity equation)

$$\rho s\frac{d\Omega_2}{dt}+\rho\frac{\partial u_{\phi,2}}{\partial t}+2\rho\Omega u_{s,2}=0, \tag{33}$$

(which is the ϕ component of the Navier–Stokes equation).

The time constant T of the motion is very large when compared to one day (Section 2.2). Then, from (32), $u_{z,2}\ll u_{\phi,2}$ (the constant part of $u_{z,2}$ is zero because it is antisymmetrical) and from (33) $u_{s,2}\ll u_{\phi,2}$; we can neglect $\partial u_{s,2}/\partial t$ and $\partial u_{z,2}/\partial t$ when compared to $2(\mathbf{\Omega}\times\mathbf{u}_{\phi,2})$.

The ϕ-component of the vorticity equation reduces to

$$\partial(\rho u_{\phi,2})/\partial z=0, \tag{34}$$

which allows to reduce its s-component to

$$s\frac{d\Omega_2}{dt}\frac{d\rho}{dz}+2\Omega\frac{\partial}{\partial z}(\rho u_{s,2})=0,$$

$$\rho u_{s,2}=\langle\rho u_{s,2}\rangle-\frac{s}{2\Omega}\frac{d\Omega_2}{dt}(\rho-\langle\rho\rangle), \tag{35}$$

where, for example

$$\langle\rho u_{s,2}\rangle=(2z_1)^{-1}\int_{-z_1}^{z_1}\rho u_{s,2}\,dz.$$

The flux $-\Phi(s)$ of $(\rho \mathbf{u}_2)$ through the lower and upper caps closing each coaxial cylinders, makes up for the flux $\Phi(s)$ of $(\rho \mathbf{u}_0)$ through these caps:

$$\Phi(s) = -2\int_0^{2\pi}\int_0^{\theta}\rho(s,z_1)g(s)a^2\sin\theta\,d\theta\,d\phi;$$

$-\Phi$ is balanced by a flux through the lateral surface of the cylinder:

$$\Phi(s) = \iint_{\Sigma_s}\rho u_{s,2}s\,dz\,d\phi.$$

Identifying these two expressions for Φ, we obtain

$$s\langle\rho u_{s,2}\rangle = -\frac{1}{(a^2-s^2)^{1/2}}\int_0^s\frac{\rho(s,z_1)gas\,ds}{(a^2-s^2)^{1/2}}. \qquad (36)$$

From (25) (with $\theta_0=\pi/2$) and (8), we infer

$$\Phi(a) = \iint_{r=a}\rho\mathbf{u}_0\cdot\mathbf{n}\,dS = 0.$$

Hence the expression (36) for $\langle\rho u_{s,2}\rangle$ is regular at $s=a$. We can reduce the expression for $\langle\rho u_{s,2}\rangle$ to the simple form (from (25) and (8))

$$\langle\rho u_{s,2}\rangle(s) = \frac{1}{2\pi\sin\theta}\int_0^{2\pi}h\frac{\partial}{\partial r}(\rho u_{r,0})\,d\phi(\theta).$$

This expression can be transformed to take into account all the terms in the expansions of h and $\partial(\rho u_{r,0})/\partial r$, including those leading to a function g which is not symmetrical and zonal;

$$\langle\rho u_{s,2}(s)\rangle = \frac{1}{4\pi\sin\theta}\left[\int_0^{2\pi}h\frac{\partial(\rho u_{r,0})}{\partial r}\,d\phi(\theta) + \int_0^{2\pi}h\frac{\partial(\rho u_{r,0})}{\partial r}\,d\phi(\pi-\theta)\right]. \qquad (37)$$

Knowing $\langle\rho u_{s,2}\rangle$, the acceleration terms $\partial u_{\phi,2}/\partial t$ and $d\Omega_2/dt$ are obtained through (33) and (35)

$$\rho s\frac{d\Omega_2}{dt} + \rho\frac{\partial u_{\phi,2}}{\partial t} = -2\rho\Omega u_{s,2},$$

$$\langle\rho\rangle s\frac{d\Omega_2}{dt} + \rho\frac{\partial u_{\phi,2}}{\partial t} = -2\Omega\langle\rho u_{s,2}\rangle. \qquad (38)$$

A change in the core angular momentum σ_c is associated with these terms:

$$\left(\frac{d\sigma_c}{dt}\right)_2 = \iiint s\left(\rho\frac{\partial u_{\phi,2}}{\partial t} + \rho s\frac{d\Omega_2}{dt}\right)dv$$

$$= -2\Omega \iiint s\rho u_{s,2}\, dv = -\Gamma_{\mathrm{cor.},2},$$

$\Gamma_{\mathrm{cor.},2}$ being the Coriolis torque associated with the perturbation $\mathbf{u}_2$. Since $(\mathbf{u}_0+\mathbf{u}_2)\cdot\mathbf{n}=0$

$$\Gamma_{\mathrm{cor.},1}+\Gamma_{\mathrm{cor.},2}=0,$$

$\Gamma_{\mathrm{cor.},1}$ being the fictitious Coriolis torque computed in Section 3.3. Then

$$\left(\frac{d\sigma_c}{dt}\right)_2=\Gamma_{\mathrm{cor.},1}=\Gamma_p,$$

which means that the Coriolis torque introduced in Section 3.3 is to be related to a real change of angular momentum of the core (i.e. an exchange of angular momentum with the mantle). Together with (38), the angular momentum budget of the mantle determines acceleration terms:

$$-I_m\frac{d\Omega_2}{dt}=\left(\frac{d\sigma_c}{dt}\right)_2.$$

We can obtain $u_{z,2}$ from $u_{s,2}$ through the continuity equation. Finally

$$p_2=\int_0^s 2\rho\Omega u_{\phi,2}\, ds.$$

This zonal pressure is not small when compared to the zeroth order pressure but it does not exert any torque on the mantle.

The pressure torque has been shown to have the same effect on the core as a body torque. The resulting first order perturbation is made of annular motions $\mathbf{t}_1+u_{\phi,2}\phi(\mathbf{t}_1$ being induced by the gravity torque and $u_{\phi,2}$ by the pressure torque).

4. COMMENTS ABOUT POSSIBLE NUMERICAL STUDIES OF THE PRESSURE TORQUE

We have argued above that the gravity and the pressure torques could be the acting torques on the decade timescale (Section 1), generating accelerations of the coaxial cylindrical annuli (Section 3). We may hope to be soon provided with the data necessary to compute the pressure torque on the core (Section 2). Moreover, formulae (37) and (38) allow to compute the corresponding inertial acceleration of each cylindrical annulus. Setting $\theta=\sin^{-1}(s/a)$, $s\omega_2(\theta)=u_\phi$ and

$$F = -\frac{\Omega}{2\pi\rho \sin^2\theta}\left[\int_0^{2\pi} h\frac{\partial(\rho u_{r,0})}{\partial r}d\phi(\theta) + \int_0^{2\pi} h\frac{\partial(\rho u_{r,0})}{\partial r}d\phi(\pi-\theta)\right],$$

we obtain, at the core surface

$$\frac{\partial\omega_2(\theta)}{\partial t} + \frac{\langle\rho(s)\rangle}{\rho(s,z_1)}\frac{d\Omega_2}{dt} = F(\theta). \tag{39}$$

We have thus got a tool to examine whether the pressure torque is responsible for the exchanges of angular momentum between the core and he mantle by comparing the excitation function F with the observed time changes of the angular rotation $\omega(\theta)$ on short time spans. Numerical studies of the pressure torque acting on the core cannot be limited to computing the global change of the core angular momentum, but must include a detailed analysis of the effects of the pressure on each cylindrical annulus.

It appears that models of the flow computed in the geostrophic approximation (Gire, 1989) acting on the topographies calculated by Morelli and Dziewonski (1987) induce pressure torques ($\Gamma_p \simeq 2\,10^{19}$ N m) 50 times larger than the torque necessary to explain the changes in the length of the day. Although the knowledge of the topography is so poor that we cannot even assert the sign of the torque, its magnitude challenges us. Three explanations can be suggested:

- The seismologists may have overestimated the altitude of the bumps.
- The gravity torque may oppose the main part of the pressure torque. From the z-component of the balance (9) at the top of the core and the formulae (3) and (4) for the pressure and gravity torques, we can compare the typical magnitudes $|\Gamma_{pf}|$ and $|\Gamma_g|$ of the pressure and gravity torques:

$$|\Gamma_g| = (|h_U|/|h|)|\Gamma_{pf}|$$

  where $|h_U|$ is a typical value for the deviations of the level surfaces of the gravity field from an ellipsoid and $|h|$ a typical altitude of the topography on the CMB. Ignoring the gravity torque may be the cause of large miscalculations.
- The core may have got jammed in a stable position, where the topographic torque is minimum. This last hypothesis is attractive, since it would allow to constrain the core-mantle topography knowing the flow at the top of the core and would indicate a long term stability of the flow upwellings and downwellings with respect to the mantle. If this hypothesis is the good one, small changes in the flow at the top of the core would be associated with large variations of the pressure torque. It would explain the sudden changes in $d\Omega/dt$ which occur in a few years.

5. THE LORENTZ FORCES

Nothing new has been brought, in this paper, about the part played by the

Lorentz forces in the exchanges of angular momentum between the core and the mantle and between the cylindrical annuli inside the core. But we have to mention them since the differential rotation of the cylinders of radius s induced by the gravity and pressure torques generates magnetic fields opposing the differential rotation itself. This mechanism takes place even if the mantle is supposed to be perfectly insulating; when the weak conductivity of the mantle is taken into account, Lorentz forces appear which tend to oppose the motions of the core relative to the mantle.

Braginsky (1970) discussed the role of the restoring Lorentz torque inside the core. The Lorentz torque is a body torque, as well as the gravity torque, the former discussion of the gravity torque then applies and the differential rotation of the cylindrical annuli plays again the main part. Actually, the Lorentz torque can be considered as another perturbation of the zeroth-order equations (11–12) since it takes place when the Taylor condition (12) is not exactly obeyed. Using Braginsky's work the differential angular rotation $\omega(s)$ is shown to be governed by the system:

$$I(s)\,\partial\omega(s)/\partial t=\Gamma(s)+\Gamma_g(s)+\Gamma_{pf}(s), \tag{40}$$

$$\Gamma(s)=\frac{2\pi}{\mu}\int_{-z1}^{z1}\frac{\partial}{\partial s}\,(s^2\langle B_sB_\phi\rangle)\,dz, \tag{41}$$

(which is his equation 2–13), where

$$\langle B_sB_\phi\rangle=\frac{1}{2\pi}\int_0^{2\pi}B_sB_\phi\,d\phi,$$

$$\partial B_\phi/\partial t=sB_s\,\partial\omega/\partial s. \tag{42}$$

(See his equation 2.9′.) The mantle is supposed to be insulating, $I(s)\,ds$ is the moment of inertia of the cylindrical annulus of radius s and width ds. $\Gamma(s)\,ds$ is the total electromagnetic torque exerted on the cylinder. We have introduced in Eq. (40) the exciting gravity and pressure torques Γ_g and Γ_{pf} discussed in this paper. Equations (40) to (42) describe the evolution of a forced oscillator; The time constant τ of the oscillation τ is on the order of $10^{-1}\,l/b$, where l is a typical length scale of $\omega(\theta)$ and b a typical value of the cylindrical radial component of the magnetic field. Knowing the time changes $\omega(\theta,t)$ of the zonal motions would allow to study the respective parts played by te topographic torque (see Section 4.2) and the Lorentz torque, and then to infer the time constant τ and the characteristic value of the cylindrical radial field b.

We have preferred the gravity and pressure mechanisms to the electromagnetic core-mantle coupling as the leading mechanism for the decade changes in length of day. However the electromagnetic torque provides us with the dissipative mechanism necessary to make the model complete, explaining why the rotation of the core relative to the mantle does not grow indefinitely.

CONCLUSION

Topographic coupling appears to be a good candidate that allows exchanges of angular momentum between the core and the mantle and the related changes in the length of the day to take place on the decade timescale. Unfortunately it may be difficult to compute for at least two reasons. First, given a model of the topography of the CMB inferred from seismic tomography and a model of the pressure field at the top of the core derived from S.V. data, the topographic torque appears to be the sum of a rather large number of terms; the uncertainty on each of these terms is such that even the sign of the sum is difficult to assert. Second, this "topographic torque" is probably not acting alone; a gravity torque may also exist which cannot be calculated but which could be on the same order of magnitude as the topographic torque. These difficulties of course do not reduce the need for better models of topography and pressure field at the CMB.

Such improved models are expected in the coming years. The main objective of the present study has been to check that the tangentially geostrophic hypothesis, which is required to compute the surface flow from geomagnetic S.V. data and to derive theoretically the pressure field from the surface flow, is consistent with changes in the core angular momentum of the core. If a time varying non-zonal flow exists at the core surface and if the CMB presents departures from axisymmetry, then a secondary flow, organized in cylindrical annuli, arises under the action of the pressure field associated with the primary flow (and also the buoyancy forces). The resulting time varying annular motion deserves further attention. More detailed studies of the latitude dependence of the zonal toroidal flow at the top of the core should be undertaken as soon as more precise S.V. data are available. Such studies would allow to estimate the contribution of the pressure torque to the exchanges of angular momentum between the core and the mantle; they coud also provide upper bounds on the torques exerted by the Lorentz force on the cylindrical annuli, and as a consequence, on the radial component B_s of the magnetic field inside the core.

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4. CONTRAINTES SUR LA FRONTIERE NOYAU-MANTEAU DEDUITES DE L'ETUDE DU COUPLE DE PRESSION AGISSANT ENTRE NOYAU ET MANTEAU

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CORE-MANTLE BOUNDARY SHAPE: CONSTRAINTS INFERRED FROM THE PRESSURE TORQUE ACTING BETWEEN THE CORE AND THE MANTLE.

Dominique Jault and Jean-Louis Le Mouël

Summary

In the last few years models of the flow at the top of the Earth's core and of the related pressure field have been calculated from the secular variation of the geomagnetic field, and core-mantle topographies have been computed by seismologists. A pressure torque results from the action of the pressure field on the core topography which can theoretically be computed from models of both the pressure field and the core mantle interface. Small scale features of the flow and of the topography are shown to be capable of contributing strongly to the pressure torque; it is thus impossible to calculate the exact value of the torque from the knowledge of only the long wavelength components of the models. But the interaction between the large scale components generates by itself torques two orders of magnitude larger than the torques inferred from the irregularities of the length of the day. It is nevertheless possible to reconcile the topographic coupling mechanism with the length of the day observations, keeping the amplitude of the core topography proposed by seismologists, if an orthogonality relation between the geometry of the fluid upwellings and downwellings at the top of the core and the topography is satisfied. It is shown how to compute such a topography, for a given flow, and close to an original topography provided by seismic tomography. Some consequences of the so-inferred link between the fluid flow at the top of the core and the core-mantle boundary topography are discussed.

Key words: Corè-mantle boundary, Earth's rotation, Pressure torque, Core surface motions

1 Introduction

The description and analysis of the geophysical phenomena affecting the Earth rotation have been deeply improved during the past five years. Seasonal changes (Rosen and Salstein,1983) as well as irregular changes in the atmospheric angular momentum -related to El-Nino signals (Rosen et al.,1984)-, and the quasi-biennal oscillation of the stratosphere (Chao,1989) have been shown to be responsible for the changes in the length of the day (l.o.d.) up to periods of about 4 years. Tidal braking (Stephenson and Morrison,1984; Christodoulis et al,1988) and non-tidal acceleration related to the viscous rebound of the solid Earth after the last deglaciation (Yoder et al.,1983) have also been quantitatively discussed. As for the "decade" changes in the length of the day, they are attributed to core-mantle coupling (e.g. Lambeck,1980) but are not yet well understood. The amplitude and time constants of these variations are however known more accurately: the time constants vary from five to a few hundred years (see e.g. Stephenson and Morrison,1984), the maximal amplitude of the torque acting on the mantle for the past 100 years is about 10^{18} N.m while the amplitude of this torque since 1970 is about 2 10^{17} N.m.

Motions at the core surface may be inferred from the secular variation (s.v) of the Earth's magnetic field (Roberts and Scott,1965). The motions inside the core and in particular the toroidal zonal motions which carry the core angular momentum cannot be inferred from magnetic data. However the changes of this zonal toroidal flow, with time constants of the order of 10 years, may consist of changes in motions organized in cylindrical annuli rotating rigidly about the Earth's axis of rotation (Jault, Gire and Le Mouël ,1988). Hence, changes in the core angular momentum can be inferred from changes in the zonal toroidal part of surface core motions. It has been verified that the changes in the mantle angular momentum, as observed during the past twenty years, may be balanced by the changes in the core angular momentum computed this way (Jault et al.,1988 but see also Jackson,1989).

The forces responsible for the exchange of angular momentum between the core and the mantle need now be elucidated. In a former paper (Jault and Le Mouël,1989) we undertook a theoretical study of the pressure torque. If, at the core surface, the Lorentz force is small compared to the Coriolis acceleration, there is an equilibrium between the tangential components of the pressure force and of the Coriolis acceleration. This hypothesis which requires (Le Mouël, Gire and Madden ,1985)

$$\frac{\partial Q}{\partial r} \ll \sim 10^{-6} tesla.m^{-1}$$

is unfortunately difficult to test since the radial gradient of the toroidal field Q, at the top of the core, is unknown. The tangentially geostrophic hypothesis allows computation of the pressure at the top of the core from the surface motions (e.g. Hide,1989) and strongly reduces the ambiguity pointed out by Backus (1968) when deriving the surface motions from the radial component of the induction equation (Backus and Le Mouël,1986). In our previous paper, we showed how to reconcile the tangentially geostrophic hypothesis and the existence of a non-zero pressure torque (related to a non-zero inertial acceleration) acting on the mantle; the pressure torque (and a possible counteracting gravity torque) induces a time-varying flow, organised in cylindrical annuli. The latitude dependence of the acceleration of the zonal toroidal flow can be in principle derived from a model of the topography of the core surface h and a model of the motions $\vec{u}$ at the surface of the core. Each annulus (as depicted in figure 1) experiments the angular acceleration $\partial\omega(\theta,t)/\partial t$ such as:

$$\frac{\partial\omega(\theta,t)}{\partial t}+\frac{d\Omega}{dt}=F(\theta,t)$$

where ρ the core density is taken uniform for the sake of simplicity, Ω is the Earth's rate of rotation, and F is given by:

$$F(\theta)=-\frac{\Omega}{2\pi a\sin^2(\theta)}\left(\int_0^{2\pi}h\frac{\partial(u_r)}{\partial r}d\phi(\theta)+\int_0^{2\pi}h\frac{\partial(u_r)}{\partial r}d\phi(\pi-\theta)\right) \qquad (1)$$

θ is colatitude, ϕ longitude, a the core radius, and u_r is the radial component of $\vec{u}$; F can be referred to as the angular acceleration in an inertial frame. It is then possible to compare the theoretical distribution with latitude of the acceleration induced by the topographic torque with the distribution inferred from s.v. data.

2 Computation of the pressure torque

Morelli and Dziewonski have calculated a spherical harmonic expansion of the topography h of the core-mantle boundary (CMB) to degree 4 (Morelli and Dziewonski, 1987) from travel times of body waves; their model disagrees to some extent with a previous model constructed from the same kind of data (Creager and Jordan,1986). Free oscillations (Giardini et al,1987), Earth's nutation data (Gwinn et al,1986), geoid anomalies (Hager et al.,1985) and the Earth's gravity coefficients C_2^1 and S_2^1 (Wahr,1987) should also be used to constrain the core-mantle boundary shape. The model proposed by Morelli and Dziewonski will be used , in this paper, as an example of a possible topography of the CMB.

The models of the flow at the core surface (Le Mouël et al.,1985; Voorhies,1986; Whaler and Clarke,1988; Gire and Le Mouël,1989; Lloyd and Gubbins,1989 ; Jackson,1989) have been computed with different assumptions but are perhaps more constrained than the topography models.

To compute the acceleration induced by the pressure torque (1), we shall use, together with the model of the CMB topography of Morelli and Dziewonski, the motions calculated by Gire and Le Mouël (1989) in the tangentially geostrophic assumption which is necessary to derive the surface pressure from the surface flow. To ensure geostrophy, a tangentially geostrophic basis has been used. It is worth noting that there is no zonal part in the poloidal component of a tangentially geostrophic flow. The quantity $\partial(u_r)/\partial r$ is directly derived from the poloidal component of the flow. Gire and Le Mouël also advocated symmetry properties in the flow: the non-zonal velocities are identical at two antipodal points and the zonal velocities are symmetrical about the equator plane. These symmetries have been imposed on the computed flow. As a consequence, only the components of the CMB topography that are symmetrical ($h(\theta,\phi) = h(\pi-\theta,\phi+\pi)$) about the center of the Earth and non-zonal do contribute to the torque (see formula 1); they constitute the efficient part of the topography. Then, only eight terms (non-zonal terms of degree n=1 and n=3) in the expansion of the topography calculated by Morelli and Dziewonski can generate a torque acting between core and mantle and accelerations of the zonal annular flow, when interacting with the flow calculated by Gire and Le Mouël. Figures 2 and 3 illustrate the toroidal zonal accelerations F(Θ) calculated from formula 1. The pressure torque acting between core and mantle would be on the order of 2 10^{19} N.m, two orders of magnitude larger than the torque acting on the mantle that is responsible for the fluctuations in the length of the day for the past twenty years. The different models of flow (for the epochs 1970,1980,1985) lead to similar accelerations of the zonal annular flow when acting on the topography h. On the other hand, as illustrated by fig.4, large changes in the resulting acceleration occur when random perturbations δh are added to the CMB topography (these perturbations δh are inside the incertitudes affecting the expansion proposed by Morelli and Dziewonski). The largest incertitudes in the computation of the torque come then from the topography.

The acceleration F(Θ) depends also strongly on the truncation level of the flow (compare figures 2 and 3) and topography expansions. Let these expansions be:

$$h(\theta,\phi)=\sum_{n=1}^{\infty}\sum_{m=0}^{n} h_n^{mc,s} Y_n^{mc,s}(\theta,\phi) \qquad (2)$$

$$s(\theta,\phi)=\sum_{\substack{n=1\\ n\,odd}}^{\infty}\sum_{m=1}^{n} s_n^{mc,s} Y_n^{mc,s}(\theta,\phi)$$

$Y_n^{mc,s}$ is the fully normalized surface harmonic of degree n and order m. s is the scalar whose the poloidal part of the flow is the gradient; its expansion takes into account the symmetry of the non-zonal flow claimed by Gire and Le Mouël. The coefficients of the expansion of $\partial(u_r)/\partial r$ are then $n(n+1)s_n^{mc,s}$. In order to illustrate the non-convergence of F (Θ) with respect to the truncation level, let us suppose the coefficients of the models of topography and flow statistically independent and introduce the related quantity G (a global weighted torque):

$$G=-\frac{2\pi a}{\Omega}\int_0^{\frac{\pi}{2}} \sin^3(\theta) F(\theta)\, d\theta$$

then:

$$G = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{0}^{2\pi} h \frac{\partial(u_r)}{\partial r} dS$$

and:

$$G = 4\pi \sum_{\substack{n=1 \\ n odd}}^{\infty} \sum_{m=1}^{n} n(n+1)(h_n^{mc} s_n^{mc} + h_n^{ms} s_n^{ms})$$

We do not know the core surface topography spectrum. Let us represent it tentatively by the Earth's surface topography spectrum:

$$h \approx n^{-1}$$

As for the spectrum of s, we derive from figure 8 of Gire and Le Mouël (1989), taking into account the change in the normalization of the surface harmonics $Y_n^{mc,s}$:

$$s \approx n^{-1,5}$$

then

$$G \approx \sum_{n odd} n^{0,5}$$

This estimate is certainly a rough one, but is enough to show that it is impossible to derive the topographic torque from the long-wavelength components of the motion and of the topography alone.

3 The amplitude of the topographic torque

We have pointed out that the calculated topographic torque is two orders of magnitude larger that the torque responsible for the fluctuations in the length of the day for the past twenty years. This is not a distinctive feature of the torque generated by the surface pressure obtained from the flow of Gire and Le Mouël acting on the CMB topography computed by Morelli and Dziewonski. We have got similar values of the topographic torque with a set of random topographies with similar amplitude. Indeed, the simple examination of the interaction of one term in the expansion of the topography with one term in the expansion of the pressure makes clear that the potential torque is too large (Hinderer et al.,1989). Also, the unknown short-wavelength components of the flow and of the topography are likely to induce larger torques. Different explanations can be advanced to account for the discrepancy between the calculated and the observed values of the torque:

-The CMB topography may be overestimated. The amplitude of the topography which can be dynamically supported by the large-scale mantle convection depends upon the poorly known viscosity of the lower mantle (Ricard, Fleitout and Froidevaux,1984; Hager et al.,1985). Hager and his coauthors obtained a model of the dynamically supported topography about a factor four smaller than the model of the CMB topography proposed by Morelli and Dziewonski. This motivated Creager and Jordan (1986) to propose the alternative hypothesis that the differences in travel times of body waves are due to chemical boundary layers at the CMB. Recently, Doornbos and Hilton (1989) have included bottomside reflections (PKKP) of body waves in the inversion and they have found that most of the large scale structure producing differences in travel times is located above the CMB. As a consequence, their model of CMB topography (on the same order of magnitude as the dynamic model of Hager et al.) is smoother than the previous model of Morelli and Dziewonski and yet it would lead to a pressure torque acting on the mantle (5 10^{18} N.m) still far too large when compared with the fluctuations in the length of the day.

-It is tempting to introduce the idea of an ad hoc "effective" topography (Hide,1989), which could be different from the topography associated with delays in travel times of body waves. Hide has noticed, for example, that a stable layer of low-viscosity liquid may separate the metallic core from the solid mantle; the horizontal gradients of the pressure field acting on the solid mantle would be weakened and the effective topographic height would be lower than the actual height. However we are aware of no quantitative physical model describing such a mechanism.

-The gravity torque is likely to oppose an important part of the pressure torque (Jault and Le Mouël,1989).

-In order to compute the topographic torque, we have used different hypothesis, among which geostrophy, that may prove false.

Instead of considering either one of these explanations, we choose in the present paper to look for the ultimate consequences of our model. In particular, we keep the order of magnitude of the topography used so far. Then the topographic torque actually acting on the CMB is two orders of magnitude smaller than the potentially available torque. Now, the CMB topography is not likely to change on the range of periods 10-100 years since it is associated with slow process such as lower mantle convection (Hager et al.,1985) or chemical differenciation (Ruff and Anderson,1980). As for the motions $\vec{u}$ at the core surface, the different geostrophic flows derived by inverting the 1970, 1980, 1985 s.v. models present the same morphological features (Gire and Le Mouël,1989); an extrapolation of the changes calculated from 1970 to 1985 leads to time constants of about 100 years. On the other hand, the torque responsible for the so-called "decade" fluctuations in the length of the day changes quickly and a few years only can separate two peaks of the torque, proportional to $d\Omega/dt$. The discrepancy between the time constants of the flow and the time constants of the torque responsible for l.o.d. fluctuations suggests that only a small part, quickly varying, of the flow contributes to the torque acting on the mantle. Stix and Roberts (1984) proposed previously a similar view when interpreting the l.o.d. fluctuations as the result of an electromagnetic core-mantle coupling whose time variations originate in the changes of the fluid flow at the surface of the core. In order to obtain a torque varying rapidly enough, they had to remove from the calculated torque an average torque attributed to flux leakage from the core into the mantle. In the same way we propose to interpret the changes in the length of the day as resulting from small fluctuations of the topographic torque around an equilibrium position.

In other words, the toroidal zonal acceleration of each annulus induced by the pressure force acting on the non-axisymmetrical core-mantle interface is near zero. Or (see formula 1):

$$\forall\theta \qquad \int_0^{2\pi} h\frac{\partial(u_r)}{\partial r}d\phi(\theta) + \int_0^{2\pi} h\frac{\partial(u_r)}{\partial r}d\phi(\pi-\theta) = 0 \qquad (3)$$

4 A constraint on the CMB topography models.

The CMB models seem to be at the present day less well determined than the flow models. We will then try to compute a CMB model close to the model of Morelli and Dziewonski and such that the pressure associated with the flow model of Gire and Le Mouël acting on this boundary induces a topographic torque of the same order of magnitude (i.e. almost zero) as the torque inferred from the l.o.d. observations.

We noticed above that a computation of the topographic torque limited to a low truncation level has little sense. However, a strong topographic torque generated by the low degree terms in the spherical harmonic expansions of the CMB topography and of the pressure is unlikely to be balanced, for a long time, by an opposite torque induced by higher degree terms. Moreover, we are interested here in the existence part of the problem: Can we find a low degree topography with a large amplitude ($h \approx 5km$) such that a given pressure induces almost no torque when acting on it?

4.1 Dimension of the space T_0 of the topographies satisfying identity (3).

We suppose the core surface motions, with the symmety properties claimed by Gire and Le Mouël, known up to degree N (N=2p+1) and we look for a topography with the same truncation level. The pressure associated with the motion acts on the topography and generates an acceleration F of the annular toroidal zonal motions inside the core. Geostrophy and symmetry properties of the motion make the zonal and even degree terms of the CMB topography ineffective. Then only the non-zonal and odd degree terms, which form the "efficient" topography, generate torques on the mantle. Identity (3) can be written:

$\forall\theta$ $$F(\theta)=\sum_{0\leq l\leq N-1}a_l\cos^{2l}(\theta)=0$$

or

$\forall l$ $$a_l=0 \qquad (4)$$

The a_l are linear forms on the set of coefficients $h_n^{mc,s}$ (formula 2) of the expansion of the efficient topography. They span a $2(p+1)^2$ dimensional space T_{eff}. Then the dimension of the space of topographies which give a zero acceleration F when interacting with a motion $\vec{u}$ (The a_l depend on $\vec{u}$) is:

$$dim(T_0\cap T_{eff})\geq 2(p+1)^2-(2p+1)=2p(p+1)+1>0$$

Then topographies exist in T_{eff} which satisfy identically equation (3). If we ask for $F(\theta)$ to be zero for different flows (relative to different epochs), $dim(T_0\cap T_{eff})$ is reduced.

4.2 An example: A CMB topography close to the model of Morelli and Dziewonski and giving a small amplitude topographic torque.

Let us consider the case p=2 and use simultaneously three models of the core surface motions (for the epochs 1970, 1980, 1985) to constrain the CMB topography. We have 18 unknown parameters and 15 equations and the non-uniqueness remains. As we need additional constraints to build a particular topography, we chose to look for a topography as close as possible to the model

h_0 of Morelli and Dziewonski (we can keep unchanged the zonal and the even degree parts of the Morelli and Dziewonski model as they generate no torques). Furthermore we replace the linear identity (F=0) by the identity:

$$F = -0.02\sin^2(\theta)(°/yr)/10yrs$$

It roughly accounts for the changes, inferred from s.v. inversion, of the toroidal zonal motions which have been showed to balance the changes in mantle angular momentum from 1970 to 1980 (Jault et al.,1988). The results do not depend heavily upon the exact value of F and the choice (F=0) leads to similar results. The model h_1 close to the original topography is obtained using the Newton method. Table (1) summarizes the new coefficients which can be compared with the original ones. The fit to the acceleration data has been made excellent while the amplitude of the topography is only slightly reduced after the inversion. The map (figure 5) of the so-calculated efficient topography looks similar to the map of the efficient part of the Morelli and Dziewonski topography (figure 6). The similarity is of course far more striking when looking at the maps (figures 7 and 8) of the total topographies. This model h_1 is our preferred example among the various models we have calculated for different p (p=1,2,3) and different models of the core surface motions. Table (2) gives an example h_2 of a large amplitude topography with the same truncation level as for the model h_0 of Morelli and Dziewonski (p=1); but only the model of the core surface motion for the epoch 1980 has been used. The correlation ratio between the coefficients of either h_1 or h_2 with h_0 are larger than the correlation ratio between the coefficients of h_1 and h_2. This is not surprising since we cannot hope to get any indication of convergence with so low truncation levels.

4.3 A criterium to decide the closeness of a given topography to the space T_0 of the topographies generating no torque on the mantle

We carried the same kind of inversion for other models of CMB topography which have approximately the same amplitude; the results are not so good and the topography of Morelli and Dziewonski seems close to a satisfying topography, from the topographic torque point of view. We have tried to quantify this property. We can define, for each topography h, its distance d(h) from the linear space T_0 of the topographies generating no torques on the mantle. The space T_0 is defined by the linear forms (a_l) on the set of coefficients $h_n^{mc,s}$ and depends on the truncation level (p=2) and on the models of the core surface motions (for the epochs 1970,1980,1985) we have used. Each linear form is the equation of an hyperplane of the space of the topographies, and T_0 is the intersection of these hyperplanes. We transform the set of linear forms (a_l) into a set of orthogonal linear forms (b_l). Again T_0 is the intersection of the hyperplanes defined by each linear form b_l. Then the distance d(h) between a given topography h and the space T_0 can be calculated from the distances (d(h,b_l)) between the topography and the orthogonal hyperplanes defined by the linear forms (b_l):

$$d(h)^2 = \sum_l d(h, b_l)^2$$

We have calculated the distance d in the case of the model h_0 of Morelli and Dziewonski (d=1.18km) and in the case of the topography h_1 calculated above (d=0.17km). Then we have compared these distances with the distances to T_0 of a set of 300 random topographies, the coefficients of which follow a centered gaussian law with variance ($\sigma = 0.66$) (the variance of the coefficients of the effective part of the Morelli and Dziewonski model is 0.66). The distribution of the so-calculated distances is represented by the histogram of fig.9: the model of Morelli and Dziewonski is, possibly by chance, among the 60 out of 300 closest topographies to the space T_0. No topography among the 300 random topographies is as close to the linear space T_0 as our calculated topography h_1. It illustrates again how it is unlikely that a given topography generates by chance a weak torque on the mantle.

5 A stabilizing mechanism responsible for the small amplitude of the topographic torque.

Let us forget for the moment the differential rotation of the core annuli induced by the pressure torque and consider only the average torque on the core (and the opposite on the mantle). It makes the whole core rotate with respect to the mantle and, as a consequence, the torque itself changes. The amplitude $\Gamma(\phi)$ of the average topographic torque can be represented as a function of the phase shift ϕ between the flow pattern and the CMB topography. Taking as an example the motion calculated by Gire and Le Mouël for the epoch 1980 and the CMB model of Morelli and Dziewonski $\Gamma(\phi)$ has been computed for $0 \le \phi < 2\pi$. The results are illustrated by fig.10. Some points ($\phi = \phi_c$) of the curve of fig.10 -such as the point marked 1- are stable points: the flow pattern is drawn back by the topographic torque after it has drifted by $\delta\phi$ from ϕ_c. ($\phi = 0$) is such a point for the topography h_1 computed in the former paragraph (fig.11). Now the curve $\Gamma(\phi)$ of fig.10 looks like a sine curve sin(mϕ) (($\phi = 0$) is supposed to be a stable point):

$$\Gamma(\phi) = -\Gamma_0 \sin(m\phi) \qquad (5)$$

Then:

$$I_c \frac{d^2\phi}{dt^2} = -\Gamma_0 \sin(m\phi) \qquad (6)$$

I_c being the axial moment of inertia of the core; for $m\phi \ll \pi$, equation (6) can be linearized

$$I_c \frac{d^2\phi}{dt^2} = -\Gamma_0 m\phi \qquad (7)$$

it governs an oscillating system with angular frequency λ and period T.

$$\lambda^2 = -\frac{m\Gamma_0}{I_c}$$

With the values m=3 and Γ_0=2 10^{19}N.m corresponding to fig.10, this becomes:

$$\lambda = 3 \quad 10^{-9} s^{-1}, \qquad T = 60 \quad years$$

Hence these oscillations have indeed periods comparable with those of the l.o.d. fluctuations. This 60 years figure is only an order of magnitude and should not be compared with a 60 years oscillation of the geomagnetic field as for example the one proposed by Braginsky(1984). The amplitude of the maximal torque observed during the past 100 years, 10^{18} N.m corresponds to a shift ϕ of only 2.9° from the equilibrium position (thus linearizing is justified) and the corresponding maximal angular rotation $\omega = d\phi/dt$ amounts to 0.03°/year; these values are reasonable. A dissipative mechanism such as the electromagnetic torque can attenuate the oscillations around stable points and make the description complete. This locking mechanism of the flow pattern with respect to the mantle is capable of playing an important part in the history of the core surface motions. Other mechanisms, such as lateral variations of temperature at the bottom of the mantle (Bloxham and Gubbins, 1987), are capable of making physical quantities at the core-mantle boundary drive the surface flow. As a matter of fact, the above mechanism demands that the flow pattern is not directly determined, on the decade timescale, by the topography or by lateral variations of temperature; thermal coupling is a slow process and Anufriyev and Braginsky (1977) have showed than the slope of the spherical boundary of the core allows the flow to accomodate to large scale bumps.

Actually in our theory each annulus is accelerated independently from the others by the topographic torque and we could extend to each annulus the locking mechanism described in the case of a rigid core if the flow upwellings and downwellings were carried by the cylindrical annuli. To decide this question, we have to examine which forces could make the pattern of the flow change at the core surface. The Lorentz forces certainly tend to make the core rigid on long periods of time. On the other hand, short term changes in the pattern of the flow may result from changes in the density heterogeneities ρ inside the core. As the symmetry of the flow, as detected by Gire and Le Mouël, supposes density heterogeneities (driving the non zonal flow) antisymmetrical with respect to the center of the Earth.

$$\vec{u}(r, \pi-\theta, \pi+\phi) = \vec{u}(r, \theta, \phi)$$
$$\rho(r, \pi-\theta, \pi+\phi) = -\rho(r, \theta, \phi) \tag{8}$$

the convection term $-\vec{u}.\vec{\nabla}\rho$ induces density heterogeneities $\partial\rho/\partial t$ changes with the opposite symmetry. Only the toroidal zonal motions keep symmetry (8). Thus the rotations of cylindrical annuli, which can be associated with the toroidal zonal motions at the core surface, may play a preponderant part in the density heterogeneities changes inside the Earth's core and, as a consequence, in the changes of the pattern of the flow at the core surface.

6 Concluding remarks

The topographic coupling remains a good candidate for exchanges of angular momentum between the core and the mantle. But, if the core topography amplitude is as big as proposed by seismologists, the integrated product, along a parallel of the core surface, of the topography and of the radial derivative of the radial component of the flow at the top of the core must be nearly zero. Then the topographic torque can both have the required order of magnitude and present the rapid changes -achieved in a few years- which are needed to explain the observed variations in the length of the day. Computing the torque associated with different topographies could then provide a tool to discriminate between them (Voorhies,1988). An important and rather puzzling consequence of the ideas developed above is -which was actually suggested many years ago on different lines- that the flow should tend to be stationary with respect to the mantle; the locking mechanism could be provided by the topographic torque itself.

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Table caption:

Table 1: Spherical harmonic coefficients of both the calculated topography h_1 of the CMB and the topography model of Morelli and Dziewonski (km).

Table 2: Spherical harmonic coefficients of the calculated topography h_2 (km).

Figure captions:

Fig.1: Sketch of various surfaces and parameters used in the text.

Fig.2: Angular acceleration of the cylindrical annuli (degree yr^{-1} 10 yr^{-1}) for truncation level n=4 and different epochs: dotted line, 1970 ; dashed line, 1980 ; solid line, 1985.

Fig.3: Same as for Fig.2, but for n=5.

Fig.4: Changes in the angular acceleration with random perturbations of the Morelli and Dziewonski model.

Fig.5: Part of the calculated topography h_1 capable of generating a pressure torque on the mantle.

Fig.6: Effective part of the Morelli and Dziewonski model

Fig.7: Calculated topography h_1

Fig.8: Morelli and Dziewonski topography model.

Fig.9: Histogram of the distances of a set of random topographies from the space T_0 of the topographies generating no torques on the mantle.

Fig.10: Changes in the torque acting on the core with an imaginary rotation of the pattern of the flow with respect to models of the CMB: Morelli and Dziewonski model.

Fig.11: Changes in the torque acting on the core with an imaginary rotation of the pattern of the flow with respect to models of the CMB: calculated topography h_1.

Table 1 :

n	m	Calculated topography h_1		Model of Morelli and Dziewonski	
		c_n^m	s_n^m	c_n^m	s_n^m
1	1	0.09	-0.25	0.07	-0.03
3	1	-0.03	0.09	0.30	-0.04
3	2	0.56	-0.19	1.15	-0.31
3	3	-0.25	-0.95	-0.11	-1.25
5	1	0.01	-0.01	-	-
5	2	-0.17	0.06	-	-
5	3	-0.24	-0.08	-	-
5	4	0.45	-0.17	-	-
5	5	-0.03	-0.44	-	-

Table 2 :

n	m	Calculated topography h_2	
		c_n^m	s_n^m
1	1	-0.11	-0.21
3	1	0.17	0.27
3	2	1.16	-0.31
3	3	0.40	-0.87

Figure 1

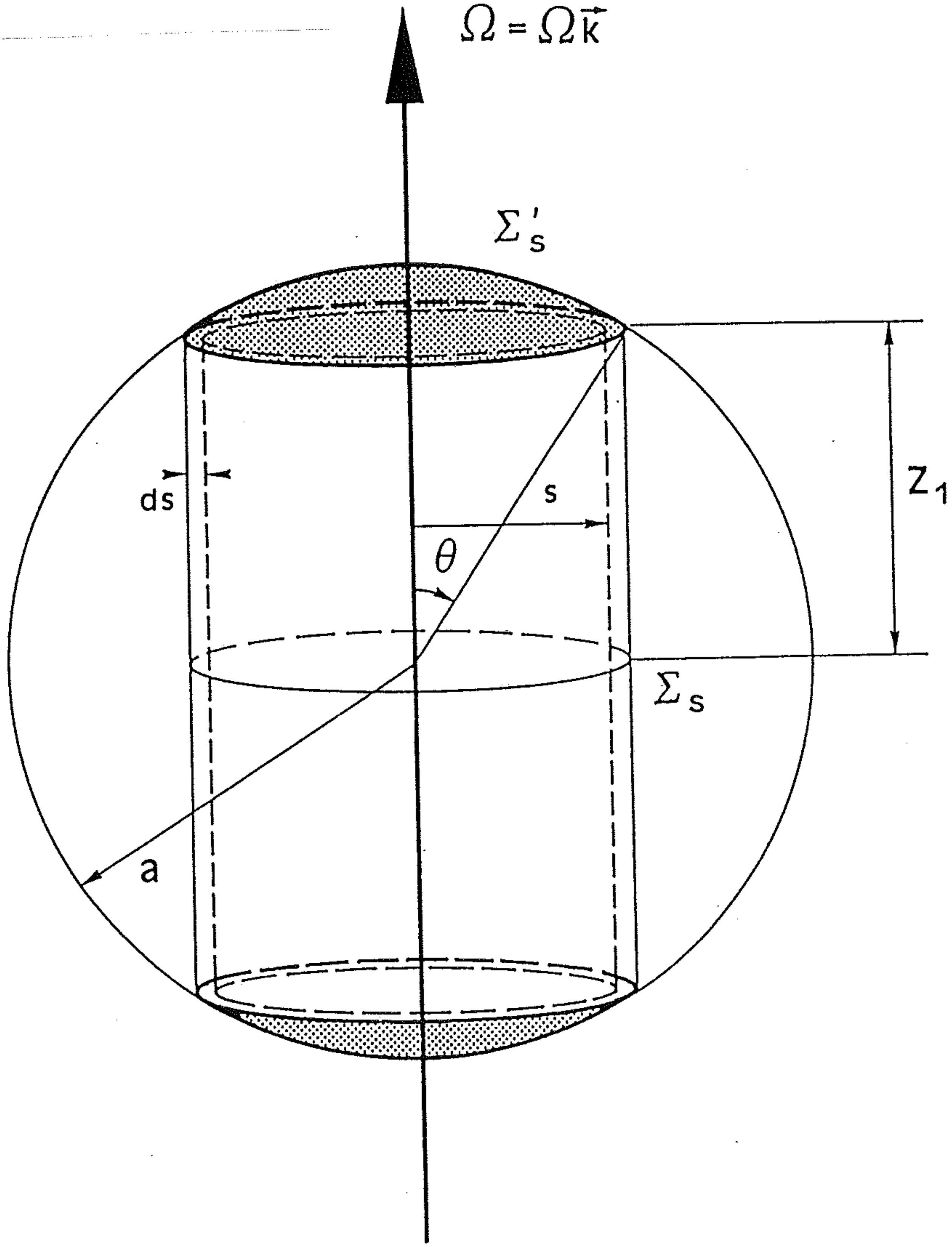
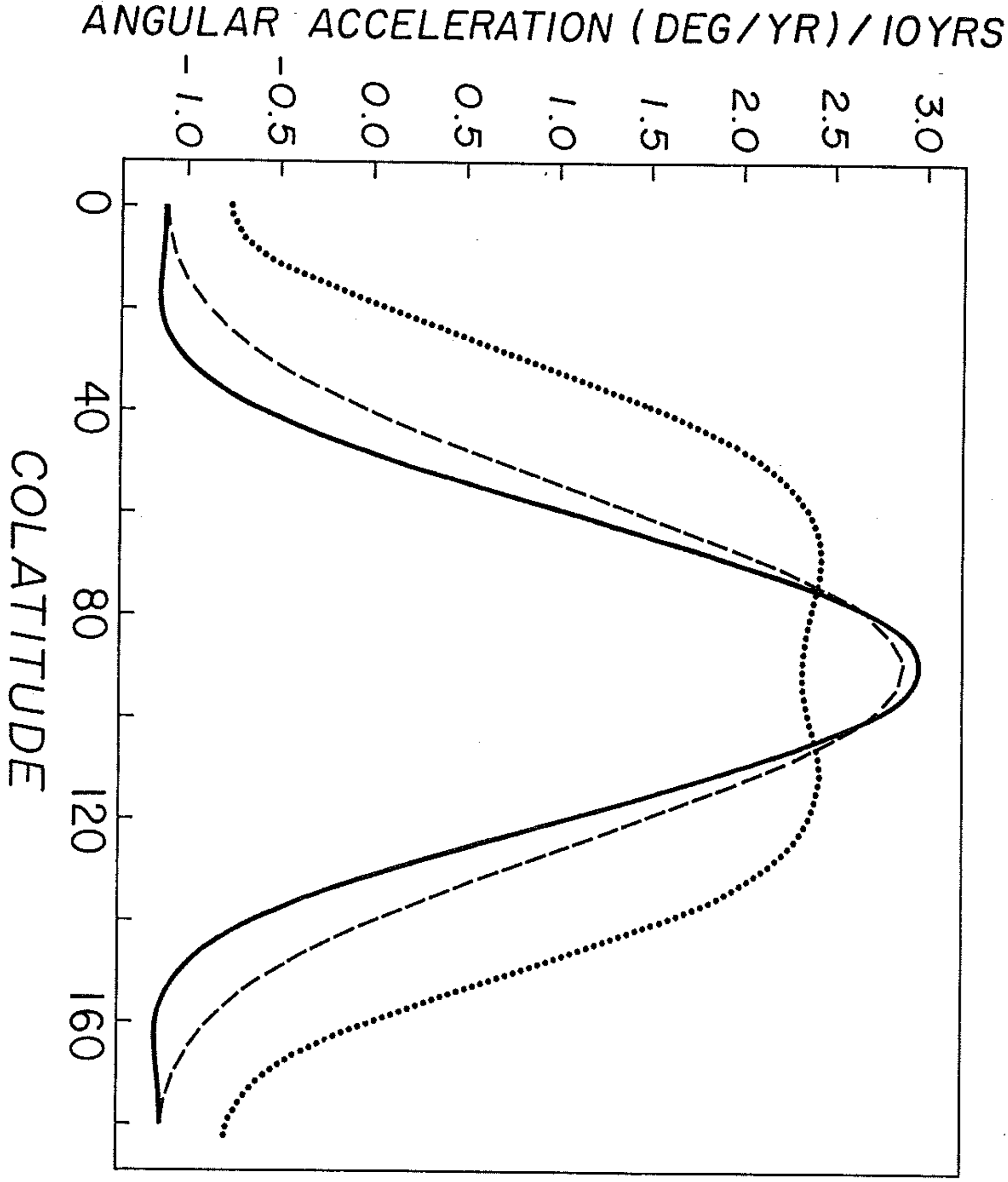


Figure 2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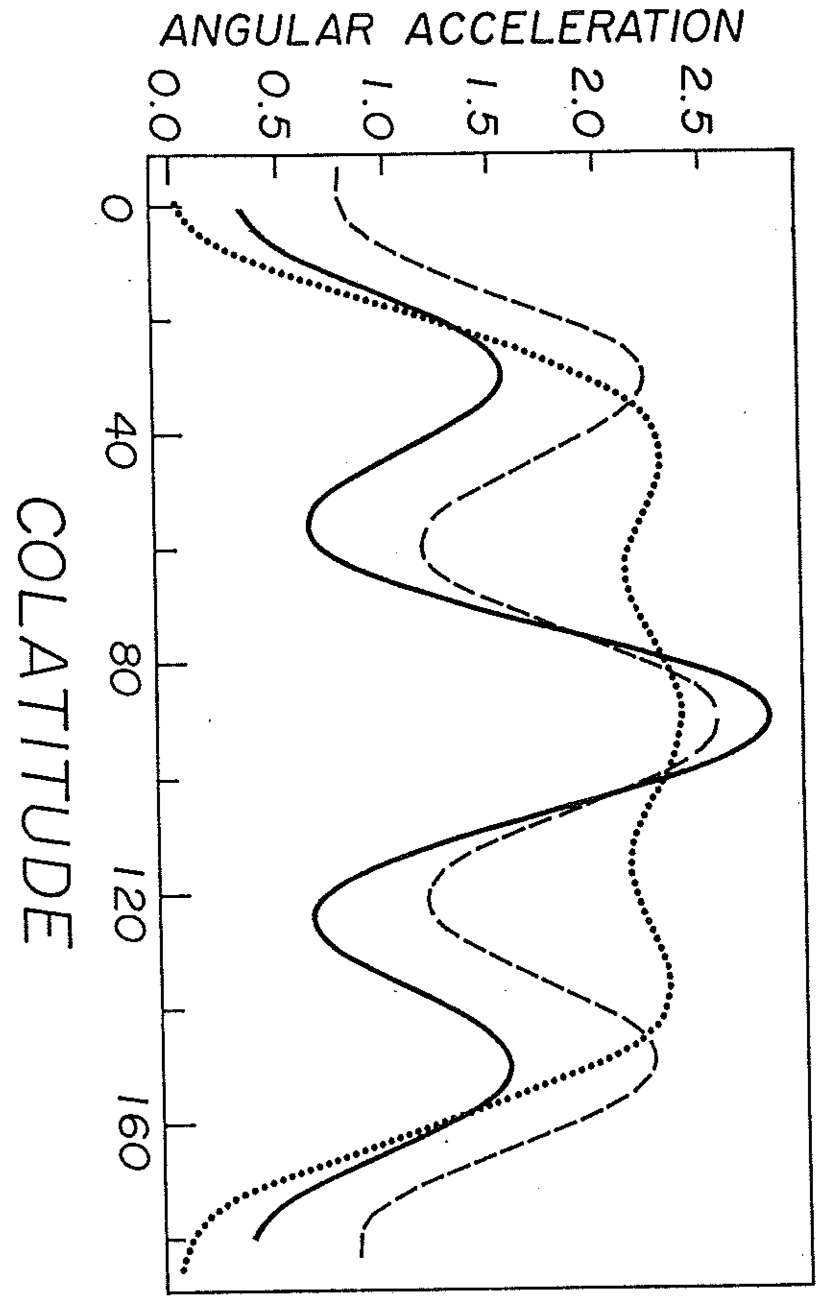
40
80
120
160
COLATITUDE

Figure 3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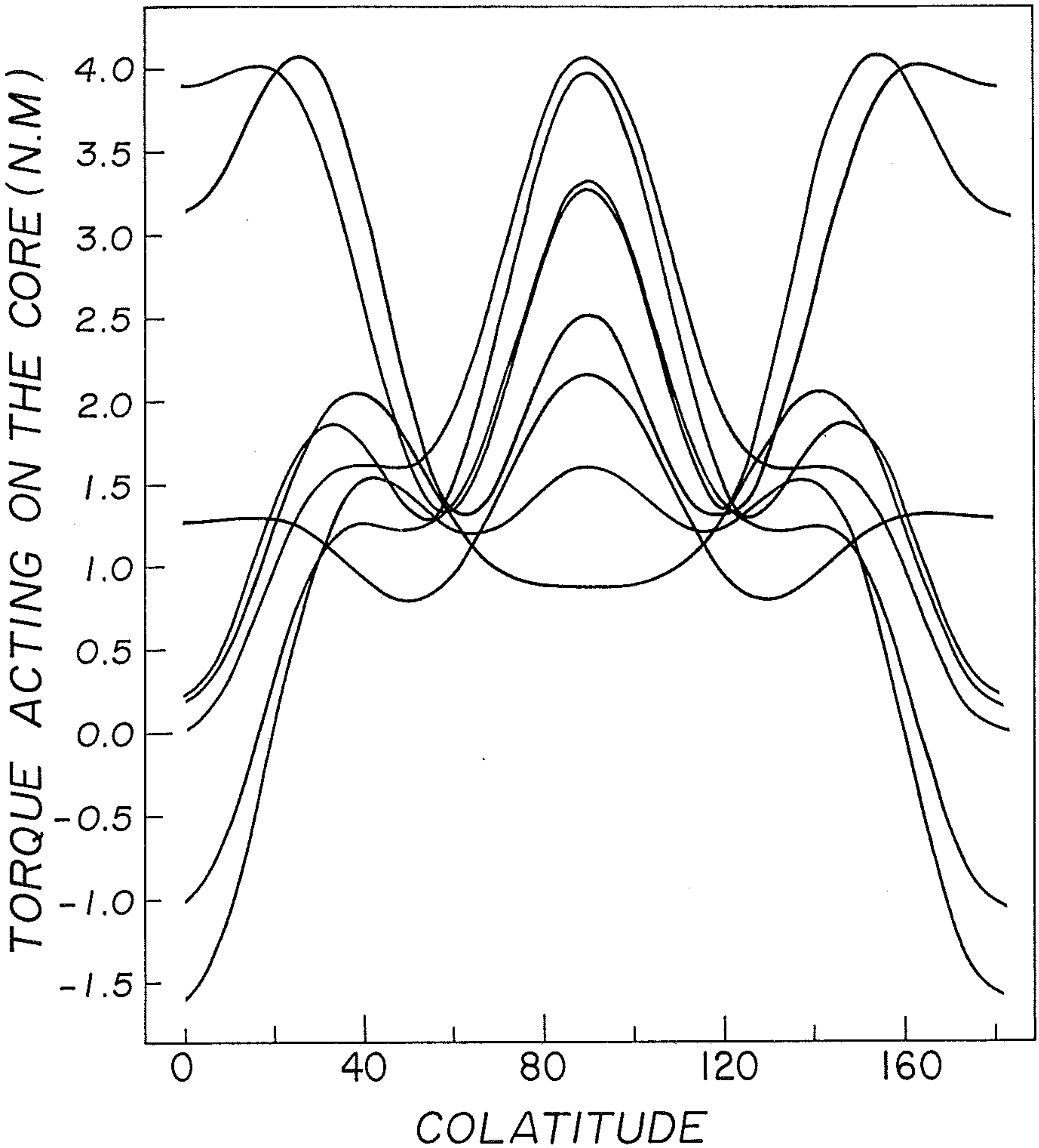
COLATITUDE
0
40
80
120
160

Figure 4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3.0
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0
40
80
120
160
COLATITUDE

Figure 5

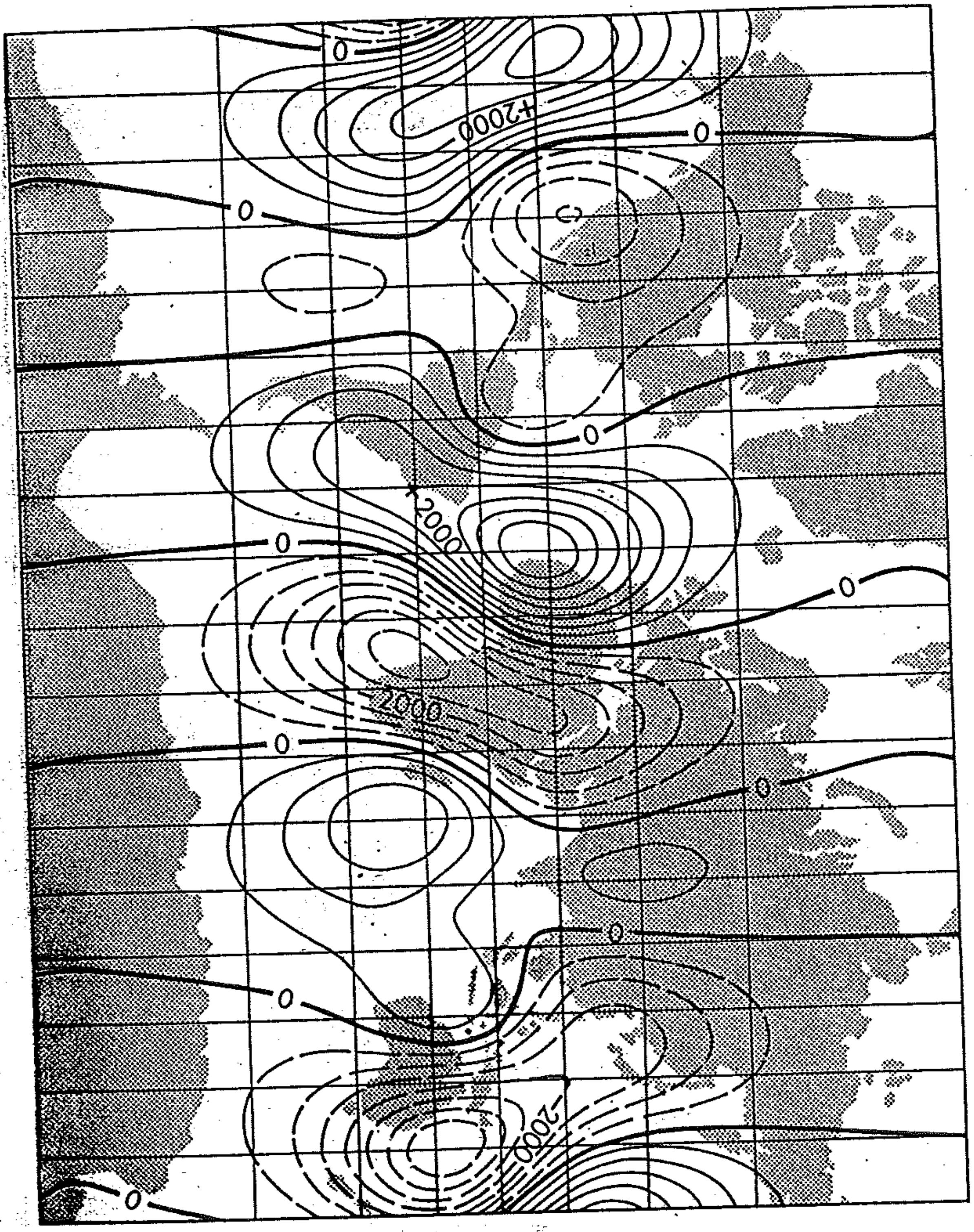


Figure 6

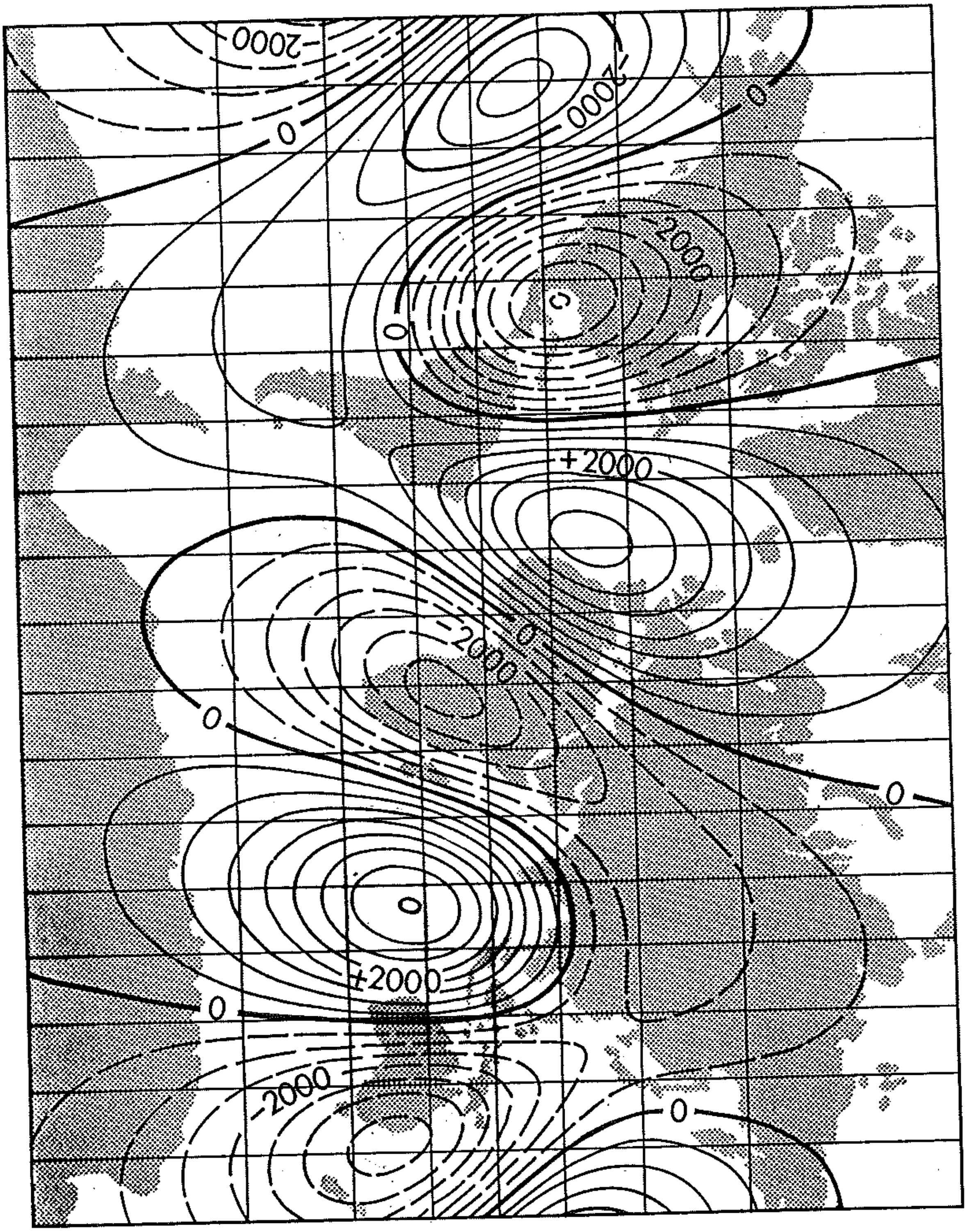
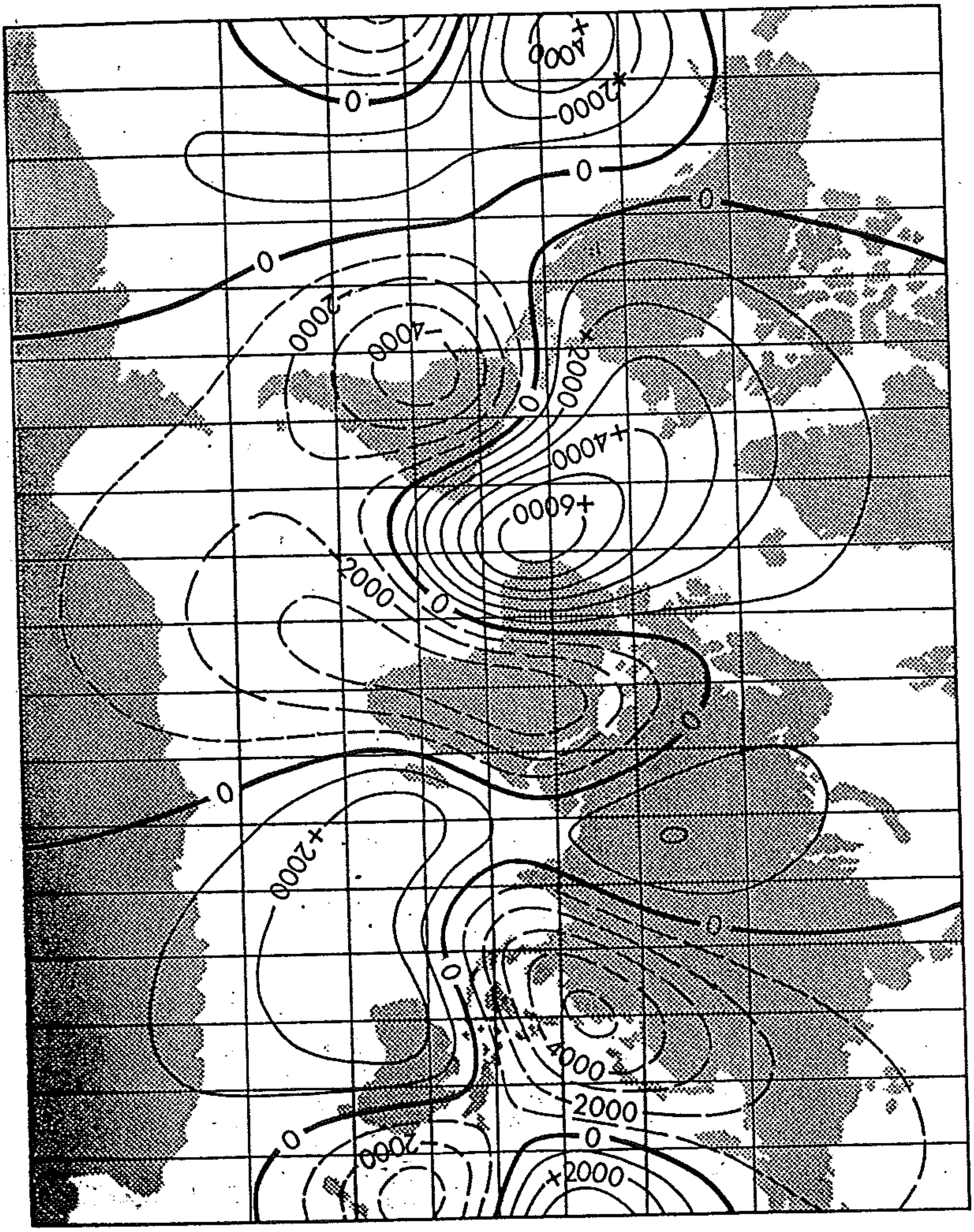


Figure 7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+2000

Figure 8

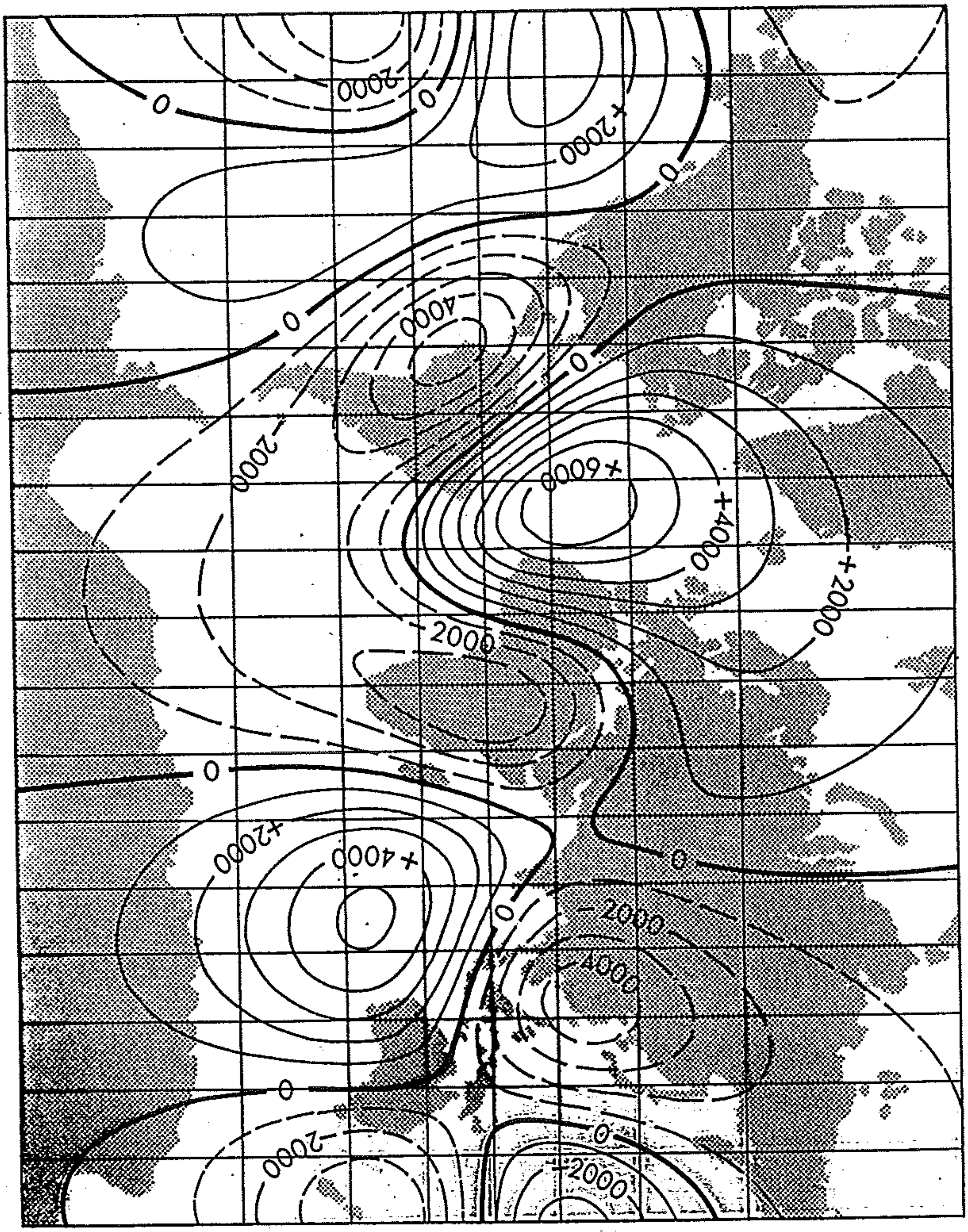


Figure 9

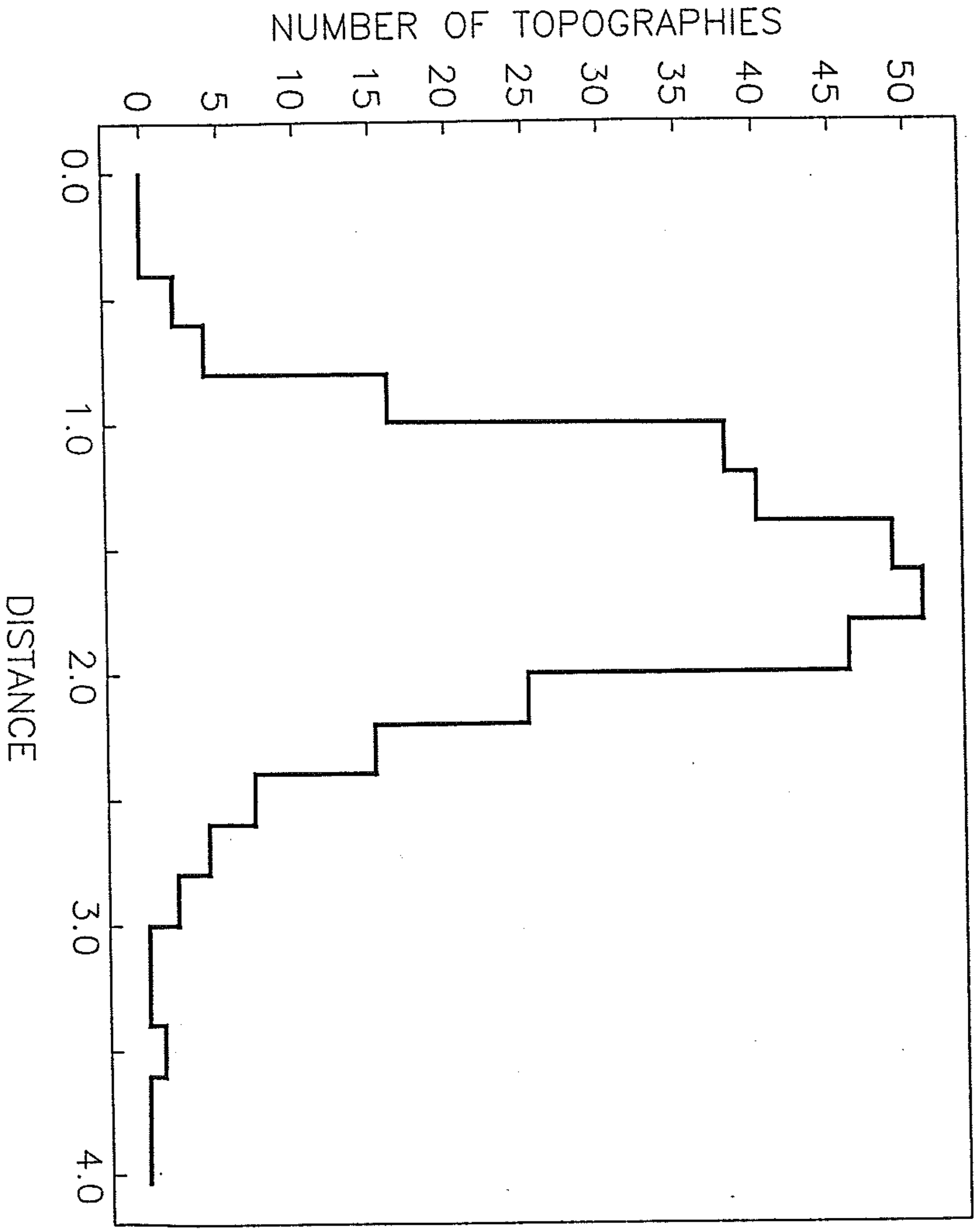

NUMBER OF TOPOGRAPHIES
0
5
10
15
20
25
30
35
40
45
50
0.0
1.0
2.0
3.0
4.0
DISTANCE

Figure 10

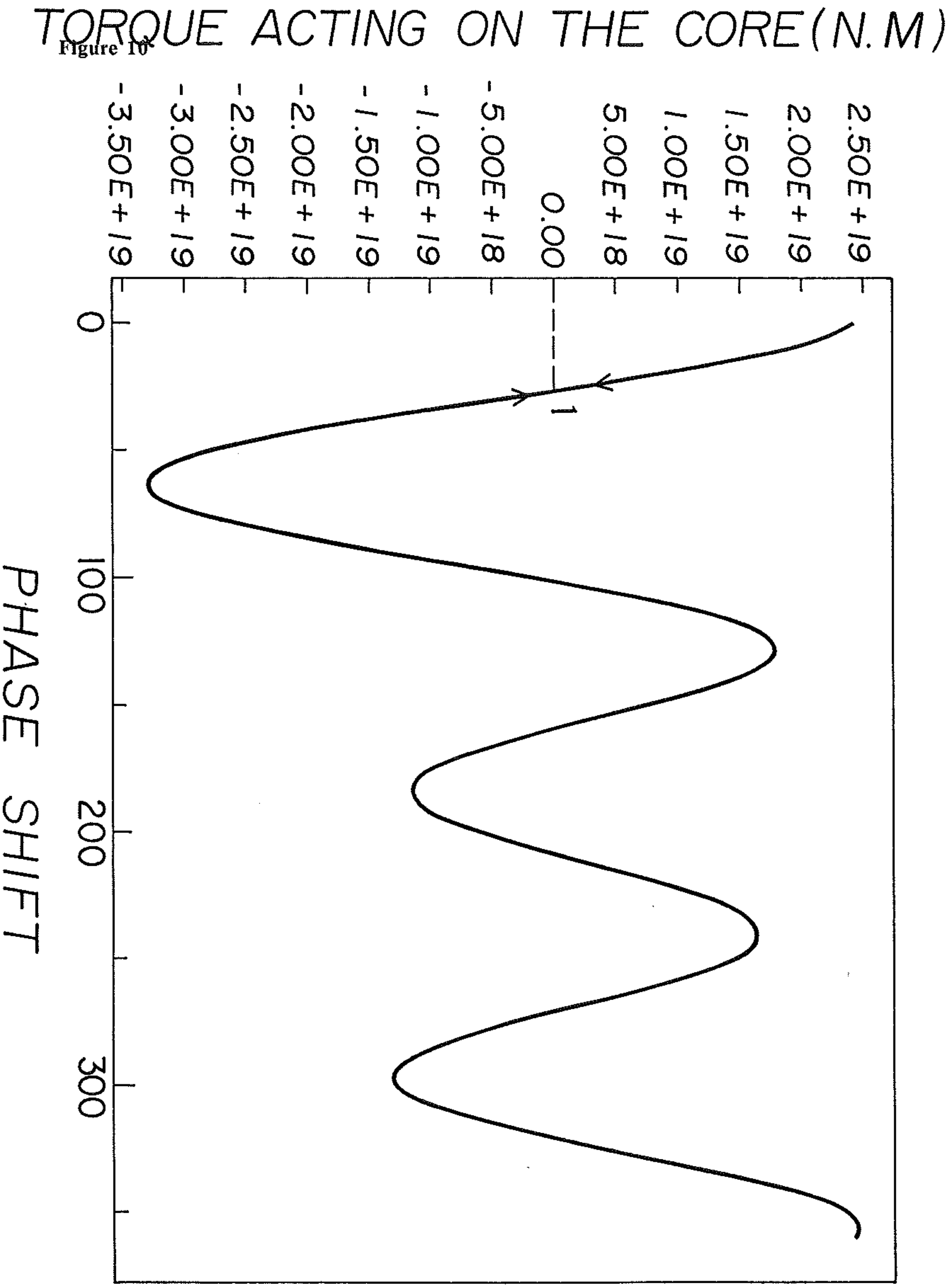
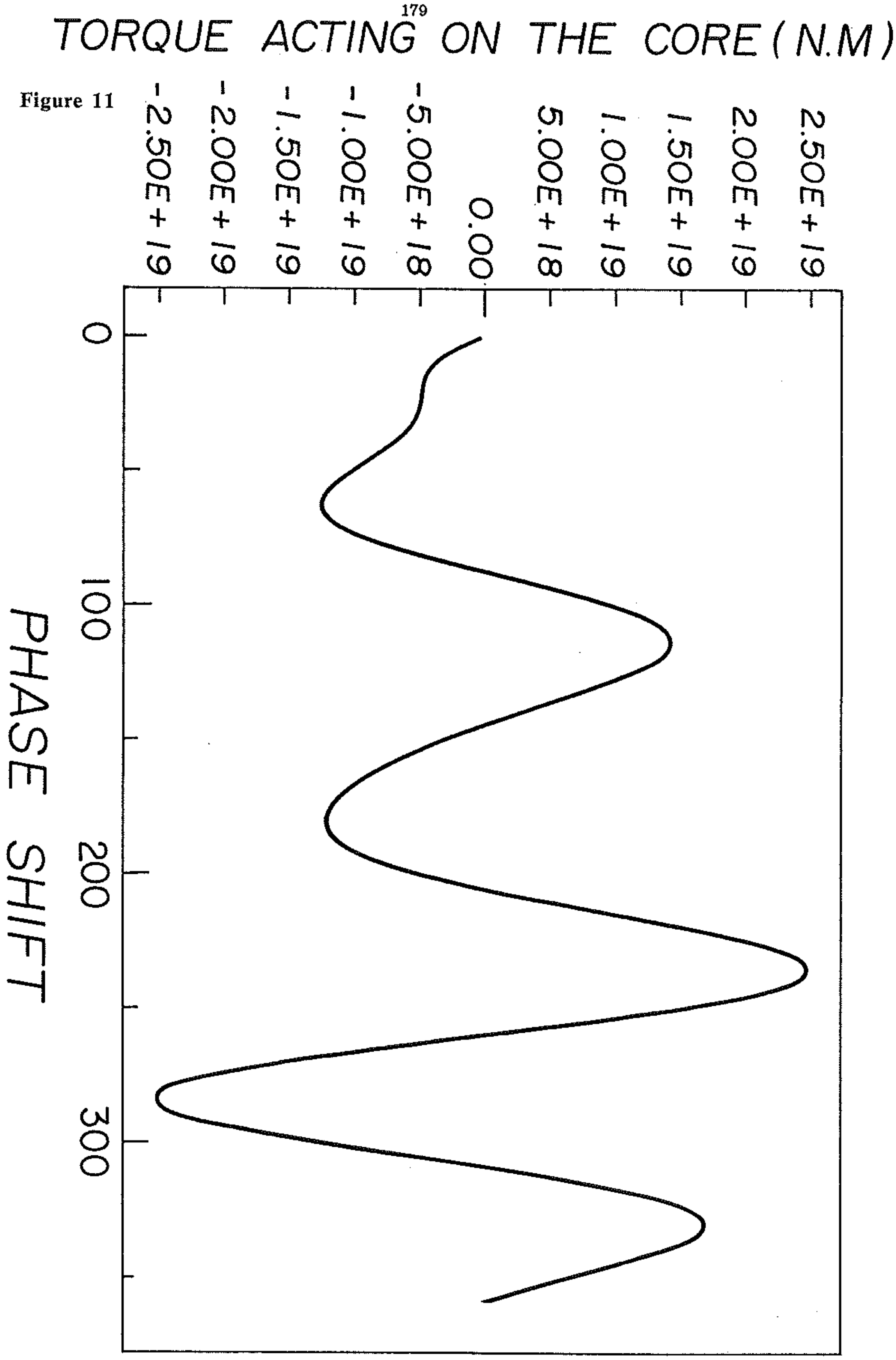

TORQUE ACTING ON THE CORE(N.M)
2.50E+19
2.00E+19
1.50E+19
1.00E+19
5.00E+18
0.00
-5.00E+18
-1.00E+19
-1.50E+19
-2.00E+19
-2.50E+19
-3.00E+19
-3.50E+19
1
0
100
200
300
PHASE SHIFT

Figure 11


TORQUE ACTING ON THE CORE (N.M)
2.50E+19
2.00E+19
1.50E+19
1.00E+19
5.00E+18
0.00
-5.00E+18
-1.00E+19
-1.50E+19
-2.00E+19
-2.50E+19
0
100
200
300
PHASE SHIFT