Drawing seismic rays in layers with velocity gradients using only ruler and compass

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Because of its widespread use in seismic ray tracing, the case of a linear increase of velocity with depth in a given layer has drawn seismologists' attention for many years. The ray integrals, which give the horizontal distance and the corresponding traveltime, can be evaluated in a closed form; the radius of curvature of the ray, the distance above the surface where the center of the circular ray lies and the shape of the wavefront are given by well-known analytical equations.

The purpose of this paper is to present an entirely geometrical alternative to this computational aspect. We will deal here with the following problem: how can a ray be drawn in a stack of horizontal layers with a linear increase of velocity within each bed, using only a ruler and a compass? Except for Snell's law and the important assumption that the raypath is circular in a given layer, no prior knowledge of the ray geometry is required.

In a time of numerical methods and of almighty computer science, this challenge may look odd. But we feel that it could prove of some interest from the pedagogical viewpoint — besides the fact that only two common drawing instruments are necessary to bring a solution. It is interesting in today's "high tech" environment that simple geometrical considerations will lead to a rediscovery of some of the equations mentioned above.

First let's consider the simple case of a single layer with a linear increase of velocity from the value V_0 in surface to the value V_1 at depth z_1 (Figure 1). Shooting from S on the surface, we seek to draw the ray ST which has its turning point at depth z_1. This ray leaves the shotpoint at the angle i_0 given by

$$\sin i_0 = V_0/V_1.$$

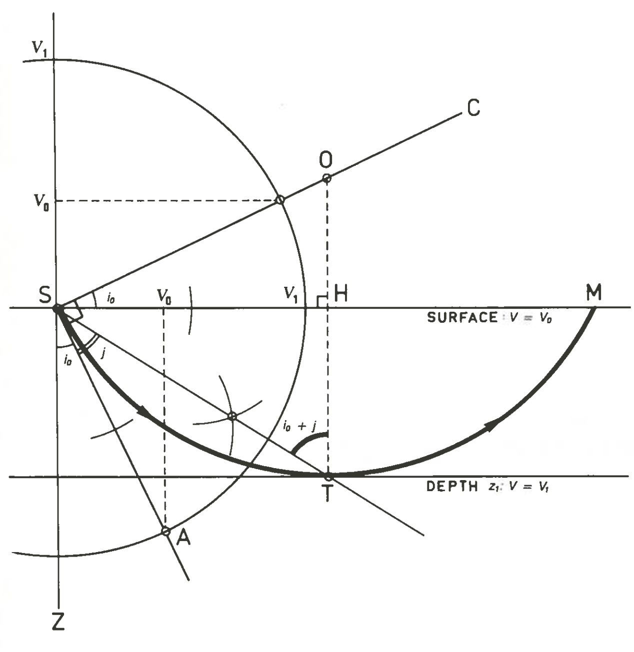

Figure 1.

This angle can be easily drawn using a circle with center S and radius V_1, provided a given scale is ascribed to the velocity. Using this scale, a vertical corresponding to the velocity in surface V_0 intersects the circle in A and

$$ZSA = i_0.$$

A similar construction can be used to draw the perpendicular SC to SA, where the center O of the circular ray is sought.

Suppose now the problem is solved and denote by T the turning point, by H the intersection of OT with the surface and by j the angle AST. A very easy way to determine the point T can be found by a careful examination of the figure. We prefer here to do it differently in order to introduce the construction used in the next section. The triangle SOT being isosceles, ST can obviously be considered as the bisectrix of the angle ASX.

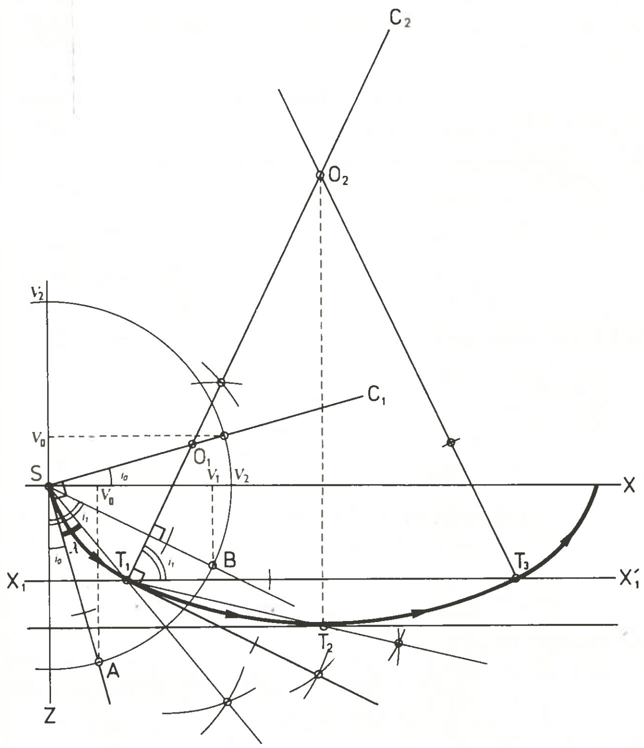

Figure 2.

This remark is the key to the geometrical construction;

- First draw SA and SC;
- Draw the bisectrix on angle ASX;
- This bisectrix reaches depth z_1 at point T,
- From T draw a vertical which intersects SC: O is the point of intersection.

If R denotes the radius of curvature OS, it follows that

$$R = z_1 (1 - \sin i_0).$$

Introducing now the velocity gradient

$$k = (V_1 - V_0)/z_1$$

and the ray parameter

$$p = \sin i_0 / V_0 = 1/V_1.$$

we find the well-known equations of the radius of curvature R and of the distance OH above the surface where lies the center of the ray:

$$R = 1/pk; \; OH = V_0/k.$$

Consequently, the horizontal distance SM can be expressed as

$$\Delta = 2R \cos i_0 = 2(1 - p^2 V_0^2)^{1/2}/pk$$

which provides an alternative to the more usual way to obtain the last equation, using the differential calculus.

We'll now consider the case of only two layers — a multi-layered medium being a mere generalization. The main difference with the previous case is the fact that the propagation of the ray in the upper layer cannot be determined as easily, unless it is ascertained that the center O of the circular ray lies again above the surface at a distance V_0/k_1, where V_0 is the velocity at the surface and k_1 the velocity gradient in the upper layer.

Instead of that, suppose again the problem is solved (Figure 2). We call O_1 the center of the circular ray ST_1 in the upper layer, i_0 the angle ZSA given by $\sin i_0 = V_0/V_2$, i_1 the angle ZSB given by $\sin i_1 = V_1/V_2$, λ the angle AST_1. Note that i_0 and i_1 can be drawn using the same construction as in the previous case.

The following angles can be evaluated:

$$XST_1 = \pi/2 - i_0 - \lambda = ST_1X_1,$$
$$O_1ST_1 = \pi/2 - \lambda = O_1T_1S,$$
(the triangle O_1ST_1 being isosceles)

and

$$O_1T_1X_1' = i_1.$$

Summing these angles and equating the result to π yields

$$\lambda = (i_1 - i_0)/2.$$

ST_1 is therefore the bisectrix of the angle ASB.

The following construction can be used to draw the ray in the upper layer: first draw SA and SB such that ZSA $= i_0$ and ZSB $= i_1$; then draw the perpendicular SC_1 to SA; next draw the bisectrix of the angle ASB; this bisectrix intersects the bottom of the layer at the point T_1; finally,. from T_1, draw a perpendicular to SB. The intersection with SC_1 gives the point O_1, center of the ray.

Drawing the ray in the lower layer is easily done using the construction proposed in the previous section. The upgoing part of the ray in the upper layer is straightforward. LE

(Author's Dedication: This geometrical construction is dedicated to Camille Girard who taught me geometry.)

François Thouvenot received his degree in geophysics from the Institut de Physique du Globe de Strasbourg in 1976 and his doctorate from Grenoble University in 1981. He has been a teaching assistant at Grenoble University since 1982 and his main interests are deep seismic sounding problems and the tectonophysics of the Alpine arc. He is also currently involved with the French deep vertical sounding program (ECORS) in the western Alps.